Computational Complexity

Class 1

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Wednesday 16h15-18h00
Room 3150

Grading
- Homework 25%
- Mid-term exam 25%
- Final (first) exam 50%

→ give a first mark m_1.

- Homework 25%
- Second exam 75%

→ give a second mark m_2.

The final mark will be the maximum of m_1 and m_2.

Encoding

Exercise 1. Give an efficient way to encode a pair of binary words as a single one.

Exercise 2. In the standard representation, every positive integer n is represented by a binary word of length $\lfloor \log_2(n) \rfloor + 1$. Show that it is optimal, that is to say, for every one-to-one function $\alpha : \mathbb{N} \to \{0,1\}^+$, there are infinitely many n satisfying $|\alpha(n)| > \lfloor \log_2(n) \rfloor$.

Exercise 3. A directed graph with n vertices can be represented by its adjacency matrix, or equivalently by a binary word of length n^2. Show that some infinite sets of graphs are encoded by languages that are not context-free. Can you find some graph properties encoded by complements of context-free languages?

Exercise 4. Propose an alternative encoding of undirected graphs using adjacency list.

Turing Machines

Unless explicitly mentioned, we consider the deterministic model of Turing machine with one tape, infinite only on the right.

Exercise 5. Give a Turing machine accepting the language of palindromes (words that are equal to their mirror) and stopping on every word.

Exercise 6. Give a Turing machine with two tapes accepting the language of palindromes and stopping on every word. Compare the computation time of this machine with the one of Exercise 5.

Exercise 7. Are Turing machines with a tape infinite both on the left and on the right more expressive than the ones with one tape, infinite only on the right.

Exercise 8. Are the following modifications of Turing machines equivalent to the original one?

- The machine's head cannot go left (either stays in place or goes right).
- The machine can only write on the input word, not on empty cells.
- The machine can only write on empty cells.
- The machine rejects immediately if it visits the same configuration twice.

Exercise 9. We consider a variant of Turing machines where the head can start in any place on the input word. A word is accepted if there exist at least one input position such that the word is accepted with the head starting in this position. Is this model equivalent to the original one?