Computational Complexity

Class 11

Alternating Turing machines

Exercise 1. Show that the problem of evaluating a boolean circuit is in the class AL.

Exercise 2. The problem QBF if the following one: given a formula $\varphi(x_1, x_2, \ldots, x_n)$, check if

$$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \ldots \varphi(x_1, x_2, \ldots, x_n)$$

Prove that this problem is in AP and in PSPACE.

Exercise 3. Let A be a finite nonempty set equipped with a binary operation $\cdot$. Given a subset B of A, the closure $\langle B \rangle$ of B is the smallest $C \subseteq A$ such that $B \subseteq C$ and C is closed under $\cdot$, i.e. for all $a, b \in C$, $a \cdot b \in C$. Prove that the problem of determining, given A, $\cdot$ and B as above and $a \in A$, whether $a \in \langle B \rangle$, is in AL. Prove that this problem is in fact P-complete.