Computational Complexity

Class 12

Probabilistic classes

Exercise 1. Show that $P \subseteq RP \subseteq NP$ and $RP \subseteq BPP$.

Exercise 2. Consider the class PP such that: (see definitions on the board)

$$x \in L \quad \text{if and only if} \quad P(R(x, y)) > \frac{1}{2}$$

Prove that $NP \subseteq PP$.

Exercise 3. Show that BPP is closed under union, intersection and complement.

Exercise 4. Show that BPP is closed under concatenation.

Exercise 5. A word w of a language L is said to be in the border of L if there exist a letter in w such that changing this letter gives a word that does not belong to L. Prove that if L is in BPP then the border of L too.

Exercise 6. Show that if the problem SAT is in BPP then it is also in RP, and conclude that if $NP \subseteq BPP$ then $NP \subseteq RP$ too.