Computational Complexity

Class 3

Turing machine: configurations

Exercise 1. We call a configuration c of a Turing machine *reversible* if there is at most one configuration d, such that c is reached from d in one step. A Turing machine is said to be *weakly reversible* if any configuration reachable from some initial configuration is reversible. Note that if it is the case then we can trace the computation back. For a given Turing machine, construct a weakly reversible machine recognizing the same language. Estimate the time overhead in your construction.

Exercise 2. Given an off-line Turing machine, using $S(n)$ space on input of size n, prove that if $S(n) \geq \log(n)$ then there exists an off-line Turing machine, halting on every input, accepting the same set and working in space $\mathcal{O}(S(n))$.

Turing machine: complexity

Exercise 3. A function $f : \mathbb{N} - \{0\} \to \mathbb{N}$ is *space constructible* if there is an off-line Turing machine which, for an input of length $n \geq 1$, writes in exactly $f(n)$ cells of the auxiliary tapes. Show that the following functions are space constructible: n, $2n$, n^2 , $n^2 + n$, 2^n, $\lceil \log_2(n) \rceil$.

A function $f : \mathbb{N} - \{0\} \to \mathbb{N}$ is *time constructible* if there is a Turing machine which, for an input of length $n \geq 1$, makes exactly $f(n)$ steps and halts. Show that the following functions are time constructible: n, $2n$, n^2 , 2^n , 2^{2^n} .

Exercise 4. Assuming that natural numbers k, m, n are given in binary representations, estimate the time to compute $m + n$, $m \mod n$, mn, $mn \mod k$.

Exercise 5. Estimate the computation time of the Euclide algorithm implemented on Turing machine.

Exercise 6. Give a one-tape Turing machine accepting the set $\{0^n 1^n \mid n \in \mathbb{N}\}$ in time $\mathcal{O}(n \log(n))$.

Exercise 7. Remind the complexity classes given in the lectures, and their hierarchy.