Computational Complexity

Class 4

Complexity classes: L

Exercise 1. Prove that the Dyck language consisting of balanced words of brackets is in L.

Exercise 2. Consider a variant of an off-line Turing machine with no working tape but with several reading heads on the input word. Prove that these machines accept exactly the languages in L.

Complexity classes: P and Sat problem

Exercise 3. A propositional formula is said to be in disjonctive normal form (DNF) if it is of the form:

$$(x_1 \wedge x_2 \wedge \cdots \wedge x_k) \vee (y_1 \wedge y_2 \wedge \cdots \wedge y_\ell) \vee \cdots \vee (z_1 \wedge z_2 \wedge \cdots \wedge z_m)$$

and in conjonctive normal form (CNF) if it is of the form:

$$(x_1 \vee x_2 \vee \cdots \vee x_k) \wedge (y_1 \vee y_2 \vee \cdots \vee y_\ell) \wedge \cdots \wedge (z_1 \vee z_2 \vee \cdots \vee z_m)$$

The satisfiability problem consists in determining if, given a formula, there is a valutation of the variables which makes the formula true.

(1) Prove that the satisfiability of a DNF formula is in P.

(2) Generally speaking, it is not known if the satisfiability of a CNF formula is in P. Prove that if every variable appears at most twice, then it is the case.

(3) Prove that the satisfiability of CNF formulas for which every clause has at most one positive literal is in P.

(4) Prove that the satisfiability of CNF formulas for which every clause has at most two literals is in P.

(5) For every propositional formula φ, prove that there exist a CNF formula with a size linear in the one of φ and which is satisfiable if and only if φ is satisfiable.

Boolean circuits

Exercise 4. The parity function $P_n : \{0,1\}^n \to \{0,1\}$ is defined by:

$$P(x_0 \cdots x_{n-1}) = \left(\sum_{i=0}^{n-1} x_i \right) \mod 2$$

Construct a boolean circuit computing P_3 and do the computation for the input 011.