Computational Complexity

Class 6

Boolean circuits: parameters

Exercise 1. Show that, for sufficiently large n, there is a function $f : \{0,1\}^n \to \{0,1\}$ that cannot be computed by a Boolean circuit with fan-in 2 (see exercise 2 of the previous class) with $\frac{2^n}{2n}$ gates.

Exercise 2. Let $A_n : \{0,1\}^{2n} \to \{0,1\}^{n+1}$ be the mapping which adds two binary sequences $x_0, \ldots, x_{n-1}$ and $x_n, \ldots, x_{2n-1}$. Construct a circuit with $O(n)$ gates with $n+1$ outputs computing A_n. Can you construct one with a polynomial number of gates and constant depth?

Exercise 3. Consider a Boolean circuit with three input gates, but no negative input gates. Can you negate the three inputs, i.e. construct a Boolean circuit with three output gates giving the value of the negation of the input gates, using AND and OR gates as usual, plus two NOT gates?

Boolean circuits: complexity classes

Exercise 4. A sequence of Boolean circuits $(C_n)_n$ is said to be L-uniform if there is a Turing machine which, on inputs of length n, outputs a representation of C_n and runs in logarithmic space. Show that if a language is recognised by a L-uniform sequence of Boolean circuits with fan-in 2 and logarithmic depth (class NC^1), then it is in the class L.

Exercise 5. Prove that for all $i \in \mathbb{N}$, $NC^i \subseteq AC^i \subseteq NC^{i+1}$, and that $AC = NC \subseteq P$.

Exercise 6. Show that the accessibility problem in graphs is in the class AC^1.

Exercise 7. Show that the membership problem of a FO-definable class of finite graphs is decidable in the class AC^0.