

# Computational Complexity

## Class 7

### Boolean circuits

**Exercise 1.** A sequence of Boolean circuits  $(C_n)_n$  is said to be  $L$ -uniform if there is a Turing machine which, on inputs of length  $n$ , outputs a representation of  $C_n$  and runs in logarithmic space. Show that if a language is recognised by a  $L$ -uniform sequence of Boolean circuits with fan-in 2 and logarithmic depth (class  $NC^1$ ), then it is in the class  $L$ .

**Exercise 2.** Show that the accessibility problem in a directed graph is in the class  $AC^1$ .

### Non deterministic complexity

**Exercise 3.** Show that if  $P = NP$  then  $NEXPTIME = EXPTIME$ .

**Exercise 4.** Show that the accessibility problem in a directed graph is  $NL$ -complete.

**Exercise 5.** Show that all the languages in  $L$  except the empty and the full languages are  $L$ -complete.

**Exercise 6.** Consider the language of the formulas in conjunctive normal forms with two literal in each clause, which are not satisfiable. Show that this language is  $NL$ -complete.

**Exercise 7.** Show that the problem SAT is  $NP$ -complete (Cook theorem).