Computational Complexity

Class 8

Reduction and completeness

Exercise 1. Show that all the languages in L except the empty and the full languages are L-complete.

Exercise 2. Show that the accessibility problem in a directed graph is NL-complete.

Exercise 3. Consider the language of the formulas in conjunctive normal forms with two literals in each clause, which are not satisfiable. Show that this language is NL-complete.

Exercise 4. Knowing that SAT is NP-complete, show that the following problems are NP-complete.

- 3-SAT:
  Input: a formula in conjunctive normal form in which all the clauses contain three literals,
  Output: "yes" if the formula is satisfiable, "no" otherwise.
- Clique: (a clique in a graph is a set of vertices that are pairwise connected)
  Input: a graph G and a positive integer k,
  Output: "yes" if G contains a clique with at least k vertices, "no" otherwise.
- Independent set: (an independent set in a graph is a set of vertices that are pairwise not connected)
  Input: a graph G and a positive integer k,
  Output: "yes" if G contains an independent set with at least k vertices, "no" otherwise.

Exercise 5. Show that the following problem is $PSPACE$-complete:
Input: a non deterministic finite automaton $\mathcal{A}$,
Output: "yes" if $\mathcal{A}$ is universal, that is to say if the language recognised by $\mathcal{A}$ is full, "no" otherwise.