Computational Complexity

Class 9

Reduction and completeness

Exercise 1. Consider the language of the formulas in conjunctive normal forms with two literals in each clause, which are not satisfiable. Show that this language is NL-complete (use exercise 2 from class 8).

Exercise 2. Knowing that SAT is NP-complete, show that the following problems are NP-complete.

- 3-SAT:
  Input: a formula in conjunctive normal form in which all the clauses contain three literals,
  Output: "yes" if the formula is satisfiable, "no" otherwise.
- Clique: (a clique in a graph is a set of vertices that are pairwise connected)
  Input: a graph G and a positive integer k,
  Output: "yes" if G contains a clique with at least k vertices, "no" otherwise.
- Independent set: (an independent set in a graph is a set of vertices that are pairwise not connected)
  Input: a graph G and a positive integer k,
  Output: "yes" if G contains an independent set with at least k vertices, "no" otherwise.

Exercise 3. Show that the following problem is $PSPACE$-complete:
Input: a non deterministic finite automaton $\mathcal{A}$,
Output: "yes" if $\mathcal{A}$ is universal, that is to say if the language recognised by $\mathcal{A}$ is full, "no" otherwise.

P/poly

Exercise 4. Prove that $P \subseteq P/poly$.

Exercise 5. Show that the class $P/poly$ is closed under Kleene star.

Exercise 6. Prove that all unary languages (i.e. languages over a one letter alphabet) are in $P/poly$.

Exercise 7. Show that $P/poly$ contains undecidable languages.

Exercise 8. A language L in Σ^* is said sparse if $|L \cap \Sigma^n| = n^{O(1)}$. Show that all the sparse languages are in $P/poly$.