Mathematics and Logic: the View from Prague 1800

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Mathematics and Logic: the View from Prague, 1800

- Prague University and Knowledge of European Mathematics and Logic
- Knowledge of Leibniz, Lambert, and Kant's Views

- Mathematical Interests of Bolzano's Teachers and Colleagues
- Let's Open Bolzano's Library

- Bolzano's effort to become a Professor of Mathematics of Charles-Ferdinand University

- Mathematics and Logic through Bolzano's Eyes
  Contributions to a Better-Grounded Presentation of Mathematics, Caspar Widtmann 1810, Prague

- Bolzano – a member of the Royal Czech Society from 1815
  Lectures in the Royal Czech Society of Sciences, Bolzano's Legacy

- The Atmosphere of Bolzano's Prague's Scientific World

Bolzano's Studies

- School of Arts, part of St. Mary Tyn Church Main School
- 1791 - 1796 Piarist's College "gymnasium"

  some Bolzano's experiments in poetry in the Latin language (about the Czech mythical Princess Libussa consoling the poet, who grieves over the sad fate of his homeland) - patriotism of Bolzano independent on the language

  introduction to critical philosophy of Immanuel Kant (1724 Königsberg – 1804)
- 1796 - 1799 Bolzano visited Faculty of Arts (Faculty of Philosophy)
- Professor of Mathematics Stanislav Vydra (1741) "cordatus Bohemus" Kästner's textbook *Anfangsgründe der Mathematik*, Göttingen 1758 – 1769
- Abraham Gotthelf Kästner , (1719 Leipzig - 1800 Göttingen), a teacher of Carl Friedrich Gauss
- 1799 - 1800 - Bolzano studied of higher mathematics - Professor Franz Joseph Gerstner (1756 - 1832)

  - making his career choice – to be a priest or not
- 1800 - 1804 - studies of theology
- 1805 - doctoral degree in philosophy

Abraham Gotthelf Kästner (1719 – 1800)

- Carl Friedrich Gauss on A.G. Kästner :

"He was a great mathematician among poets

and

a great poet among mathematicians"

Bolzano's first meeting with Kant

- 18 years - Kant and his **Kritik der reinen Vernunft**.
- Important inspiration for developing his own views
- J.A. Stoppani, Bolzano's friend and schoolmate *„Kants Nahmen lernte ich zuerst aus Ihrem Munde kennen"*. (Bolzano to Stoppani, 1835, August, 13th, see in Winter, E. Ueber die Perfektibilität des Katholicismus, Berlin 1971).
- Robert Zeithammer reconstructed the list of Bolzano's readings from records unknown today. The Critic of practical reason (**Kritik der praktischen Vernunft)** by Kant is one of them.

Bolzano's Professional Development

- 1804 – First mathematical paper
  Betrachtungen über einige Gegenstände der Elementargeometrie

- 1804 - Competitions
  1. free position of the Professor of Elementary Mathematics - no
  2. new position of the Professor of Science of Religion (Religionslehre) - yes

- 1805 - 15th April – Bolzano was appointed to the position
  Duties:
  1. To read lectures for all students of the university (3 years' course of Science of Religion) according to the prescribed textbook of Bishop Jakob Friedrich Frint, the Emperor's confessor
  2. Exhortae during Sundays and holidays (about 1000 exhortae in 15 years)
     Erbauungsreden für Akademiker, 1813
- 1807 – Confirmation of his position of the Professor of Science of Religion
- 1810 - *Beyträge zu einer begründeteren Darstellung der Mathematik, Caspar* Widtman, Prag

Bolzano became aware of internal relation between logic and matematics.

Bolzano's Teachers

- 1797 – 1799 Bolzano – philosophical course, see autobiography
- His teachers: logic, metaphysics, ethics - Karl Heinrich Seibt
  elementary mathematics - Stanislav Vydra
  physics - Franz Schmidt
  biology and technology - Vincenc Blaha.

The last two teachers were very kind - according to Bolzano's memories.

practical mathematics - Franz Anton Herget
higher mathematics - Franz Joseph Gerstner

- Bolzano became interested in mathematics early. He used the well-known and widespread textbook **Kästner's Compendium** as an introduction to mathematics (recommended by Stanislav Vydra). Bolzano studied especially the speculative part of mathematics. He was not satisfied with lectures of Stanislav Vydra, who probably read lectures without rigorous proofs.

Stanislav Vydra 1741 – 1804

- The member of the Jesuit Order from 1757, a priest
- The Czech patriot – Hero of Alois Jirasek's historical novels
- From 1772 Professor of Mathematics at Prague University
- As teacher of Elementary Mathematics he read lectures in Latin, German and Czech, he created new terminology
- 1789-1799 - the Dean of the Philosophical Faculty
- 1800 – the Rector of the University
- The author of the first Czech algebra textbook ,*"Počátkové arythmetiky (Beginnings of Arithmetics)",* which includes also parts of combinatorics, probability, applied mathematics

Karl Heinrich Seibt (1735-1806)

- the concept of “die schönen Wissenschaften”, which he introduced in his inaugural lecture “Von dem Einflusse der schönen Wissenschaften auf die Ausbildung des Verstandes …“
- Seibt further developed ideas of “die schönen Wissenschaften” presented at the more advanced Protestant universities.

Higher mathematics and Gerstner

- Gerstner had prepared the programme divided usually into three years.
- 1st year – Euler: Introduction to analysis, calculus
- 2nd year - mechanics and hydraulics according to Karsten
- 3rd year – optics and theoretical astronomy according to the French astronomer de Lalande . This year Bolzano studied together with the first year of theology.
- Bolzano managed to complete the standard two-year plan in just one year.
- Gerstner supported and encouraged advances of the talented pupil with great interest, reading Bolzano's mathematical essays. Probably at this time Gerstner invited Bolzano to his own library.

Franz Joseph Gerstner (1756 – 1836)

From České Budějovice (South Bohemia) to Linz (Upper Austria)

- The founder of Prague Polytechnique, 1806 according to l'Ecole polytechnique in Paris
- The designer of the first horse railway in Austria
- The founder of theory of waves

Gerstner's Theory of Waves

- 1802 Gerstner was the first to formulate the basics of Wave Theory.
- In his observations he wrote how water particles in a wave move in circles. The water in the crest of the wave moves in the direction of the wave and that in the trough (the lowest point of the wave) moves in the opposite direction.
- He experimented in South Bohemian ponds (tommorrow we could see some of them).
- This theory has many applications today with different media: water (ocean waves), the earth (seismic earthquake waves), electromagnetic field (radio waves), air (sound waves) etc.
- The study was not easy, because for instance ocean waves are not strictly sinusoidal or any other purely mathematical curve.

Competitions

The mathematical competition was declared in September 6th, 1804.

- October 25th, 1804 - professor Gerstner prepared exams for two candidates – Bernard Bolzano and Joseph Ladislav Jandera, the pupil of the blinded Professor Stanislav Vydra.
- Gerstner prepared 9 questions from prescribed Kästner's textbook, Land's Presidium choosed only 3 and sent them in the sealed envelope back to the Director for Studium. The written exam was started at 9 o'clock.

  All professors of the Philosophical Faculty participated. Candidates worked all day. Jandera closed his work at half past four and Bolzano finished at six o'clock.
- The following day, October 26th, they continued by an oral exam.
- 14th theorem of the Kästner geometry was the theme of the exam : *Parallelogrammes between the same parallels and above one base have the same area.*
- Both candidates had about half an hour for their explications . Bolzano went more to the depth, but his explication was not the same as Kästner's procedure. In the sequence of his proof - he used theory of similarity of triangles in different order than Kästner - earlier.

The winner of this competition

- Joseph Ladislav Jandera, from the Order of Praemonstrats, was the winner of this competition.
- The comission consisting of Professors of the Prague Philosophical Faculty wrote *"Jandera has a better voice for reading lectures". He was recommended by the old respected Professor Stanislav Vydra.*
- Jandera spent at the faculty as the Professor of the Elementary Mathematics half a century. He wrote only one textbook and no scientific papers. He was probably a good teacher and Bolzano and Jandera remained friends for the rest of their lives.
- Bolzano is part of the intellectual sequence:

Stepling, Tessanek, Vydra, Gerstner -> Bolzano

Bolzano's Private Library

- Kästner, Abraham Gotthelf: *Anfagsgründe der angewandten Mathematik*, Göttingen 1769 - 1794, 10 volumes
- Bolzano, Bernard: *Leben Franz Joseph Ritters von Gerstner ... beschreiben von Dr.B. Bolzano ...,* Prag 1837, 75 B 745
- Joseph Stepling, Johannes Tessanek – classical sources published in Prague
- Wydra, Stanislaus: *Sätze aus der Mechanik, die ...der angewandten Mathematik vorzutragen pflegt Stanislaus Wydra*, Prag 1795, 75 B 141
- Bartels Johann Martin Christian: *Disquisitiones quatuor ad theoriam functionum analyticarum pertinentes* ...Dorpati, 1822, 75 B 350
- L. Euleri: *Opuscula varii argumenti,* Berolin, 1746, 166 pages
- Euler, Leonhard. *: Lettres de M. Euler a une princesse d'Alemagne...* Nouvelle édition, Avec des Additions, par MMle Marquis de Condorcet et De La Croix, Tom 1,2,3 - 1787 - 1789, Ex libris!
- Euler, Leonhard: *Vollständige Anleitung zur Differenzial-Rechnung,* Aus Lateinischen übersetzt und mit Anmerkungenund Zusätzen begleitet von Andreas Christian Michelsen, Berlin Lagarde 1790 - 1793

2 papers of Gerstners

- Gerstner, Franz Joseph: *Einleitung in die statische Baukunst von Franz Gerstner,* Prag, Normalschul-Buchdruckerei 1789, 75 B 738
- Gerstner, Franz Anton (son of Bolzano's teacher) : *Lehrgegenstände der practischen Geometrie am k.k. polytechnische Institute, nach dem Lehrplane desselben bearbeiten und vorgetragen von Franz Anton Ritter von Gerstner*, Wien 1818, 67 pages, 75 B 99

Bolzano's Private Library 2

- Kulik, Jacob Phillip: *Untersuchungen über die Kettenbrüchenlinie.* Mit 2 Steintafel, ...75 B 705 (special print from Abhandlungen)
- Lacroix, Silvestr Francois: *Handbuch der Differential und Integral Rechnung* ...Berlin Realschulbuchhandlung, 1817, 75 B 264
- Lacroix, Silvestr Francois: *Lehrbuch der Wahrscheinlichkeitsrechnung ...,* Erfurt 1818, 75 B 387
- Lagrange, Joseph Louis, comte: *Theorie der analytischen Functionen, in welcher die Grundsätze der Differentialrechnung vorgetragen werden, unabhängig von Betrachtung der unendlich kleinen oder verschieden Grössen der Grenzen oder Fluxionen, und Zurückgeführt auf die algebraische Analysis*. Aus dem Französischen übersetzt von Johann Phillip Crüson (?), Berlin Lagarde 1798 - 1799, two volumes, 75 B 656
- Lagrange, Joseph Louis, comte: *Lecons sur le calcul des fonctions,* nouvelle édition, Paris 1806
- Moth, Fr. X.: *Die Lagrange'schen Relationen dargestellt von Fr. X. Moth*, Prag Schönfeld 1829, 75 A 8
- Lambert, Johann Heinrich: ... 2 volumes with notes, 75 B 548
- Ohm, Martin: 3 volumes
- Hallaschka, Franz Cassian: *Elementa eclipsium ...1816* (+ next 3 volumes)
- Vlacq, Adrian: *Tabellen der Sinuum Tangentium und Secantium: Wie auch der Logarithmorum Vor der Sinus Tangentes, Und die Zahlen von 1. bis 10000*. In Ordnung gebracht durch Adrian Vlacq. Amsterdam 1716, 75 C 222, 48 pages
- Vorpahl, Karl Ludewig: *Versuche für die Vervollkommung der Philosophie. Erster, zweiter und dritter Versuch, die Methaphysik, Poesie und Mathematik betreffend,* Berlin 1811, 75 C 14, 15, 140 pages
- Wagner, L.: *Grundriss der reinen allgemeinen Logik zum Gebrauch fü Vorlesungen auf Gymnasien und gelehrten Schulen herausgegeben von L.H. Wagner,* Hof 1806, 75 C 139, (notes), 75 C 106 (many notes), 212 str.

Bolzano's Private Library 3

- Weingaertner, Johann Christoph: Textbook - Combinatorial Analysis, Leipzig 1800-01 (2 volumes)
- Bolzano interested in Kant' philosophy yet in fourties , see Weisse, 75 B 29
- Wendel, Johann Andreas: *...skeptische Logik oder Darstellung der vermeintlichen Wissenschaft der Logiker von ihrer schwachen Seite, vornehmlich in Hinsicht auf Begriff, Satz and Schluss,* Coburg - Leipzig 1819, 96 pages, 75 B 22
- Whewell, William: *Geschichte der inductiven Wissenschaften, der Astronomie, Physik, Mechanik, Chemie, Geologie etc. von der frühesten bis zu unserer Zeit.* Nach dem Englischen des W. Whewell, mit Anmerkungen von J.J. Littrow ...Stuttgart 1840-1841, 75 B 540 (3 volumes)
- Willing, Friedrich Adolf Hennig: *Wissenschaft der Mathematik nach heuristisch-genetischer Methode,* Berlin 1838, 75 B 373 (1st volume – operations with real numbers)
- Wittstein, Theodor*: Neue Behandlung des mathematisch-psychologischen Problems von der Bewegung einfacher Vorstellungen, welche nach einander in die Seele eintreten. Zugleich als Beitrag zu einer schärferen Begründung der mathematischen Psychologie Herbart's,* Hannover 1845, (notes), 75 B 727
- Papers of Wolff (Christian Wilhelm Friedrich August)

  for instance *Cosmologia generalis,* Fracofurti- Lipsiae 1731, 1737
- Wurstisen, Christian: *Theoricae novae planetarum Georgii Purbachii Germani. ...Quaestiones vero in Theoricas Planetarum Purbachii, authore Christiano Urstisio Basil. aedem forma damus. ...*Basileae 1569, 75 C 431
- Wroński - Hoené, Józef Maria: *Introduction a la philosophie des mathématiques, et technie de l'algorithmie,* Paris 1811, 75 B 755 (notes), brochure
- Young, John Radford: *The Elements of the Differential Calculus; comprehending the General Theory of Curve, Surfaces and of Curve of Double Curvature,* London 1831, 75 B 199
- Fechner papers

Bolzano's Legacy

- Bolzano – a member of the Royal Czech Society of Sciences from 1815
- 1841 - Lectures in the Royal Czech Society of Sciences
- Christian Doppler, Professor of Practical Mathematics of the Prague Polytechnic
  On Coloured Light of Double Stars, 1842
- Franz Xaver Moth (1802-1879), a friend of Bolzano, he taught at the University of Vienna
- Czech patriot society – family of the historian František Palacký, and František Ladislav Rieger, his son in law, and
  family of the first Professor of Mathematics in the Czech language Professor František Josef Studnička at the Charles – Ferdinand University, etc.
- For example in 1881: Studnička organized the celebration of 100th Bolzano's birthday and he published the Czech translation of the Bolzano's paper "*Rein analytischer Beweis des Lehrsatzes, zwischen je Werthen, die ein entgegegesetztes Resultat gewähren , wenigstens eine reelle Wurzel der Gleichung liege, 1881*(*Ryze analytický důkaz ...*, in the Časopis pro pěstování mathematiky a fysiky , ČPMF – the journal of the Union of the Czech Mathematicians and Physicists . The Union was founded in 1862 as the first one in Austria)
- Otto Stolz, University of Innsbruck, *B. Bolzanos Bedeutung in der Geschichte der Infinitesimalrechnung,* Mathematische Annalen Bd. 18, 1881, etc.

Some of Bolzano's descendants in Prague, October, 5th 2007, SEDMA

(SEDMA = 7 – the abbreviation of the Prague's Seminar for History of Mathematics, Computer Science, and Astronomy)

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