CS419: Quantum Computing – Assignment 1

Problem 1

Formally state the following postulates of quantum mechanics:

1. State space (superposition).
2. Measurement (in a general basis).
3. Unitary evolution.

Problem 2

Consider the following unitary matrices, known as the Pauli matrices.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

For each one of the Pauli matrices, find out where the transformation maps the states $|0\rangle, |1\rangle, |-\rangle, |+\rangle$. (That is, compute $X|0\rangle$, $X|1\rangle$, $X|-\rangle$, $\ldots$, $Z|-\rangle$, $Z|+\rangle$.)

Problem 3

Consider the state $|\psi\rangle = \sqrt{\frac{3}{5}}|0\rangle + \sqrt{\frac{2}{5}}|1\rangle$.

1. Compute the inner product $\langle\psi|\psi\rangle$.
2. Compute the **matrix** $|\psi\rangle\langle\psi|$.
3. What will be the outcome of measuring ψ in the computational ($\{|0\rangle, |1\rangle\}$) basis?
4. What will be the outcome of measuring ψ in the $\{|+\rangle, |-\rangle\}$ basis?

Problem 4

Consider the the rotation gate

$$R_\epsilon = \begin{pmatrix} \cos(\epsilon) & -\sin(\epsilon) \\ \sin(\epsilon) & \cos(\epsilon) \end{pmatrix} .$$

1. If we measure $R_\epsilon|0\rangle$ in the $\{|0\rangle, |1\rangle\}$-basis, what is the probability we will observe $|1\rangle$?
2. If we measure $R_\epsilon|1\rangle$ in the $\{|+\rangle, |-\rangle\}$-basis, what is the probability we will observe $|+\rangle$?