CS419: Quantum Computing – Assignment 2

Problem 1

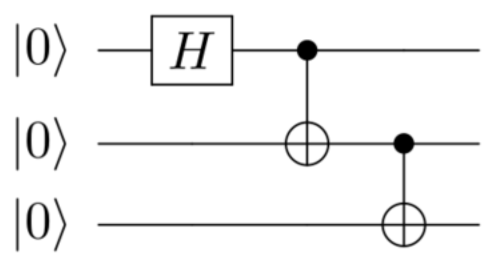
Compute the state of the following composite qubit systems, and express them in the computational basis.

1. $|+\rangle \otimes |-\rangle$.
2. $|0\rangle \otimes |-\rangle \otimes |+\rangle$.
3. $H^{\otimes 2}(|+\rangle \otimes |-\rangle)$.

In each of the systems above, what is the probability of measuring (in the computational basis) the all-zero qubit state?

Problem 2

Consider the following circuit.



1. Determine the state of the three qubits after applying each one of the gates.
2. Is the final state entangled or separable?
3. Suppose the top two qubits are measured. Determine the probabilities of the possible outcomes, and what state the third qubit collapses to in each of these cases.

Problem 3

Recall that in the quantum teleportation protocol, after Alice measures her qubits we have that:

- If Alice measured $|00\rangle$, then Bob has the state $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$.
- If Alice measured $|01\rangle$, then Bob has the state $\beta\,|0\rangle + \alpha\,|1\rangle$.
- If Alice measured $|10\rangle$, then Bob has the state $\alpha\,|0\rangle - \beta\,|1\rangle$.
- If Alice measured $|11\rangle$, then Bob has the state $-\beta\,|0\rangle + \alpha\,|1\rangle$.

Answer the following questions:

1. After Alice sends Bob the outcome of her measurement, in each one of the cases, how can Bob perturb his state to obtain $|\psi\rangle$? (Hint, use the X and Z gates.)

2. Draw (or describe **clearly** in words) a quantum circuit that takes as input the outcome of Alice's measurements and corrects Bob's state to $|\psi\rangle$ using the CNOT and Controlled-Z gate. (See: `https://en.wikipedia.org/wiki/Quantum_logic_gate` for reference).

Problem 4

Compute the density matrix of the following mixed states:

1. $\begin{cases} |0\rangle \text{ with probability } 2/3 \\ |-\rangle \text{ with probability } 1/3 \end{cases}$.

2. $\begin{cases} |00\rangle \text{ with probability } 2/4 \\ |01\rangle \text{ with probability } 0 \\ |10\rangle \text{ with probability } 1/4 \\ |11\rangle \text{ with probability } 1/4 \end{cases}$.

3. $\begin{cases} i\,|0\rangle \text{ with probability } 1 \end{cases}$.

Finally, consider the density matrix

$$\begin{pmatrix} 5/9 & 1/9 & -i/9 \\ 1/9 & 2/9 & -(2i)/9 \\ i/9 & (2i)/9 & 2/9 \end{pmatrix}$$

What is the probability of measuring $|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$?