Quantum Computers: What are they and why am I talking about tensors?

with Natalie McHugh

Motivation

Clarke's three laws

3) Any sufficiently advanced technology is indistinguishable from magic.

~ Arthur C. Clarke

… I want friends to talk about my work with

I have 1 minute, what is a Quantum computer?!

In Practice

A big, extremely cold fridge with lasers and mirrors (some of the time).

Why do we care?

Mainly, the *Quantum Fourier Transform* (QFT) is faster than the classical Fourier transform.

Other stuff like atom simulation but more open of a question

I've gained a second minute, tell me more

In Theory :)

- Everything gets Quantum or Qu- slapped on the front (e.g. quantum bit is a Qubit)
- We store probability distributions* on qubits instead of just a standard, singular bitstring
- We can't look at the state of the computer (and therefore make copies "easily")

This talk...

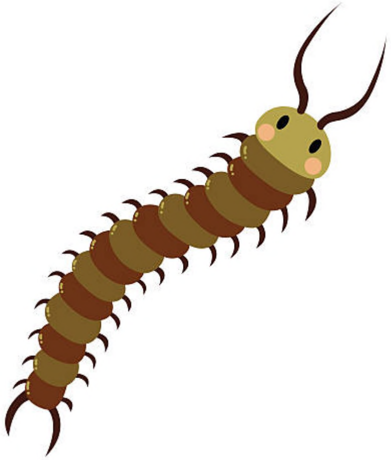
1. Tensor network basics
2. Quantum basics
3. But what if we had both at the same time!

What is a tensor?

- The n-dimensional generalisation of a matrix.

- The *rank* of a tensor is how many indices/“inputs”/ “legs” it’s over.
  - i.e. Rank 0 is a scalar, rank 1 is a vector, rank 2 is a matrix.
  - NB: different to the definition of ‘rank’ for a matrix.

- We can *evaluate* a tensor by assigning values to all legs and taking the output .



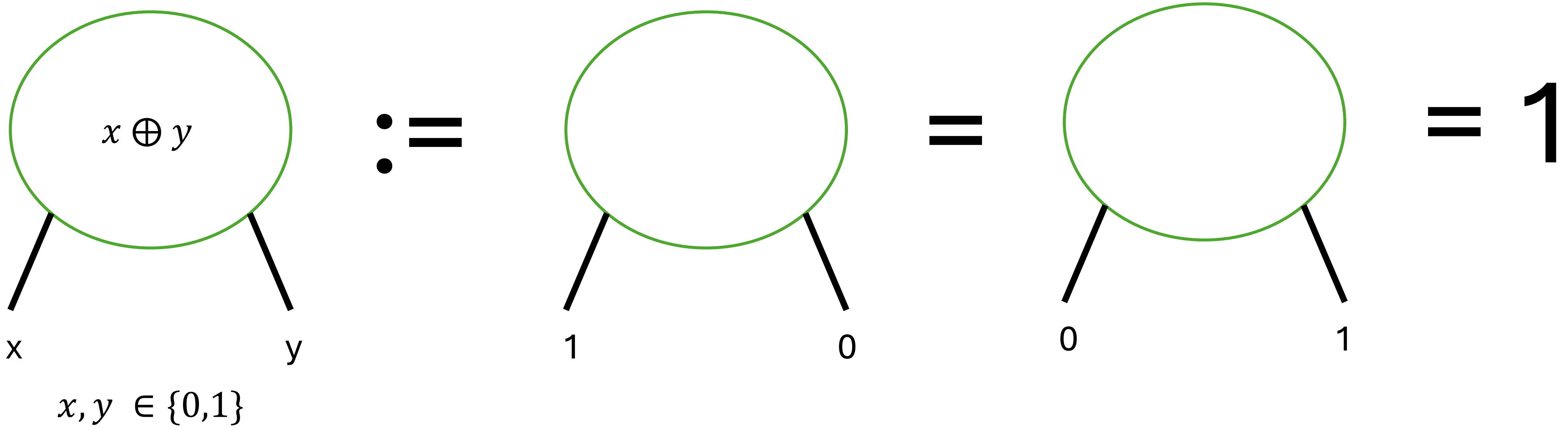
How do I represent them?

- Need to represent D^r values (D = # of choice for each input e.g. 2 for binary, r = rank of the tensor)

- Option 1: Group inputs into two groups so we can write it as a large matrix (flattening)
  - Normally done when representing on a computer or ML/neural nets

- Option 2: As a node with legs

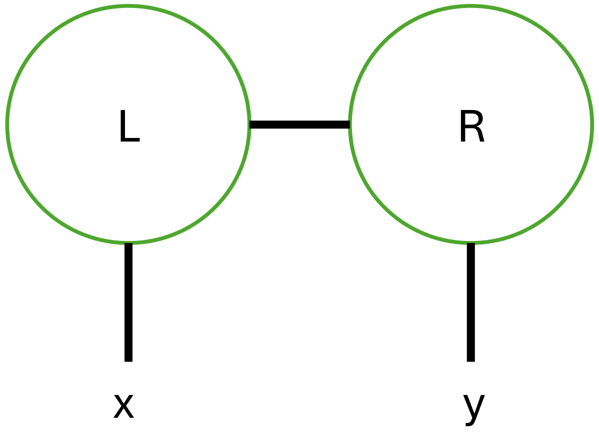
Example: XOR



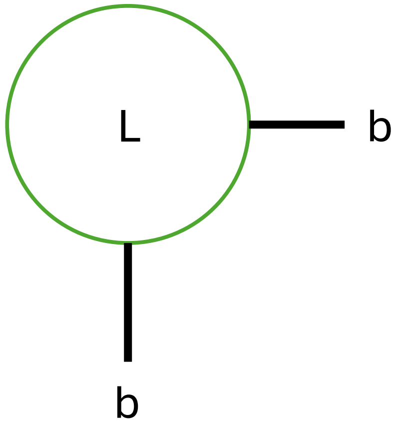
And 0 otherwise

Tensor Networks

- Connect one tensor to at least one other
- Congrats, we have a tensor network



Where



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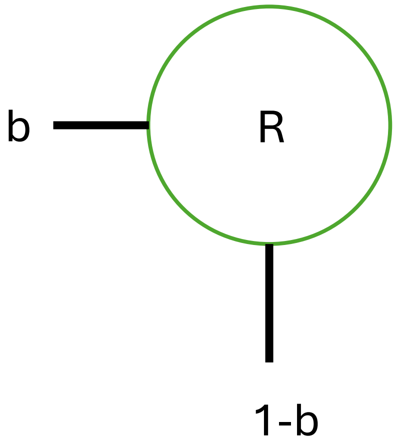

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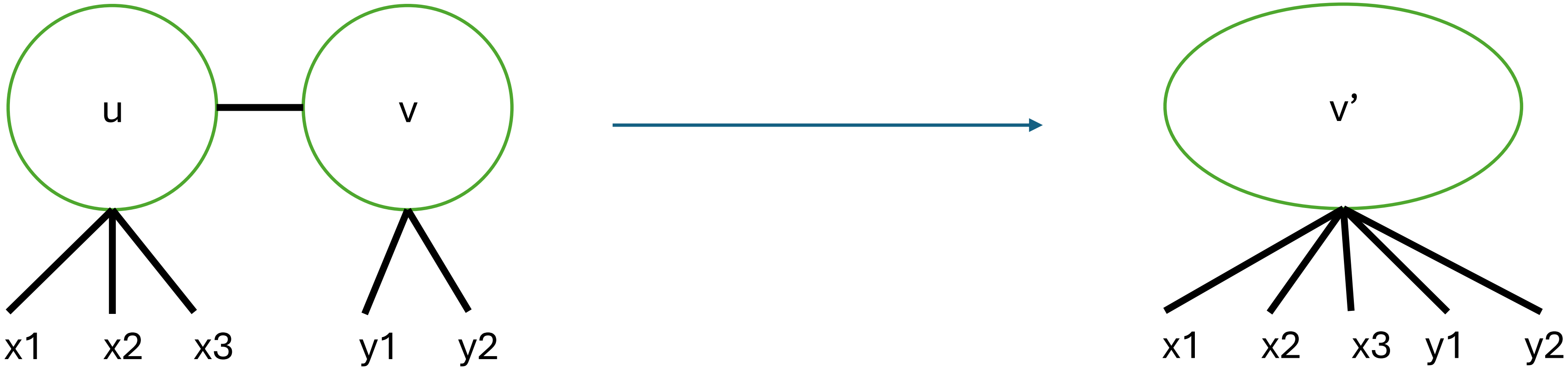
$b \in \{0,1\}$

Contraction

- Select an edge (u,v)
- Merge its two endpoint together to make v'
- $v'(\boldsymbol{x}, \boldsymbol{y}) \coloneqq \sum_i u(\boldsymbol{x}, i) v(\boldsymbol{y}, i)$
  - Need to calculate for all $(\boldsymbol{x}, \boldsymbol{y})$



Cost of a contraction: deg(v) + deg(u) - 1 = deg(v')

Rest stop #1

Don’t be scared, the quantum bit is next

Whistle-stop Tour of Quantum Computing

We often use $|\psi\rangle$ for an arbitrary quantum state, normally on n qubits

You can think of it as a probability distribution over n bitstrings.

Probability equivalent	**Quantum Stuff**
$\boldsymbol{x} \in \mathbb{R}^n$	$\|\psi\rangle \in \mathbb{C}^{2^n}$
$\Sigma x_i = 1$	$\lVert \|\psi\rangle \rVert_2 = 1$

Whistle-stop Tour of Quantum Computing

$|i\rangle$: the i^{th} computation basis state (bitstring always represents i)

Superposition: the state isn't a computational basis state

e.g. – uniform distribution

But Nat, you said we can't look at the state easily, what use does this have if we can look at it?

- Measurements give you info about the state

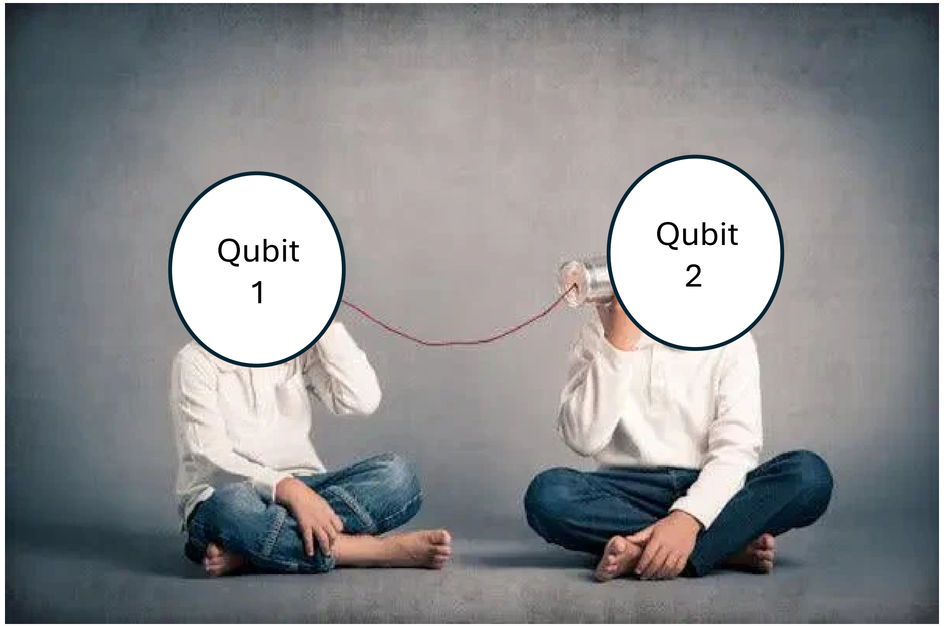
- Pick an orthonormal basis of $\mathbb{C}^{2^n}$ (normally $\{|0\rangle, |1\rangle, \ldots, |2^n - 1\rangle\}$)
- Ask $|\psi\rangle$ nicely to sample itself in that basis
- $|\psi\rangle$ turns into it's chosen sample and we can freely look at it
  - ("looking" is a measurement in case you wanted to cheat ☺)

- Lose information as we get a sample not the whole distribution

Marvel (or other sci-fi series) keeps talking about entanglement but you've not mentioned it yet

Entanglement is like *dependency* between bits

Think of string phone metaphor as the two qubits need to "talk to each other"



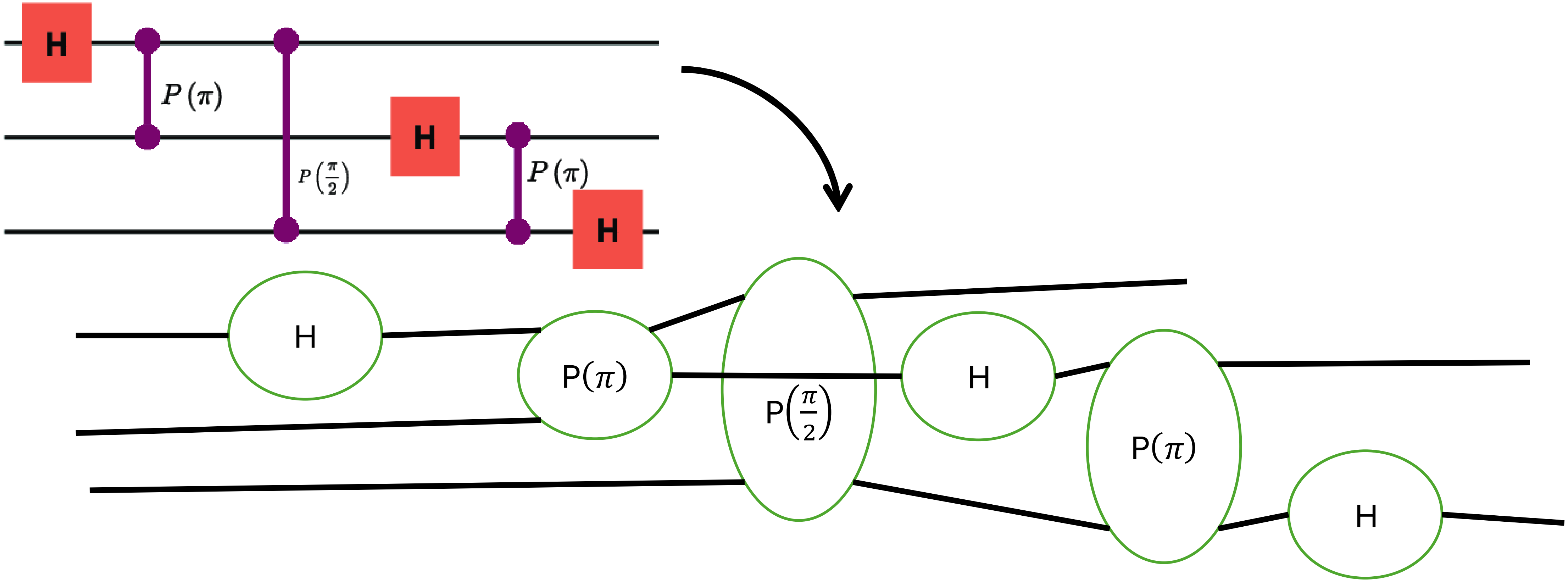
"Spooky action at a distance"
– Albert Einstein

Rest stop #2

Good stuff comes in a sec

The start of the payoff

We can represent a circuit with a tensor network



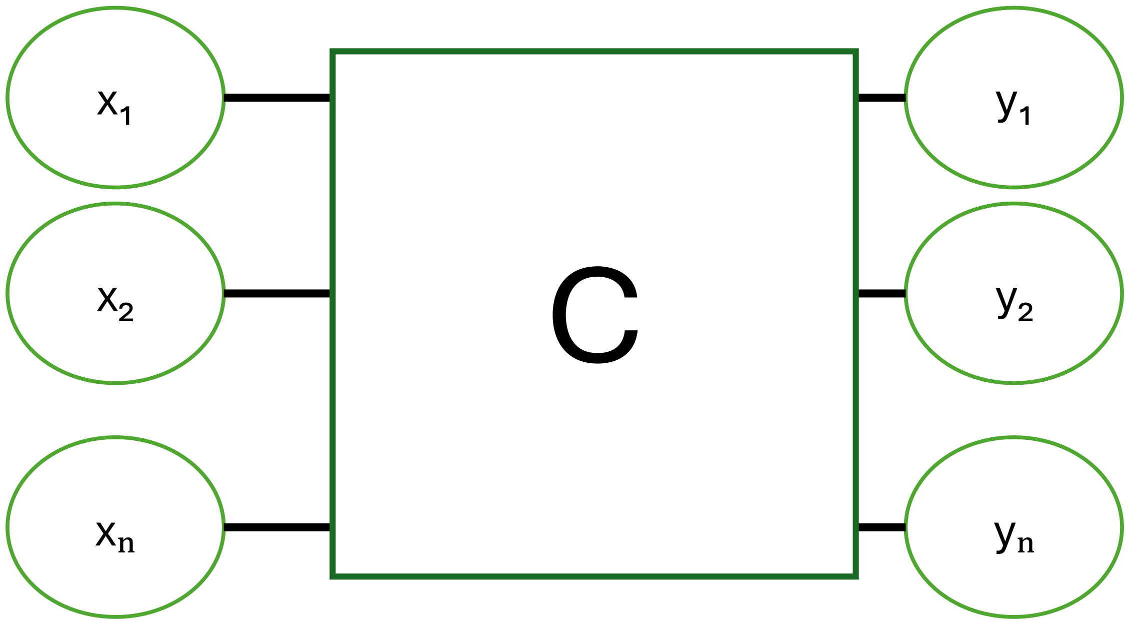
Sorry for the spaghetti diagram, writing an apology is easier than fixing it

The payoff

If:

- All gates in our circuit, C, are unitary (size preserving) and
- $\boldsymbol{x}, \boldsymbol{y} \in \{0,1\}^n$

Then



$= \Pr[\boldsymbol{y} | \boldsymbol{C} \text{ running on } \boldsymbol{x}]$

Final house keeping

- Contracting tensor network is classical
- Contraction complexity of a graph G, $cc(G)$, is the “most expensive” contraction over the “best” order
- $cc(G) \in [\max\{tw(G), \Delta(G) - 1\}, tw(G) \times \Delta(G)]$
- Calculating $\Pr[\boldsymbol{y} | \boldsymbol{C} \text{ running on } \boldsymbol{x}]$ takes $O\left(|E| \times 2^{cc(G)}\right)$ time

    - Called “strong simulation” if $\boldsymbol{x} = 0$

Recap

- What’s a tensor?
- What’s a quantum [computer, bit, state, circuit]? (delete as appropriate)
- Why use tensor networks for quantum computation

Thanks for listening

Any questions?