Tensor networks, MPS and "Entanglement Rerouting"

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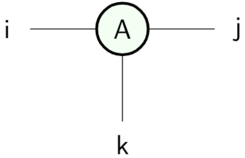

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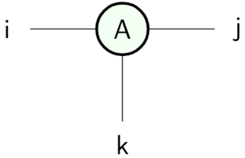

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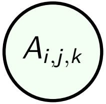
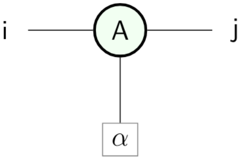
An alternative way of representing Scalars



What is a tensor? (2)

Now for vectors.
For $i, j \in [\chi]$,



$$= \begin{pmatrix} A_{i,j,1} \\ A_{i,j,2} \\ \vdots \\ A_{i,j,\chi} \end{pmatrix} \in \mathbb{C}^{\chi}$$

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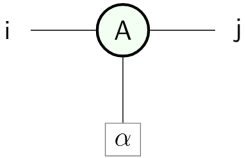

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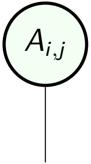
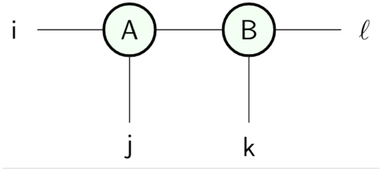


Connecting two tensors

We can connect tensors together. First, we'll see connecting two scalars. For $i, j, k, \ell \in [\chi]$,



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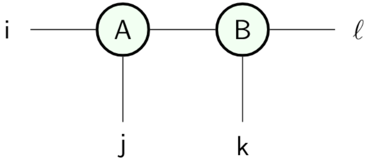

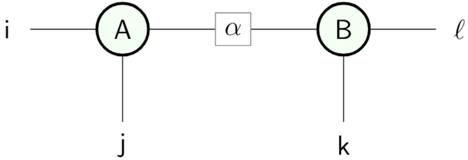

$$= \sum_{\alpha=1}^{\chi} A_{i,j,\alpha} B_{\alpha,k,\ell} \in \mathbb{C}$$

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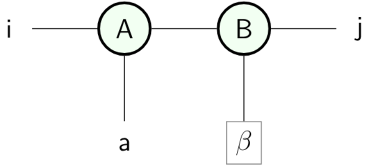
Now a similar example with B being a vector. For $a, i, j \in [\chi]$,



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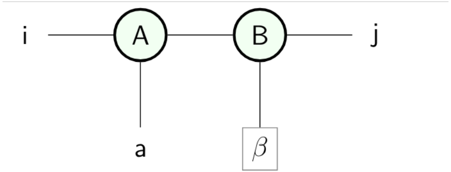

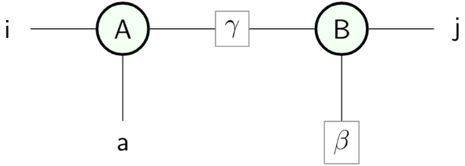

$$= \sum_{\gamma=1}^{\chi} A_{i,a,\gamma} B_{\gamma,\beta,j}, \forall \beta \in [\chi]$$
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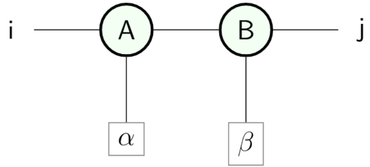

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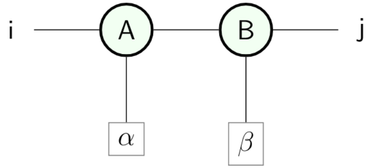
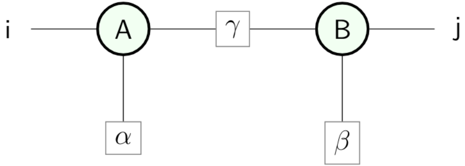
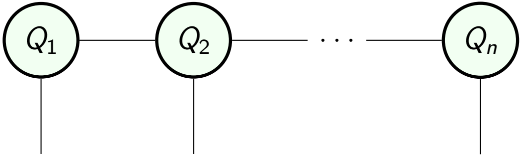




$$= \sum_{\gamma=1}^{\chi} |A_{i,\gamma}\rangle \otimes |B_{\gamma,j}\rangle$$

Example of a Tensor network: MPS

Using tensor network notation, we can represent “states with a line entanglemenet structure”, called MPS, as follows



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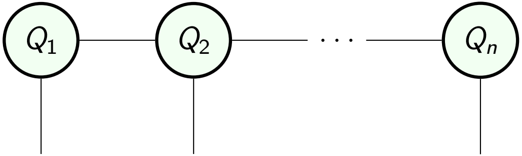

More formally, an MPS $|\psi\rangle$, with bond dimension χ, satisfies $\forall i \in [n]$, the Schmidt rank of $|\psi\rangle$ across the $\{1, \ldots, i\}, \{i+1, \ldots, n\}$ bipartition is at most χ. MPS have nice properties, for example they can be learnt using $\text{poly}(n, \chi)$ samples.

MPS preparation

State preparation: Given a ***classical*** description of a state $|\psi\rangle$, construct a circuit, C, that prepares copies of $|\psi\rangle$, i.e. $C|0\rangle = |\psi\rangle$.

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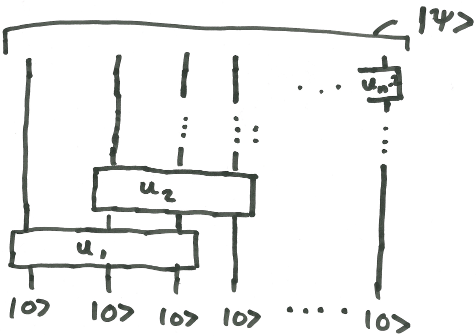

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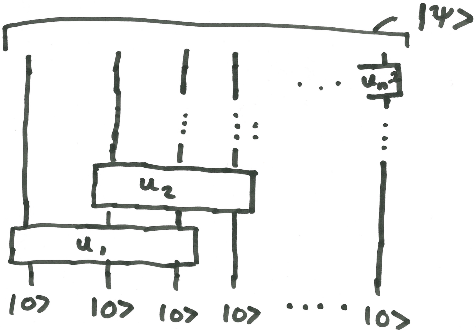

where the U_is are k-qubit unitaries, with $k = O(\log \chi)$.

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As MPS can be prepared using these very structured circuits, we can use this to help design a tomography algorithm.

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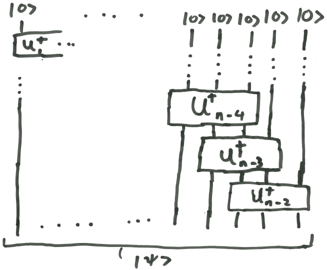
New goal: Learn $U_i^\dagger, \forall i$ as $|\phi\rangle \leftarrow U_{n-2}\ldots U_1|0\rangle$ satisfies $|\phi\rangle \approx |\psi\rangle$.



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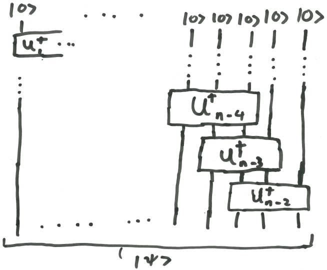

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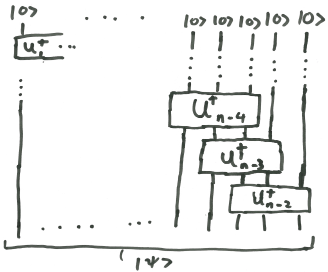

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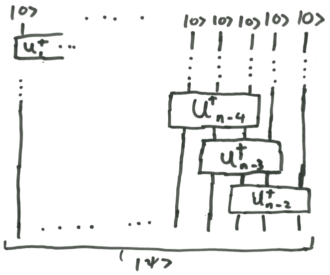

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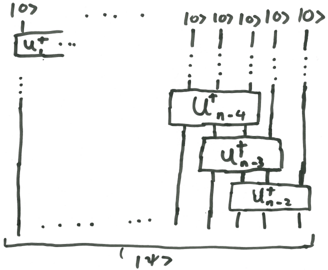

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Repeat for $U_{n-3}^\dagger, U_{n-4}^\dagger, \dots U_1^\dagger$.

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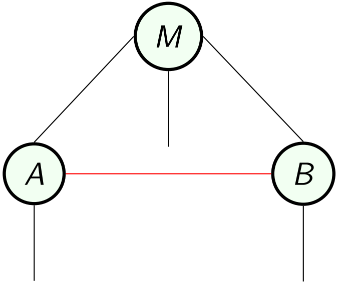

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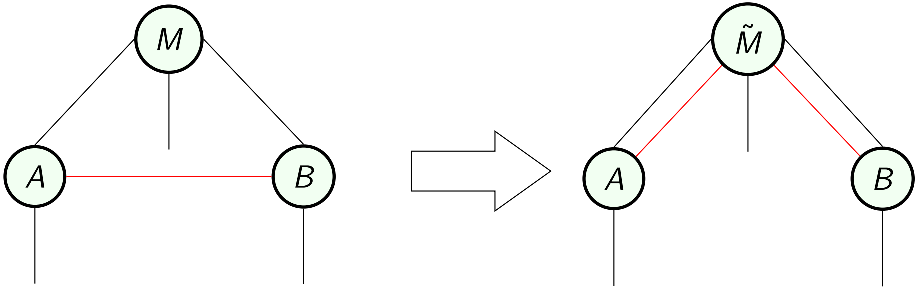

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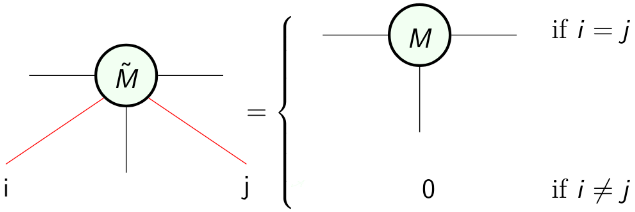

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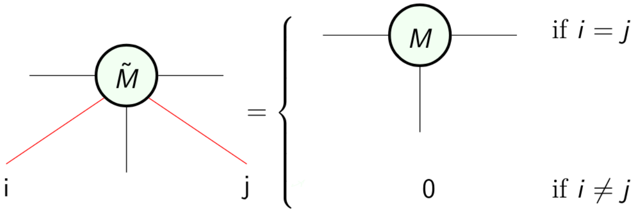

This can be extended to more than one "midpoint".

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- Could be used to "restructure" states for representing/preparing them on hardware with "limited" connectivity
- Not just restricted to states, works for arbitrary tensor networks

Thanks for your attention!

Any questions?