

Notes

Example of logic problem for

definitional display!

Examples of Interpretations

- (1) Consider the formulae:
- (i) $\forall x \forall y (A(x,y) \Rightarrow A(x,y))$
 - (ii) $A(x,y) \wedge A(y,z) \Rightarrow A(x,z)$
 - (iii) $\forall y \exists x A(x,y)$

If $A(x,y)$ means 'x is an ancestor of y' over the domain of all people, both living and dead, all the formulae become true. If $A(x,y)$ is $x > y$ over the domain \mathbf{N} , or $A(x,y)$ is $x < y$ over domain \mathbf{Z} they are also all true, but in the latter case if the domain is \mathbf{N} then (iii) is false. Also if $A(x,y)$ is 'x is mother of y' over the domain of people then (ii) is false. But (i) remains true no matter what interpretation is given - so it is logically valid (or *universally valid*).

(2) With $D = \mathbf{R}$, let $A(x,y)$ be $x \leq y$ and $f(x) = e^x$ then sequence $s = (d_1, d_2, \dots)$ of real numbers satisfies $A(f(x_2), x_4)$ iff $e^{d_2} \leq d_4$. Also $\neg A(f(x_1), c)$ is true if c denotes 0 since for any d_1 it is the case that $e^{d_1} > 0$. And clearly also $(\forall x_1) (\neg A(f(x_1), c))$ is true.

(3) With $D =$ points in the plane, $B(x,y,z)$ is 'x and y are equidistant from z' and $g(x,y)$ is 'the mid-point of the line segment joining x and y' then the sequence $s = (d_1, d_2, \dots)$ of points satisfies $B(g(x_1, x_2), g(x_3, x_1), x_4)$ iff the mid-point of the line segment between d_1 and d_2 is the same distance from d_4 as the mid-point of the line segment between d_3 and d_1 .

(4) With $D = \mathbf{Z}$, and $C(x, y, u, v)$ is ' $x.v = u.y$ ' and b denotes 2, then sequence $s = (d_1, d_2, \dots)$ of integers satisfies $C(x_3, b, x_1, x_3)$ iff $d_3^2 = 2d_1$. In this case the formula $\exists x_3 C(x_3, b, x_1, x_3)$ is neither true nor false but is satisfied for any sequence with d_1 such that $2d_1$ is a perfect square.

Final Formal Features of PDC

In general there is no effective method for testing for the logical validity of a formula of PDC. (One has to test the truth over all interpretations with possibly infinite domains.) Some formal system, which was a luxury in PC, is therefore necessary in PDC in order to systematically generate logically valid formulae, or to try to test for logical validity. For this purpose we can add the following axioms and rule of inference to those of PC :

to investigate what's happening consider fixing d_1, d_2 and d_4 - what is locus of d_3 ? (A circle, ellipse...)

Then, fix d_1 and d_4 what does the constraint mean for d_2 and d_3 ?