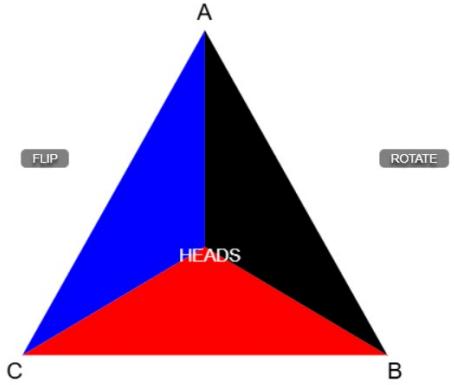
Building Bridges:
Making Sense of Mathematics and Art



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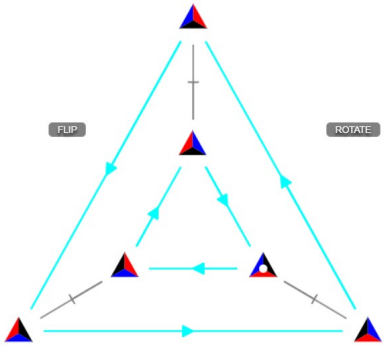


Motivating Ideas

George Hart: *What can we say about Math/Art?*

- Identifying the disciplinary character of Math/Art
- Particularly interesting and insightful because Hart is a practitioner

Themes in Hart's discussion ...

- An experiential rather than a formal perspective on mathematics
- Empirical elements in construction – a holistic process-oriented view
- **Making sense** and **communicating**
- "Constructing artefacts to say something beyond words"

... connect these with the notion of 'making construals' ...

Contents

Making a Construal

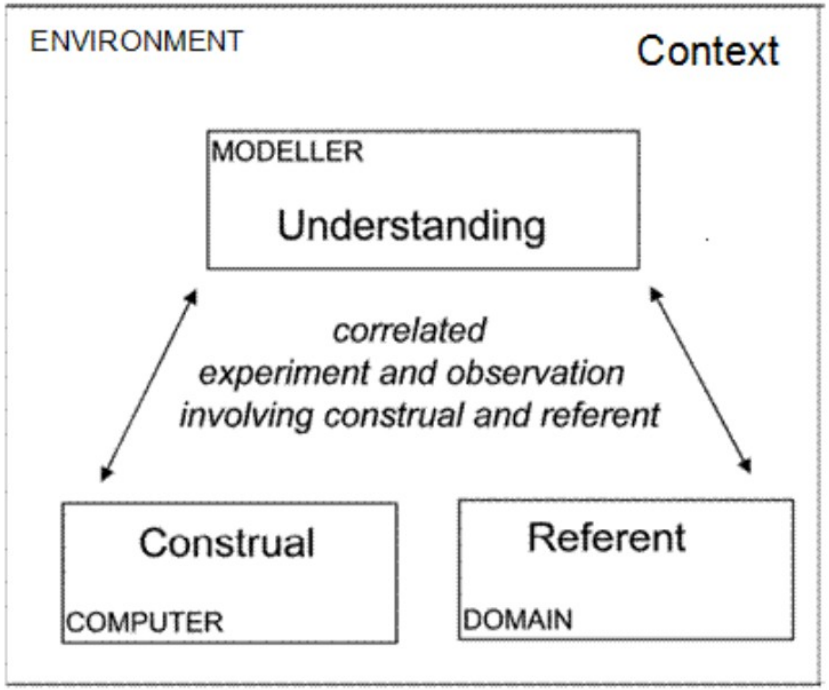


Diagram pertains to a ‘state’ ...

… as perceived by a particular agent (a ‘maker’)

Correlation as a connection-in-experience

Construal and **Referent** are evolving

Context and **Understanding** co-evolve

cf. ‘lived experience’ in its relation to ‘world view’

Versions of construals reflect evolving understanding

An Illustrative (“toy”) Construal

A **construal** is an interactive artefact that embodies its maker’s understanding of ‘how something works’. It may take the form of an environment to represent the way in which the perspectives of diverse agents on a situation are related, as conceived by its maker.

Construals are intended to promote sense-making and communication
… their primary role is to support the maker’s comprehension

cf. A **program** implements a recipe for an agent
to achieve a functional objective in a prescribed context

Programs are designed with efficient automated achievement of functional goals in mind

Illustration: A construal of S3, the unique non-Abelian group of order 6

Perspectives on the Group S3

S3 is the unique non-Abelian group of order 6, with multiplication table:

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	3	1	6	4	5
3	3	1	2	5	6	4
4	4	5	6	1	2	3
5	5	6	4	3	1	2
6	6	4	5	2	3	1

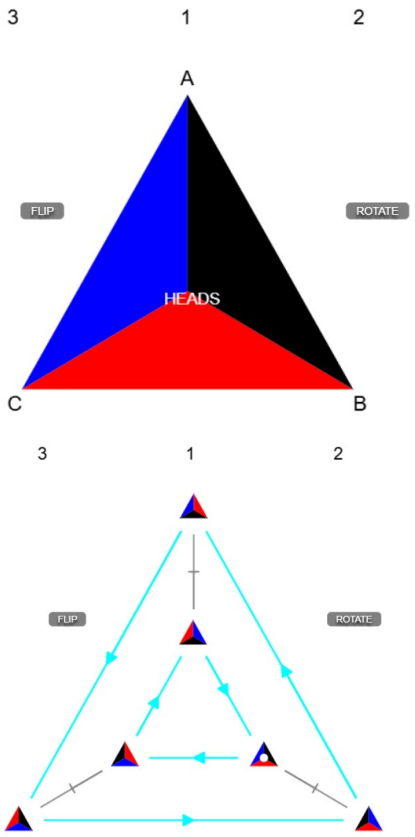
S3 is the symmetric group on 3 symbols comprising 6 permutations
1, **(123)**, **(132)**, **(12)**, **(13)**, **(23)**

S3 is the symmetry group of a solid equilateral triangle, as generated by **f**lip and **r**otate

S3 is the group generated by the presentation $\mathbf{< f,r \mid f^2 = r^3 = 1, rf = fr^2 >}$
- there is an associated Cayley diagram with 6 vertices and 12 edges

There are particular kinds of **agency** and **observation** associated with each interpretation of S3

Observables and Dependencies



perm = [3,2,1]
permstr is "e" if perm == [1,2,3] else "r" if perm == [2,3,1]
else "r^2" if perm == [3,1,2] else "f" if perm == [3,2,1]
else "fr^2" if perm == [1,3,2] else "fr"

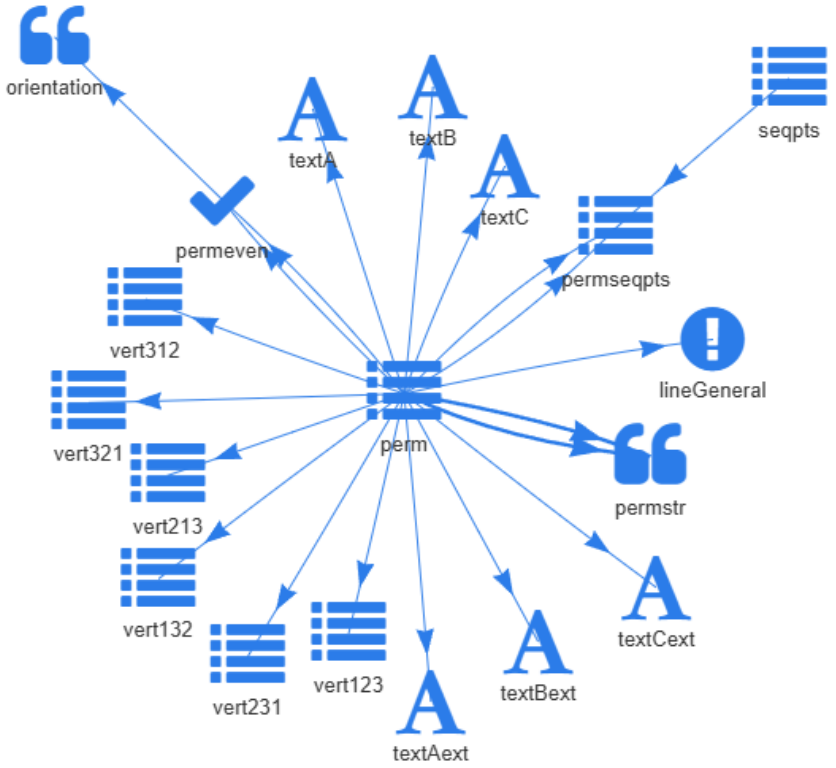
orientation is "HEADS" if permeven else "TAILS"
permeven is (perm == [1,2,3]) or (perm == [2,3,1]) or (perm == [3,1,2])

textA is Text(verts[perm[1]], originX-offsetx-13, originY+offsety, 26)

textAext is Text(str(perm[1]), originX-offsetx-13, 12*offsety, 26, textcol)
textcol is "black" if displayperm else "white"

protovert is [lineAB, lineAC, lineBC, marker]
vert123 is protovert with triangle_scale is 10, originX is 213, originY is 526,
markerrad is 3 if perm==[1,2,3] else 0,perm is [1,2,3]

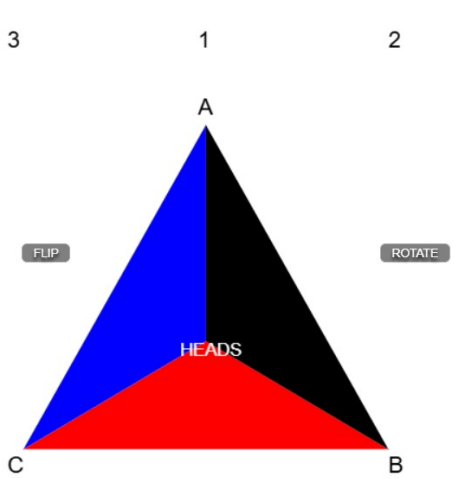
A Dependency Map

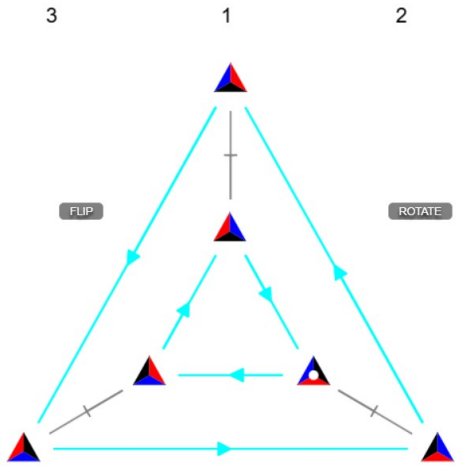


Search string: `^perm|orientation|seq|^text`

Observables dependent on **perm**

Dependencies as *expressing expectations about how changes to observables are linked*





The Concept of Construal

What interpretations have been given to the term **construal**?

Linguistic:	"Time flies like an arrow" / "Fruit flies like a banana."
Educational:	Teaching as *communicating your construal to learners*
Philosophy of Science:	… exemplified by the experimental practices of Michael Faraday

David Gooding writes: *Faraday's busy phenomenology produced a qualitative, visual theory of considerable power and generality – described by James Clerk Maxwell as mathematics "of a very high order"*

Mathematics as concerning connections-in-experience that are reliable, objective, generic

Making construals can serve: to *identify* such connections-in-experience
and: to *explore* how they may be exploited

Making *Digital* Construals

Gooding and Addis tried to use functional programming to make construals cf. developing software with the intended users and functionalities in mind – a ‘Computational Thinking’ paradigm …

… but making construals entails many other aspects of practical software development:

- Crafting the environment for computation:
  identifying the stable patterns of interaction and reliable agency prior to automation
- Determining the nature and potential for agency through sense-making and exploration
  orchestrating the interactions amongst diverse agents (cf. users, designers, developers)
- Exploiting the agency that is discovered and contrived imaginatively and safely

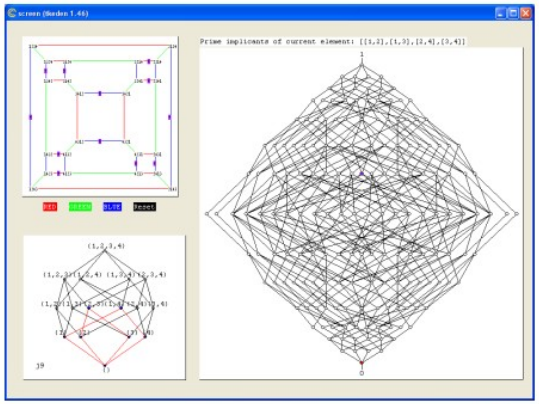
… and demands a much richer epistemic and less formal semantic framework

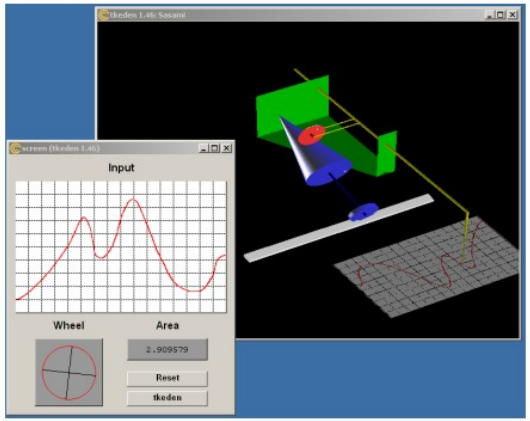
Following the precepts of William James’s ‘radical empiricism’:

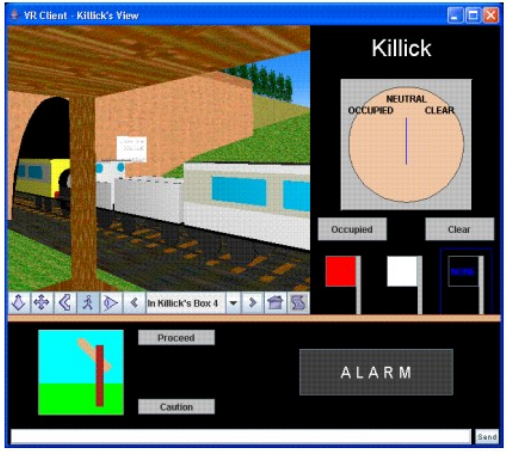
to *know* is to experience the connection between two aspects of experience

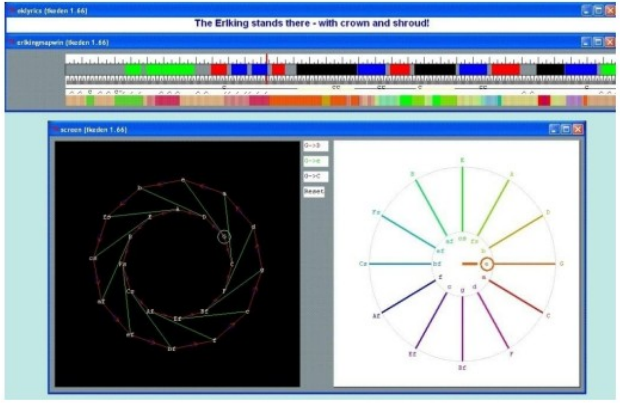
making construals is rooted in the capacity of agents to apprehend **connections-in-experience**

More Examples of Digital Construals ...









Clockwise from top-left …

The free distributive lattice FDL4

An Oppikofer planimeter (1824)

Schubert's song Erlkönig (Goethe)

The Clayton Tunnel railway accident

See warwick.ac.uk/em/publications/posters

Concluding remarks

Making construals has promise in relation to several themes significant for Bridges:

George Hart – understanding the disciplinary nature and potential of Math/Art

Paul Gailiunas – principles for communication across (“bridging”) disciplines

Edmund Harriss – a practice to support ‘genuine pretending’ in relation to diverse worlds

Bridges highlights the engagement of the mind and the senses in the process of construction

- Making construals exposes the subtlety of what might ostensibly appear to be simple agency
- Has topical interest w.r.t. the potential and potential limitations of automated construction

Acknowledgements

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Questions?