EMPIRICAL MODELLING IN PRODUCT DESIGN

Carlos N. Fischer
Departamento de Estatística, Matemática Aplicada e Computação, Universidade Estadual Paulista - UNESP, Brasil.
Rua Dez, 2527; CP 178; Santana; Rio Claro, SP. 13500-230
cfischer@rc.unesp.br

Meurig Beynon
Department of Computer Science, University of Warwick
Coventry CV4 7AL, UK.
wmb@dcs.warwick.ac.uk

Abstract. *The purpose of this paper is to introduce and illustrate the application of an approach to modelling known as Empirical Modelling (EM), a collection of principles and techniques that has been under development at the University of Warwick, UK. The environment for EM supports a high level of interaction with a model. EM tools allow the user to build a model that is always open to extension, refinement and revision. Our case study illustrates the application of EM and demonstrates its potential as an alternative approach to modelling a product in its situated context.*

Keywords: *computer-based model, computer simulation, empirical modelling, product design, product modelling.*

1. INTRODUCTION

Modelling has a very important role in modern product design. In representing a product, it is sometimes appropriate for the designer to model the product in isolation, without considering its environment. However, where the product interacts closely with its environment and with human agents, the designer must focus on representing the product in its situated context even whilst the modelling is in progress. For this purpose, the product must ideally be represented in such a way as to allow potential re-use, convenient interaction and scope for easy integration into many different contexts.

Product modelling has benefited greatly from the use of computational environments. An appropriate computational environment for product modelling must provide the designer with the high degree of interaction with the model that is needed to achieve the best representation of a product [6,11] and to reflect its interaction with its environment. The knowledge acquired from such interaction is very helpful in suggesting improvements to the product itself.

Empirical Modelling (EM) - so called because the modelling principles are based on observation and experiment - is an unconventional approach to modelling systems under development at the University of Warwick [10]. It offers an environment very appropriate for applications that demand a high level of interaction with a computer-based model. Central to the EM perspective is an emphasis on the power of the computer to represent states, in particular, states that are easily interpretable. An EM model metaphorically represents a particular state of the system under study.

In this paper, section 2 presents a brief overview about product design and relevant topics. Section 3 introduces the concepts of EM approach, describing some interesting characteristics and tools. A case study, modelling the state and interactions associated with a simple 3-dimensional block jigsaw puzzle, is presented in section 4; the objective is to show the application of the principles and tools of EM in the construction of models and demonstrate that this alternative approach is well-suited for developing tools for the field of product design. Section 5 discusses the way in which EM can impact on information management, and speculates upon its possible implication for business models.

2. PRODUCT DESIGN

Product design is the activity of generating a solution that meets the requirements imposed on a product [7,11]. Such activity relies heavily on the creativity of the designer, which derives from the designer's capacity for observation and especially from knowledge about the product or its requirements that generally evolves as the design progresses. Indeed progress in the design is intimately associated with the evolution of knowledge about the current product, so that there is positive feedback in both directions between acquisition of knowledge about the product and progress in the design [9].

Product modelling involves creating a concrete or virtual representation (a model) of characteristics of a physical product under study. Modelling is a very important activity in modern product design processes [7,8]. An appropriate model provides the designer with a richer, higher-level and more intelligible representation of a design concept than natural language affords.

In representing a product, it is sometimes appropriate for the designer to model the product in isolation. That is, the model represents the product without considering the influence of the context in which the product will be situated. However, where the product interacts closely with its environment and with human agents, the designer must focus on modelling the product in its situated context. That is, the model must take account of the context where the product is to be inserted, representing product features that are dependent on the environment.

The computer is already extensively used as a tool for modelling within product design. The computational environment for modelling must provide the designer with a high level of interaction with the model that is needed to achieve the best representation of a product. It must also offer the designer tools that allow a model to be modified in a simple and quick manner by taking advantage of potential re-use of existing models, and permit convenient interaction and integration with an external environment. Ideally, a modelling environment should also support designers in the sharing of their knowledge about the product and the integration of their ideas about its model that leads to the evolution of product knowledge and consequently to the progress of the design. In many cases, another important feature is that the modelling environment should support animation of the model.

In this paper, a computer-based model is referred to as an *artefact* and the object under study as the *referent.*

3. THE EMPIRICAL MODELLING APPROACH

Empirical Modelling (EM) is a collection of principles and techniques that has been under development at the University of Warwick [1,2,10]. EM offers a set of tools for modelling and visualisation of general systems that allows a user to build an artefact that is always open to subsequent extension, refinement and revision. Animation of an artefact is also possible, through visualisation of sequences of states of the referent subject to some behaviour.

The EM principles are based on three fundamental concepts: *observable, dependency and agency*. The term *observable* is used to refer to elements that relate to our understanding of the referent. A *dependency* is a relationship among observables that expresses expectations about how the values of observables are indivisibly linked in change. An *agent*, with its associated observables, has privileges for actions to change the values of those observables. For example, when modelling a room, the modeller might consider the light and the light switch as observables that can be on or off; might assert a dependency between the light switch and the light: when the light switch is pressed on, the light comes on at the same time; and, an agent could be a user of the room who may switch on and off the light switch.

The identification of the relevant properties (observables, dependencies and agents) of a referent is subjective, based on the observation and interpretation of the modeller. Generally, this identification is provisional as it depends on the modeller's personal experience of both the referent and the artefact. Through experimentation with the artefact, the designer gains knowledge and new ideas related to the referent, so that new properties may arise apart from those initially identified. In an EM artefact, any new property can be added to the artefact at any stage during the modelling activity without the need to revise the whole artefact or revert to an earlier version. Moreover, as the changes in observables can be executed during the simulation, modifications in the artefact are immediately executed, which allows subsequent revision of the artefact and the complete development of the artefact in an incremental fashion. This flexibility of EM is distinguishes it from many conventional modelling approaches in which the modeller has to preconceive what the inputs and outputs are going to be before starting the construction of the model in order to prescribe the required behaviour. In such modelling paradigms, the boundary of the proposed system must be determined in advance, and if there is a need for a new input or output, e.g. an additional system feature is required, then the whole system may have to be revised and re-designed at substantial cost. Part of the flexibility of EM stems from the fact that the focus is initially on the *state* of a model or domain, rather than on the behaviours the modeller wishes to produce.

The dependencies among observables are represented by *definitions*. A typical definition takes the form $x = f(y, z)$ where x, y and z are artefact observables associated with the referent observables, and f is a function that defines the relationship between those observables. Such a definition expresses a dependency relating x, y and z that is automatically maintained by the EM tools. That is, changes in the observables y or z will result indivisibly in a change in x, the dependent observable. Such dependency amongst observables obviates the need to verify whether a dependent observable is up to date in the artefact. A definition can also assign an explicit value, as in y = 5. The set of definitions forms a *definitive (definition-based) script* to be interpreted by the EM tools.

The particular values of the observables and the dependencies associated with the artefact represent one of the possible states of the referent. Changes of state occur in the artefact either through the re-definition of observables or the addition of new definitions. Through automatic re-definition of observables and consequent visualisation of a sequence of representative states of a referent, an EM artefact can be animated so as to give the impression of a behaviour.

Another important characteristic of EM is that there is a distributed version of the modelling environment [14] that allows communication between users who can access and visualise their representations of the same artefact over a network. In this way, many designers may interact with an artefact, allowing their viewpoints to be integrated during the modelling process.

The principal modelling tool that has been developed at Warwick is the *TkEden interpreter*, which is implemented in C. TkEden supports definitive scripts in which the variables represent a variety of two-dimensional graphical elements, including shapes, point and lines, text strings, windows and displays. TkEden incorporates three definitive notations: *DoNaLD*, for line drawing, *SCOUT*, for screen layout, and *EDEN*, an evaluator for definitive notations that allows definitive scripts to be formulated over scalar types, non-homogeneous recursive lists and strings. The EM artefact described in section 4.2 and illustrated in Fig. 2 makes use of two additional definitive notations: SASAMI – a geometric modelling extension implemented by Ben Carter, and ARCA, for describing artefacts such as finite state machines and combinatorial graphs. The TkEden interpreter and its SASAMI extension are available for download as open source [SF].

The EM tools are still primarily research ones, but they are sufficient for proof-of-concept of the principles of EM, and previous work [3,5,13] has demonstrated that they allow the construction of artefacts that are always open for interaction and revision. Moreover, any components from an artefact can be integrated within another artefact, thereby making modelling easier and quicker.

4. EM IN PRODUCT DESIGN

The qualities of Empirical Modelling (EM) described in section 3 are particularly significant when the modeller needs to construct an artefact with a high degree of interaction and flexibility, as in product design. To illustrate this, we present in this section two case studies of artefact development. The objective is to show that EM offers an open-ended environment that provides an alternative approach to product design processes.

4.1 The Clock Artefact

The first case study is a simulation of a clock. Various artefacts derived from a simple clock model are depicted in Fig. 1. Each artefact can be seen as representing one of many different states of the clock model. Five coloured input buttons are used to set the power on/off and to update the time. These buttons are used to change the state of the clock.

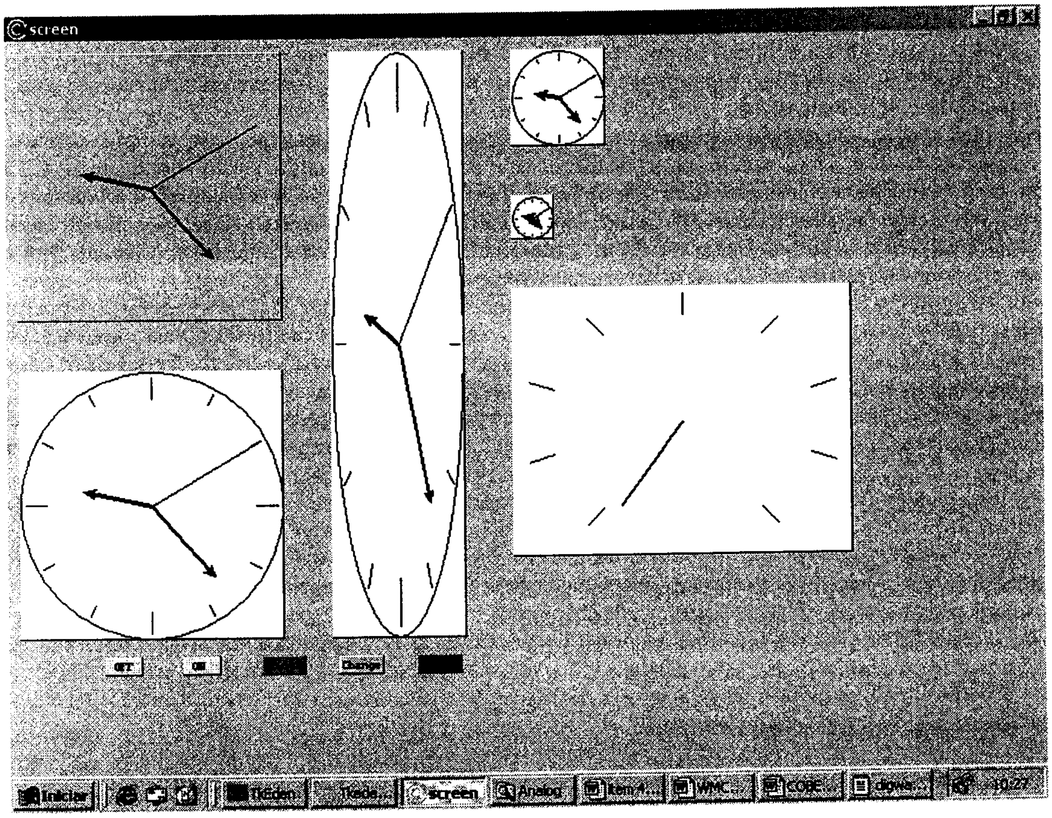

Figure 1. EM artefacts derived from a clock model.

The current values of the observables of an EM artefact represent a particular state of the referent. Changes in the values of these observables imply new representative states for the artefact, which are reflected in the visual elements of the model. These values can be changed by external and internal agents of the artefact. An example of an external agent is a user who can set the power on by pressing the *ON* input button; this action changes the value of an observable called *power_s* to a non-zero value that triggers a procedural action for switching the clock on. A similar action continuously checks the current time in the artefact and after a period of time changes the value of *power_s* to zero (simulating exhaustion of the battery) and so stops the clock. That procedural action is associated with an internal agent of the artefact. Another internal agent continually updates the value of an observable called *new_time* with reference to the computer system clock; each new value of *new_time* changes the state of the artefact. These consecutive state changes are automatically displayed on the screen, and so simulate the clock in operation.

The artefact associates states of a real clock with instantiations of agents. For example, the *clock* agent represents the clock itself and the *power* agent its power supply. In the specification of each agent, the identifiers refer to observables that are significant for the agent. In the model, the *power_s* observable represents the energy that is supplied to the clock and an observable called *live* indicates whether the agent *power* is present or absent according to the definition $live = (power\_s > 0)$.

The dependencies in the artefact reflect several different types of agency. The primary purpose of these dependencies is to bind the visual elements and the indivisible internal state of the model together so to reflect interaction with the external observables they represent. Simple instances of definitions and agencies are illustrated by the clock. In an engineering model, the hands of a clock may be coupled mechanically so that when the minute hand moves, the hour hand moves simultaneously. In the artefact, this dependency can be expressed by an explicit definition:

$$angle\_hour\_hand = (angle\_min\_hand / 2\pi) * (\pi / 6).$$

Whilst such a definition is extant in the artefact, any movement of the minute hand entails simultaneous movement of the hour hand, both in simulating the clock in its normal operation and in setting the clock. The normal operation of the clock is associated with agents that can act autonomously without the intervention of the designer. The designer can nonetheless intervene to interfere with these agents, for instance to simulate a clock running down or malfunctioning. The role of the user in setting the clock, which conforms to no preconceived pattern of interaction, can be played directly by the designer.

The relation between second and minute hands illustrates another role for agency in constructing an artefact. In our chosen clock mechanism, the motions of the second and minute hands are mostly independent but are synchronised by updating the position of the minute hand at regular intervals. In this case, the movement of the minute hand is not within the scope of the indivisible action of moving the second hand. To model this, an independent agent was introduced. This monitors the position of the second hand and updates the minute hand in discrete steps as the second hand registers that a minute has elapsed.

The building of the clock artefact illustrates some characteristic features of EM. The artefact was incrementally constructed whilst running as a background process on a PC computer. Control over dependency and agency made it possible to take account of the new views of the design process as they arose. Interactive development sessions typically involved the addition of a group of definitions to attach a new visual component to the artefact or of procedural actions to simulate the introduction of new agents. The artefact was constructed stage-by-stage: first introducing the clock geometry; then the mechanism and finally the input buttons. Several types of re-use featured in the modelling activity. For example, the codes for the structures of a coloured input button were simply replicated (with new texts and positions only) to construct the other buttons of the artefact. The script that describes the window at the bottom in Fig. 1 was replicated three times, and the positions and sizes were modified, so as to generate the other variants of the clock artefact shown in the display. Through a simple modification of the initial clock model we also derived the artefact depicted in the bottom right window in Fig. 1 that could represent, for example, a meter.

The development exercise in some ways resembled conventional engineering: it was sometimes useful to develop portions of the script independently, or to trace problems by extracting pieces of the script and exercising them in isolation; this was the case in particular for the clock mechanism, represented in the top left window in Fig. 1. The ease with which we could adapt alternative computer models on the fly contrasts with the considerable ingenuity that would be required to modify a clock mechanism.

4.2 The Jigsaw Puzzle Artefact

The case study presented here takes as its target product a four-cube jigsaw puzzle such as is suitable for a young child. An EM artefact to represent such a puzzle is depicted in Fig. 2. A four-cube jigsaw puzzle comprises 4 blocks that can be configured in six different ways so that their uppermost faces display different pictures. For this purpose, each of the six faces is textured using a different quadrant from one of six pictures. From a design perspective, one of the relevant issues is how to dispose the textures on the blocks in such a way as to make the task of transforming one solution to another more or less taxing. One way of doing this that is instructive and satisfying for a child, and does not involve the essentially independent solution of all six jigsaws, is to ensure that when one of the jigsaws is solved, each of the other solutions can be derived by performing the same symmetry operation on each of the four cubes in turn. The subtlety of the interaction between the choice of pictures, the mode of texturing, and the symmetries of the blocks leads to interesting issues both for the jigsaw design and for the manufacturing process. The implications of adopting a particular design for the manufacturer and the solver are hard to predict without modelling and experiment that embraces the situated context. In particular, the number of possible configurations of four cubes is such that, if suitable side-conditions are imposed on solutions, their difficulty can range from quite trivial to exceptionally hard.

The artefact in Fig. 2 models a design scenario that leads to jigsaws that have solutions of moderate difficulty, where there is a fixed configuration of the four blocks, in which each is associated with a particular quadrant, such that all six solutions to the jigsaw can be derived by manipulating each block *in situ.* This limits the number of abstract configurations that would have to be considered in an exhaustive search for solutions to 24^4 rather than 24^5. Coloured faces of six different colours have been used to identify the textures that make up the different pictures: these too have been disposed in such a way as to simplify the transformation of one solution of the jigsaw puzzle into another. The artefact represents four cubes with an associated state diagram in the form of a Cayley diagram [GM] that dynamically traces the effect of applying primitive symmetry operations to the cube. Explicitly, the traversal of a red, green or blue edge in the Cayley diagram is associated with a rotation of a cube about an axis that joins the midpoints of a specific pair of opposite edges. By a sequence of such primitive rotations any physical operation that permutes the vertices of the cube can be carried out. By using six input buttons (on the top left corner of the Fig. 2), we can apply primitive rotations to change the state of a specific cube or to change the state of all the cubes in the same fashion at the same time. The intepretation of the Cayley diagram displayed on the left of Fig. 2 depends upon whether a specific cube or all of the cubes are currently selected: the small white square on this diagram indicates what symmetry operations have been applied to the cubes, singly and collectively, in order to realise their current configuration. Each node in the Cayley diagram represents a possible cube state and each transition between cube states is associated with an edge that is coloured according to which input button is pressed. The diagram is viewed as a representation of reliable knowledge about how the cubes will react to inputs. All the changes of state within the diagram represent particular observed transformations of the four-cube jigsaw..

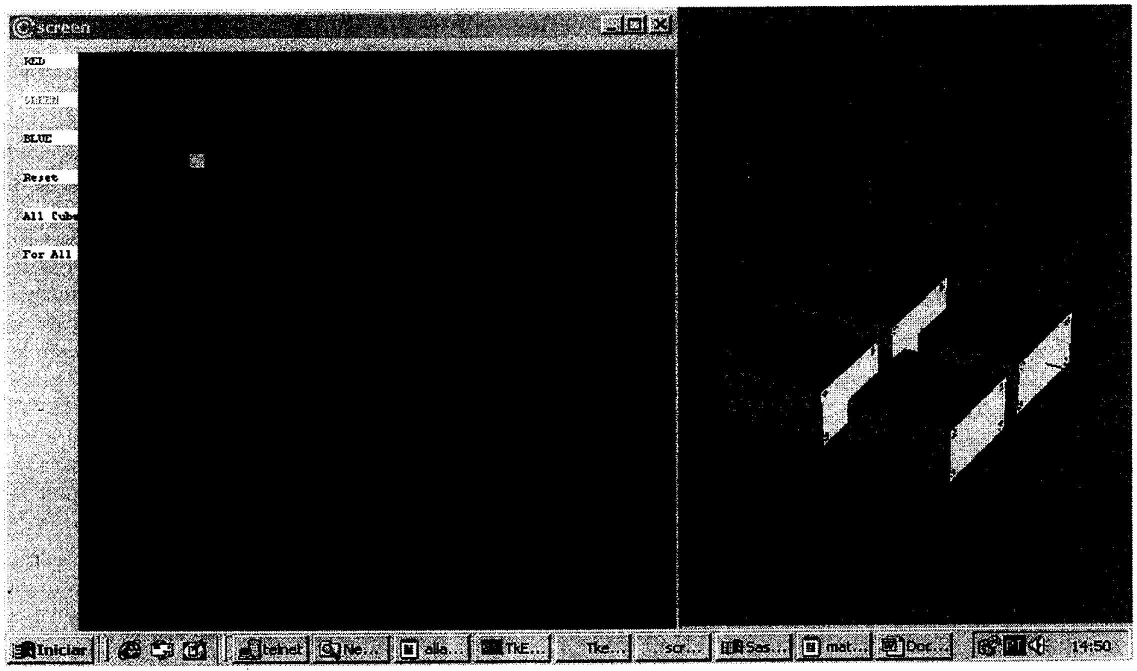

Figure 2. EM artefact of a Four-Cube Jigsaw Puzzle.

When suitable textures have been added to the blocks in Fig. 2, the artefact represents a particular state of the jigsaw, together with latent state transitions that reflect what the modeller expects to happen in response to a particular action. Each state of the referent is specified in terms of the current values of a collection of observables of the artefact. The visual elements in the model represent the current values of the observables, which can be changed by pressing the buttons. The input buttons trigger the agents that act to change those observables. Using the buttons we can simulate the direct action of a solver experimenting with the cubes.

The simple interface through which the modeller can simulate button pressing is one very limited way mode of interaction. The modeller also has access to the state of the artefact through the textual input window for the underlying TkEden interpreter for the modelling environment. This window is the interface through which the entire artefact is constructed: it gives scope to extend and modify all the definitions, functions and triggered actions interactively in a manner that illustrates the openness of EM artefacts.

The jigsaw artefact in Fig. 2 was constructed in the following way:

1. by first constructing one cube and the associated Cayley diagram;

2. by constructing the other cubes and joining them together;
3. by repeatedly texturing the uppermost faces of the four cube, then rotating all the cubes in the same direction.

Exploiting the high degree of interaction with the model allowed by the EM environment, the initial artefact was constructed in an incremental fashion: firstly, by constructing a single cube; secondly, by introducing the input buttons; and then adding the state diagram. The other three cubes were obtained by direct replication of pieces of script (for defining the geometry of the cube and associating procedural actions), and these were then combined to form a single artefact. The testing phase itself demanded a high degree of interaction leading to the revision and refinement of the artefact.

The process of developing the four-cube jigsaw illustrates issues encountered in many processes of product development, both in respect of design and construction. In developing a strategy for manufacturing such jigsaws, it is useful to be able to model the physical operations and intermediate states involved. By building a script using EM, the modeller develops a state-based representation of the entire configuration of cubes that is useful not only when considering the perspective of the solver (the 'user'), but can also be be used to track state-changing operations associated with different human agents and purposes. It is possible to organise the dependencies between cube positions, and so link the movement of one cube to another in such a way as to trace the consequences of alternating lamination of the exposed uppermost surfaces of the cubes with cutting out blocks of cubes, for instance. By such means, different ways of creating a particular puzzle can be examined in a virtual environment prior to building the manufacturing platform. Further discussion and illustration of the way in which EM can be useful in studying a manufacturing process is given in [8].

As a final illustration of the flexibility of the jigsaw artefact, it is worth mentioning a well-known puzzle that can be reproduced by making minor modifications to the artefact in Figure 2. This puzzle, popularly known as 'Instant Insanity' [PI], comprises four blocks, each face-coloured using four distinct colours to its own specific pattern, that are to be arranged in a row in such a way that when viewed from the front, the top, the back and from below faces of all four distinct colours are visible. Once the similarity between this puzzle and a four-cube jigsaw was remarked, essentially the same modifications of the artefact were simultaneously and rapidly made both in the UK and Brazil to create a virtual model of Instant Insanity. This was then used to trace the steps of the graph-theoretic solution to the puzzle described by Ivars Peterson in his Math Trek column [PI] in such a way that the physical transformations involved could be recorded using the Cayley diagrams. The difficulty of this particular puzzle underlines the richness of the potential solution spaces that are associated with four-cube jigsaws, and emphasises the usefulness of modelling that encompasses the product together with its context. It will be of interest to determine to what extent EM artefacts can capture the exceedingly complex solution spaces that can arise in connection with other commercially distributed puzzles. In this connection, an EM artefact of particular relevance is the model of a Rubik's cube that was created by Ben Carter to illustrate the features of the SASAMI 3-d geometric modelling environment. Extending the techniques applied in this paper to model the solution of the Rubik's cube puzzle is a natural subject for future work in this area.

5. EM FOR INFORMATION MANAGEMENT

Despite the dramatic fall in the cost of computer resources, product modelling by computer remains a costly activity. The main overheads in modelling are associated with capturing and managing information about the product - activities that make essential and considerable demands for human involvement and time. By their nature, traditional modelling approaches also raise major issues in information management. Each different aspect of the product modelling typically generates its own set of data and integration is complicated. It is also difficult to modify models incrementally, as is necessary if adaptation to the evolving product is to be possible.

On this basis, computer-based modelling of a product is generally confined to what is prerequisite for its manufacture, and to those factors associated with its use that are critical in respect of safety or efficiency. More comprehensive modelling is only justified for major products with particular significance for society (such as cars, planes and power plants), where large investments are involved. The usefulness of conventional computer-based modelling is also constrained by the availability of suitable mathematical modelling techniques and algorithms, and is generally targeted at specific engineering concerns (such as the aerodynamics and control aspects of an aeroplane) [12].

The technique for product modelling introduced and illustrated in this paper suggests applications of a different nature. For several reasons, the EM approach is better suited than traditional modelling approaches to developing and maintaining more comprehensive models of small products. Using EM, it is possible to model many diverse aspects of the product in a way that promotes integration, and to address issues relating to the design manufacture, structure and use in a single model. In addressing each particular aspect, the main overhead in the modelling activity is in developing the first useful model. Once this point has been reached, a large variety of simple "what-if?" experiments can be performed for little extra cost. It is also typically possible to update the model in a relatively inexpensive manner.

The motivation for comprehensive "lightweight" modelling in the context of a small product can be illustrated with reference to a fanciful but not entirely implausible e-business scenario that might be adopted for the sale of the four-cube jigsaw puzzle. The concept behind this scenario is that, at the same time as we construct and supply actual jigsaw puzzles, we also create and maintain virtual realisations of puzzles that can serve a variety of different roles. For instance, we might wish to record virtual models as one aspect of a catalogue for our puzzle designs, to use them as an interface for evaluation and marketing on the Internet, and to exploit them in connection with repair, change management and maintenance.

To elaborate on this, imagine that we develop virtual jigsaw puzzles that are available for children to manipulate across the Internet. The level of complexity of such manipulation can be constrained to rotating cubes *in situ* or such as to allow unrestricted movement of the cubes in space via a graphical interface. The content of the jigsaw puzzle can be determined by the manufacturer, but could also be to some degree under the control of the user, who could supply images of personal interest to serve as textures. The objective would be to allow a child to gain some experience of a product prior to acquiring its physical embodiment. With this in mind, it might be that certain virtual jigsaw puzzles were available to a particular user for a trial period.

There are several ways in which such an exercise in making virtual products might assist marketing. The degree and nature of interest in solving such puzzles across the net would itself be a potential indicator of the viability of the concept. If there were sufficient interest and use, the potential feedback from usage would be helpful in a whole variety of ways: indicating the popularity and difficulty ratings of different jigsaw designs; suggesting themes that might be of particular interest to children; allowing scope for the experimental investigation of different marketing strategies.

The technical details of the jigsaw modelling exercise described in section 4.2 already illustrate how the degree of flexibility in the model and character of the script representation is suited to the demands of such business activity. The direct correspondence between definitions in a script and observables in an actual four-cube jigsaw establishes a convenient direct connection between the description of the virtual product and its physical realisation. This can clearly simplify the task of cataloguing designs and retrieving their counterpart virtual puzzles, thereby reducing the information management overheads.

The potential benefits of better information management can be further illustrated by our fictitious business concept. In present manufacturing paradigms, especially where products are inexpensive to manufacture, the relative cost of maintenance is so high as to be prohibitive. Though it may be very important to a child to be able to recover a lost piece of a favourite jigsaw, it is normally uneconomic for the manufacturer to produce it. The major cost in this regard is that of identifying precisely what has to be replaced - a problem that could be addressed via Internet access. A customer could relatively easily specify the missing part on the virtual puzzle, for instance, and replacing this particular part, rather than wholesale replacement - might be a solution giving greater customer satisfaction having regard to expense and convenience. In so far as precisely targeted information can eliminate unnecessary production work, such a maintenance model might also be more efficient for the manufacturer.

A particular merit of the EM approach to product modelling in this context relates to possible future extensions of a business concept that rely on variations to existing products. In the case of the four-cube jigsaw, there is evidently scope for using the same manufacturing technology to produce another type of product that could be useful in education at a more advanced level. For instance, pupils who were able to master the manipulation of the cube through the abstract interface supplied by the complex of Cayley diagrams in Fig. 2 would be able to use this as a vehicle for illustrating key concepts in abstract group theory. To develop a product line, for instance, in the form of a software package to be used in conjunction with a physical artefact would involve little extension of the existing modelling activity.

In the above discussion, the potentially unrealistic nature of our illustrative example in section 4.2 is acknowledged, but this does not detract from the general significance of the observations that have been made. On this basis, there is some indication that EM can be helpful as an enabling technology for the new business paradigms that will emerge through applications of the Internet.

6. CONCLUSION

This paper describes an approach, Empirical Modelling (EM), that is well suited for the construction of conceptual representations of the experiential knowledge associated with real-world phenomena. Observation and experimentation are the key principles in EM. EM is essentially informed by the knowledge about the referent acquired during the model construction. This knowledge is used to extend the model and evolves as the modelling process progresses. By exploiting the flexibility offered by an EM environment we can construct models that are always open to extension, refinement and revision. This flexibility distinguishes EM from many conventional modelling approaches. Our case studies and discussion of them demonstrate that EM is a promising alternative approach to modelling products in design processes.

ACKNOWLEDGMENTS

The authors are indebted to Ben Carter, Allan Wong and Michael Evans for practical support that has contributed to the artefacts discussed and developed in this paper.

7. REFERENCES

[1] Allderidge, J., Beynon W.M., Cartwright, R.I., Yung, Y.P., 1998. "Enabling Technologies for Empirical Modelling in Graphics". Proc. Eurographics UK, 16th Annual Conference, 199-213.

[2] Beynon, W.M., 1999. "Empirical Modelling and the Foundations of Artificial Intelligence". Computation for Metaphors, Analogy and Agents. Lecture Notes in Artificial Intelligence 1562, Springer, 322-364.

[3] Beynon, W.M., Bridge, I., Yung, Y.P., 1992. "Agent-Oriented Modelling for a Vehicle Cruise Control System". Proc. ASME Conf. ESDA'92, Istanbul, 159-165.

[4] Beynon, W.M., Rungrattanaubol, J., Sun, P-H, Wright, A., 1998. "Exploratory Models for Open-Ended Human-Computer Interaction". Research Report RR346, Dept. of Computer Science, University of Warwick, UK.

[5] Beynon, W.M., Yung, Y.P., 1992. "Agent-Oriented Modelling for Discrete-Event Systems". Proc. ICC Coll. Discrete-Event Dynamic Systems, Digest 1992/138, June.

[6] Erens, F.J., McKay, A., Bloor, S., 1994. "Product Modelling Using Multiple Levels of Abstraction: Instances as Types". Computers in Industry, Vol.24, No.1, 17-28.

[7] Euwe, M.J., Schuwer, R.V., 1993. "Configuration of Complex Products". Computers in Industry, Vol.21, 1-10.

[8] Evans, M. Beynon, W., Fischer, C.N., 2001. "Empirical Modelling for the Logistics of Rework in the Manufacturing Process". In these proceedings.

[9] Fischer, C.N., Beynon, W.M., 2001 "Empirical Modelling of Products". Proc. WMC01, Phoenix, Arizona (to appear).

[10] http://www.dcs.warwick.ac.uk/modelling.

[11] http://www.lar.ee.upatras.gr/icims/noewww/sotastc.htm

[12] Koenig, J.S., Sharp, W., Linda, K.M., Wells, V., Metzger, R.P, Rodriguez, A.A., 2001. "Development of A Rotorcraft Modeling, Simulation, Animation, and Real-Time Control (MoSART) Environment. Proc. WMC01, Phoenix, Arizona (to appear).

[13] Ness, P., Beynon, W.M., Yung, Y.P., 1994. "Applying Agent-Oriented Design to a Sail Boat Simulation". Proc. ASME Conf. ESDA'94, London.

[14] Sun, P.H., Beynon, W.M., 1999. "Computer-Mediated Communication: a Distributed Empirical Modelling Perspective". Proc. of CT'99, San Francisco, August.

[GM] Grossman, I., Magnus, W. "Groups and their Graphs" The Mathematical Association of America, New Mathematical Library Series, 1965

[IP] Peterson, I. "Averting Instant Insanity", Math Trek, http://www.maa.org/mathland/mathtrek_8_9_99.html, Aug. 1999

[SF] http://sourceforge.net/projects/eden