Empirical Modelling for Learning to Understand Numbers in Non-Decimal Bases

Abstract

Instructionist and constructivist teaching methods are compared, and similarities and contrasts between constructivist learning theories of Ausubel, Papert and Kolb are examined. Problems with the current uses of technology in education, such as software inflexibility and rapid out-dating are discussed, and Empirical Modelling is proposed as a solution to some of these problems. The aims for the Tool to Aid Understanding of Non-decimal Numbers are then introduced, and its stage-by-stage development is documented with relation to educational theory.

1 Introduction

The paper begins by introducing learning and the difference between instructionist and constructivist education. Three different constructivist views are then examined and compared, followed by a discussion of the effect of age on learning and thinking style – with particular reference to the work of Jean Piaget. The concept of 'Learning to understand numbers in non-decimal bases' is then developed in more detail.

The second section of the paper looks very briefly at the current role of technology in education and problems related to IT use in schools. The Empirical Modelling (EM) philosophy is then introduced along with a discussion into how the EM philosophy could be used to transform this role.

The paper concludes with an examination of the development of the 'Tool to Aid Understanding of Non-decimal Numbers' (TAUNN), describing how EM philosophy and tools were employed. Each stage of TAUNN's progress is then documented, with evolving relationships between the model building and learning theories discussed.

2 Learning

There are two fundamental types of learning that will be discussed in this paper; they are rote learning which is the *memorisation* of a fact or procedure, and meaningful learning which is the *understanding* of a concept or principle.

Many examined topics, from long multiplication at a primary school age to search algorithms for computer science undergraduates, can be learned in a purely rote manner. This method of learning, however, does not produce a particularly portable knowledge basis for the learner. Knowing how to do long multiplication will not help when it comes to long division, knowledge of the bubble sort algorithm will not be of use if the learner wants to use insertion sort. For this kind of transferable knowledge to be acquired, the learner must *understand* the concepts behind the procedures.

There are two elementary fields of thought with regards to teaching methods in schools. They are *instructionism*, where the emphasis is on the teacher to 'supply' knowledge to the learner, and *constructivism*, where knowledge is built internally by the learner.

In the past many school-taught subjects, particularly in mathematical and scientific fields, were taught in an instructionist manner. Instructionism was preferred as a method because it allowed the teacher to control the way in which information was imparted. As each student was forced to learn in the same way, this also facilitated simple mass-testing of students' knowledge. The major problem with the instructionist method is that people learn in different ways, and although the concepts are being explained in a way the teacher understands, the learner often only undergoes rote learning of procedures, perhaps increasing their knowledge but not their understanding.

Modern educational theory has undertaken a paradigm shift to favour the implementation of at least some constructivist teaching in these areas. Constructivism promotes the building of knowledge within the learner, and is therefore much more flexible to different learning and thinking styles.

Although this shift in formal education has occurred relatively recently, constructivist views can be traced back to 450BC when Confucis made the famous statement:

> "Tell me and I will forget. Show me and I may remember. Involve me and I will understand"

Work of educational psychologist Jean Piaget (1896 – 1980) and philosopher John Dewey (1859 – 1952) led to the constructivist learning theory. Constructivism has since been developed in a variety of ways. In this paper we will take our investigations further and examine David Ausubel's Subsumption Theory, Seymour Papert's Constructionism and David Kolb's Experiential Learning.

2.1 Subsumption Theory

Ausubel's subsumption theory is concerned with the acquisition of meaningful knowledge about a concept, or 'understanding' it. It states that "the most important factor influencing learning is what the learner already knows." (Ausubel, 1968) For a learner to understand a concept he/she must firstly already have sufficient knowledge for the concept to be potentially meaningful, secondly the appropriate knowledge must be 'activated' (brought to mind) and thirdly that he/she must establish a proper relationship between the new concept and this prior knowledge.

Ausubel's theory has been further developed by instructional psychologist Charles Reigeluth who has identified different types of prior knowledge and the relationships that can be assumed between the new concept and these different types of knowledge. Table 2.1 (from Reigeluth, 1999) illustrates this in more detail.

Table 2.1: Knowledge Types

Knowledge Type	**Relationship with new concept**	**Example**

Superordinate knowledge	Broader and more inclusive.	For teaching the concept of erosion you might relate it to the superordinate concept of movement of material.
Coordinate knowledge	Same level of breadth and inclusiveness.	Erosion might be related to the opposite kind of movement of material - the coordinate concept of sedimentation (the depositing of material in layers).
Subordinate knowledge	Narrower and less inclusive.	Erosion might be related to the subordinate concept of wind erosion.
Experiential knowledge	Specific cases of the new knowledge.	Erosion might be related to the little gully that was formed in the dirt outside the school in the last big rain.
Analogic knowledge	Similar but outside the content area of interest.	Erosion might be related to sanding down some wood.
Causal knowledge	Indicates how something influences or is influenced.	Erosion might be related to its effects on transportation (e.g. washing out dirt roads.
Procedural knowledge	Indicates how something is used.	Erosion might be related to methods of contour plowing for preventing water erosion on farmland.

Reigeluth states that these different knowledge types can be related to different 'dimensions' of understanding, and that a learner is only likely to gain a full understanding if several relationships from this table have been established.

Subsumption theory is popular in schoolroom situations because although it is a constructivism-based learning method, it works well in conjunction with some instructionist learning. The learning process is still structured by the teacher, and testing of skills attained is still possible. Problems still exist with this method of learning, however. To be able to ensure each student is given the necessary knowledge to facilitate understanding, the teacher would have to individually assess each pupil at various stages, which would increase teachers' workload excessively. Alternatives would involve assumptions as to the 'average' prior knowledge of learners, but such assumptions would deteriorate the learning process for some students, and may have social or cultural bias.

2.2 Constructionism

The basic theory of constructionism and how it relates to constructivism is probably best explained by Seymour Papert himself:

> "Constructionism--the N word as opposed to the V word--shares constructivism's connotation of learning as "building knowledge structures" irrespective of the circumstances of the learning. It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it's a sand castle on the beach or a theory of the universe."(Papert, 1991)

So constructionism is a practical, usable method that promotes constructivist knowledge-building by asking the learners to physically build some entity.

Papert's most in-depth study of constructionism allowed learners to construct entities using the software tool LEGO/Logo. The software allowed users to produce lines and marks on a background, but rather than drawing with a mouse or pencil, the students had to use mathematics to produce the correct shape and length.

Although the Logo tool introduced by Papert was successful in some scenarios, it certainly hasn't been able to replace traditional instructionist teaching of mathematics. Students learning with LEGO/Logo gained understanding in some mathematical areas, but were found to be weaker in others, especially with regards to domain learning (Roe, 2003).

2.3 Experiential Learning

Experiential Learning, or learning by experience, is more broadly constructivist than the two previous examples. In this section we will discuss Kolb's experiential learning cycle and how this can be linked to Roe's Experiential Framework for Learning (EFL).

2.3.1 Kolb's Experiential Learning Cycle

Kolb's four-stage model of learning takes the user iteratively through a series of actions that result in the learning of not just the practical procedures, but also the understanding of the theoretical concepts and principles underpinning the experience.

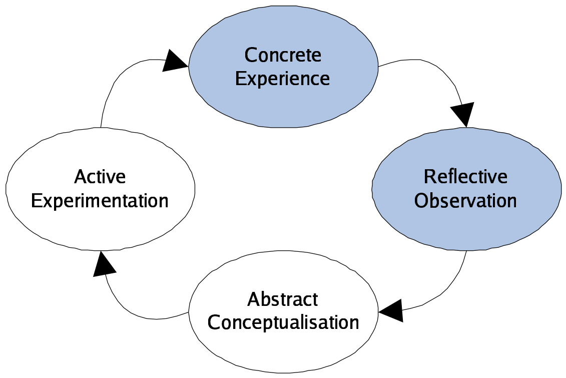

Figure 2.1: Kolb's 4-stage model

According to Kolb's model the learner experiments or plans before their experience, then after the experience a 'reflective observation' period ensues. During this observation, it is thought that the learner hypothesises and theorises to gain some form of abstract understanding of underlying principles, which he/she then uses in the experimental phase of the next cycle.

Atherton (2003) further divided Kolb's four-stage model into 'public' and 'private' experiences. The 'Concrete Experience' and 'Reflective Observation' (shaded) are described as private, whereas conceptualisation and experimentation are public. Roe (2003) suggests that the processes of abstract conceptualisation and in some cases the experimentation/planning phase can be private rather than public, depending on the environment. It is still clear, however, that experiential learning can involve both public and private activities.

The public/private divide is examined further in the Experiential Framework for Learning (EFL), described further below. Much of the background behind the EFL was described in Beynon (1997), but the framework shown was formalised by Roe (2003).

2.3.2 The Experiential Framework for Learning

The EFL describes a framework to classify learning activities on a spectrum between the 'private' and 'public' domain. The idea of the framework is not to describe the process of learning as a linear transition from the empirical to the formal, but to demonstrate how the different types of learning activity can promote one another.

private experience / empirical / concrete

interaction with artefacts: identification of persistent features and contexts
practical knowledge: correlations between artefacts, acquisition of skills
identification of dependencies and postulation of independent agency
identification of generic patterns of interaction and stimulus-response mechanisms
non-verbal communication through interaction in a common environment
directly situated uses of language
identification of common experience and objective knowledge
symbolic representations and formal languages: public conventions for interpretation

public knowledge / theoretical / formal

Figure 2.2: The EFL – from Roe (2003)

The process of moving between learning activities in the EFL can occur in either direction, but is most effective when in happens in both directions. Roe describes the movement of a learner from the empirical to the theoretical as 'abstraction' – the formalisation of learning, whereas the movement from theoretical to empirical is 'concretisation' – gaining familiarity with the practical uses of theory.

The EFL seems to provide a more realistic model for learning than the Kolbian cycle. Kolb has been criticised for ignoring the different types of learner in his cycle, which seems to imply that each individual would undergo the same cycle of experiences.

A clear relationship between the EFL and Papert's constructionism can also be seen, constructionism would be described by the upper experiences in the EFL, as detailed in Roe (2003). Ausubel's subsumption theory could be seen to be linked to the lower two rows of the EFL, particularly *identification of common experience and objective knowledge*. This area of the EFL takes the constructivist roots of Ausubel's theory but allows it to work in a personal learning environment as well as in a teacher-led situation.

2.4 Learning and Age

We have discussed the different types of knowledge and learning activity but as yet have not questioned the effect of the learner's age on their ability to learn. Piaget classified learners into four basic stages, each with several sub-stages, described in the table below (from Atherton, 2003).

Table 2.2: Piaget's Stages of Cognitive

Development

Stage	Characterised by
Sensori-motor (Birth-2 years)	Differentiates self from objects. Recognises self as agent of action and begins to act intentionally. Achieves object permanence: realises that things continue to exist even when no longer present to the sense
Pre-operational (2-7 years)	Learns to use language represent objects by images and words. Thinking still egocentric: has difficulty taking viewpoint of others. Classifies objects by a single feature.
Concrete operational (7-11 years)	Can think logically about objects and events. Achieves conservation of number (age 6), mass (age 7), and weight (age 9). Classifies objects according to several features and can order them in series along a single dimension.
Formal operational (11 years and up)	Can think logically about abstract propositions and test hypotheses systematically. Becomes concerned with the hypothetical, the future, and ideological problems

Recent educational theory has found Piaget's classification too rigid to be used in a real-world teaching situation. Many children can be shown to reach higher stages of cognitive development at a much earlier age (Carey, 1985). The fourth stage of development 'formal operation' is a particularly debated phase. It is argued that many people, and in some cases whole cultures of people, never reach this fourth stage of development – or at least never require the abilities characterised by this stage (Atherton, 2003).

It is argued further by Carey (1985) that, in some very specific areas, it can be shown that children are not "Fundamentally Different Kinds of Thinkers and Learners" than adults, sometimes even as young as age 3. Generally, however, it is clear that the brains of children work in a very different way to that of adults.

2.5 Non-Decimal Base Numbers

The aim of this paper is to look primarily at the understanding of non-decimal base numbers. The term 'understanding' in this sentence is particularly relevant – a procedure to convert decimal to binary could quite easily be learned without any true understanding of number bases, or why this procedure works.

The understanding of number base is certainly traditionally understood through instructivist methods of teaching. Unlike other mathematical concepts, such as division or simple calculus, the concept of the existence of non-decimal number bases requires a kind of 'paradigm shift' for the learner. For this reason, the Tool to Aid Understanding of Non-decimal Numbers will be aimed only at individuals in the 'Formal Operational' stage.

Section 4 of this paper will discuss the actions taken during the development of the TAUNN model to attempt to ensure the learner *understands* the concepts, rather than just memorising a procedure.

3 Empirical Modelling and Education

The TAUNN model was built using Empirical Modelling (EM) principles and tools, which are strongly based on the philosophy of William James, 'Radical Empiricism'. This section will therefore begin with a very brief overview of Radical Empiricism and its effects on EM, before discussing the use of technology, and particularly EM in the field of education.

3.1 Radical Empiricism and EM

Traditional empiricists took the view that knowledge can only be acquired through experience, especially the experience of the senses. William James took this empiricist view further, believing that reality itself was made from "pure experience", and that experiences were made up from an accumulation of past experience.

These philosophical views can be linked back to the learning theory introduced in section 2, where, for example, Reigluth described the acquisition of understanding as an accumulation of knowledge and inter-knowledge relationships. It is obvious that the EFL and its 'layering of experiences' is based on James' Radical Empiricism, and EM principles in turn have foundations in the EFL, and therefore Radical Empiricism.

3.2 Technology and EM in Education

Common problems with technology in education can be viewed from a variety of perspectives, as seen in Beynon (1997). Logistics and finance mean that schools cannot afford to have software constructed for teaching, and commercial software is often out of date too soon to make its purchase financially viable. Teachers tend to alter and combine other types of teaching materials to suit their students and teaching style, but with IT packages this simply isn't possible. From a student perspective, educational software available is often too limited in scope – particularly for those used to playing commercial games, which contain highly complex algorithms to make the player feel they have unlimited choices at every stage.

Empirical Modelling attempts to remove some of these problems by undertaking software development from a totally different perspective. Four major differences are listed below, as described in Beynon (1997)

2 Developing systems in ways that allow flexible adaptation, even by users who are not computer specialists

3 Developing techniques that create machine-independent software.

4 Emphasising the creation of 'artefacts' that exploit computer-based technology, rather than applications to produce predefined functionality based on analysis of users.

5 Supporting an open-development rather than closed-world engineering culture.

Empirical Modelling aims to produce flexible models that can be explored by the users, whoever they may be, in whichever way they choose. This philosophy hopes to encourage exploratory learning to build experiential knowledge and promote conceptual understanding.

4 The TAUNN Model

4.1 Building the Model

EM allows the modeller to create a tool or model without any traditional computer science 'specification' – with the aesthetics and functionality of the model emerging gradually during the modelling process.

The 'aim' of the modelling experience here was to produce a tool to aid 'the understanding of non-decimal numbers' – by which we mean numbers represented in a base that is not 10.

The learning theory and empiricist philosophy discussed throughout this paper was frequently referred to during the model building process and conscious links between the background study and each stage of the model are mentioned in the commentary below.

The model was built as a series of studies, each of which is described in more detail below.

4.2 TAUNN as a Learning Tool

4.2.1 Study 1 – Initial Exploration of the Tool

As tool building was undertaken in an exploratory manner, the first study is not a particularly useful learning tool. The user enters a decimal number which is then converted into binary, octal, and hexadecimal. These conversions are unlikely to make any sense to a learner who is completely unfamiliar with the concept of multiple number bases.

The tool could be used as a 'refresher' tool for learners who already have understanding of the multiple-number-base concept but have lost familiarisation due to lack of concretisation.

4.2.2 Study 2 – Counting in Number Bases

Study 2 (shown in figure 4.1) adds an 'increment' function to the Study 1 tool. We are introducing the binary, octal and hexadecimal bases to the learner – and wish, if possible, to create a relationship between these bases and the decimal base with which the learner is assumed to be familiar.

As knowledge of the decimal base would have been initially gained when the user learned to count as an infant, we would ideally like to build the knowledge of the other new bases in the same way, by incrementing or counting. This uses Reigluth's example of linking 'coordinate' knowledge.

Once knowledge of each individual number base has been imparted, we hope to create a relationship between these subordinate or experiential knowledge bases and our learning aim. In the EFL, this experience of moving from 'knowledge of a number base' to 'understanding of the concept of number bases' would be described as 'abstraction' through the framework.

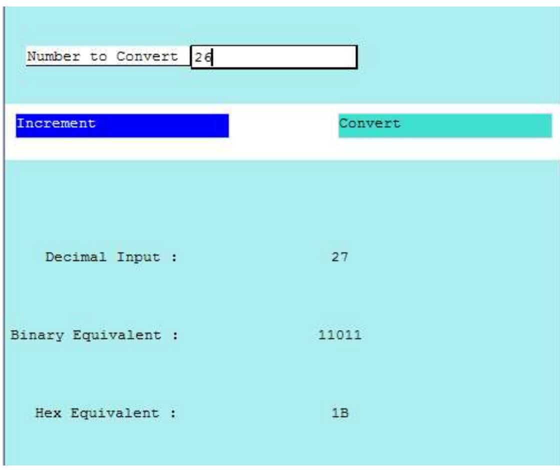

Figure 4.1: Study 2

4.2.3 Study 3 – Converting to Binary

The EFL (Roe, 2003), described in section 2, emphasises the importance of experience and involvement in learning. In the initial studies the user has been 'in control' of the environment to an extent, but has not been *involved* in the process of converting between bases.

To aid the learner in the 'reflection' stage shown in Kolb's four-stage model, the tool gives feedback, comparing the input number to the correct conversion value.

4.2.4 Study 4 – Multiple Number Bases

The most fundamental problem with the Study 3 tool is that it does not necessarily introduce the learner to the concept of 'number base', as only decimal-to-binary conversion is the only procedure considered. The user could simply rote-learn a decimal-to-binary algorithm without considering the possibility of further bases. Study 4 adds hexadecimal-to-decimal number conversions to the question list as a concretisation method for the formal concept learning required to understand.

Hexadecimal was chosen as the next number base to be introduced as it is the next most commonly used non-decimal base, and it emphasises that post-9 digits can be used in numbers.

Study 4 could possibly be used after the learner gains a background of the hexadecimal number base by using Study 2.

4.2.5 Study 5 – Focussing the Tool

Study 4 fully involves the learner in the conversion process, and has introduced multiple number bases to try to prevent rote learning. The actual conversion in Study 4, particularly to hexadecimal, however, is a fairly complicated mathematical process. The tool is not aimed to aid mental arithmetic so input digits are separated, with the value of a '1' in that digit labelled above.

If the tool was used in schools, Study 4 may be preferred as improved mental arithmetic is not necessarily an adverse co-product of the understanding of number bases. It is likely to de-motivate users without a particular enjoyment in mathematics, however, so the addition of hint tools in a general environment seems sensible.

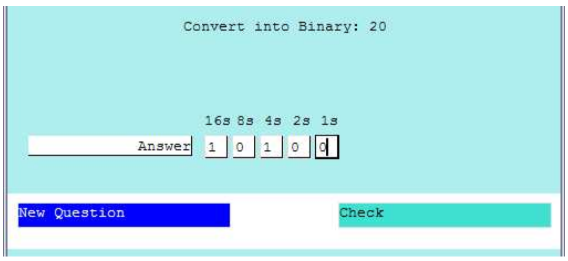

Figure 4.2: Study 5

4.2.6 Study 6 – Adding Octal

A further number base, octal, is added here to emphasise the existence of multiple usable number bases. The tool now introduces the learner to binary (almost a concept in itself, as most people have some knowledge of binary, even if only as '1s and 0s that computers understand'), hexadecimal which emphasises that 'we don't have to stop at 10 digits', and octal, showing that between binary and decimal we have 7 intermediate integer bases.

4.2.7 Study 7 – Learner structures learning

In study 7 the learner can choose to convert numbers from any one of the available bases, or be asked to convert any non-decimal based number at random. This addition to the tool could be used in either a teacher-student or learner-controlled environment as a concretisation step.

In a schoolroom environment it is likely that the teacher would want the student(s) to grasp a basic knowledge of number conversion in one non-decimal base before any conceptual abstraction can take place. Younger learners may find this kind of concretisation particularly useful if, for example, they can be described as one of Piaget's Concrete Operational learners, as formal abstraction is only 'introduced' at the Formal Operational level. The teacher could either simply ask the students to select a particular button, or make some very minor alterations to the source files to remove additional buttons altogether.

In a personal learning situation a user may find one of the three non-decimal bases particularly difficult to work with, if only for arithmetic reasons. The continued interaction with numbers in this base by selecting the corresponding button would improve the user's familiarity, and expectantly aid in the abstract conceptualisation stage.

4.2.8 Study 8 – Session Feedback

Feedback now relates to learners their overall progress rather than just providing comments at an individual question level. In a schoolroom environment the learner could now be asked to do, for example, ten questions in each base and record the totals for each. The results from this kind of exercise could then be used to structure a future learning session around the individual's needs.

4.3.9 Study 9 – 'To Decimal' Conversion

Rather than being a progression from the previous studies, the final study creates a tool that could be used in parallel with 'from decimal' tool created in Study 8. The tool now produces a binary, octal or hexadecimal number which the learner is asked to convert into the classic decimal number base. This model should ensure the learner comprehends the commutative nature of base conversion, expectantly aiding their conceptual understanding of the number base. The non-decimal and decimal numbers have been swapped exactly from the study 8 model, with "100s 10s 1s" displayed as in the previous models.

This inclusion should reinforce the fact that decimal is just another number base, as a final knowledge relationship to reinforce understanding.

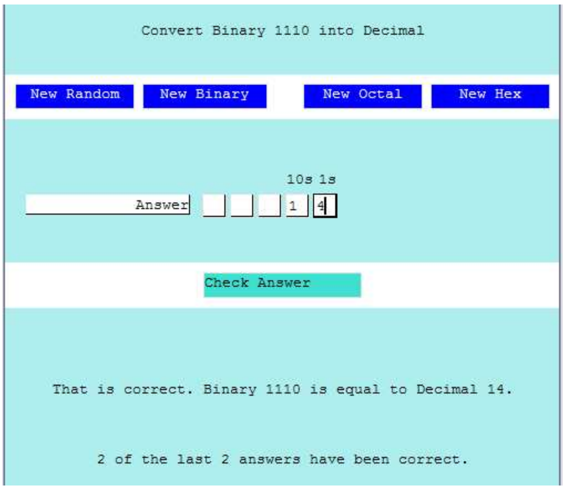

Figure 4.3: Study 9

5 Conclusions

This paper has investigated various constructivist educational theories and argues that the application of a combination of these theories is probably most likely to create a truly successful learning environment. Piaget's four-stage model of cognitive development is discussed along with some of its criticisms. It is concluded that the TAUNN model should be aimed primarily at people of 'formal operational' stage.

The current problems with the use of technology in education have been described, and the use of Empirical Modelling to overcome these problems, as suggested in Beynon (1997), is proposed. We conclude that Empirical Modelling as a concept is a very promising direction in the constructivist teaching area.

The exploratory development of the TAUNN model was discussed and the stage-by-stage commentary illustrates the use of this observation-based method. The usefulness of the tool at each stage was examined, particularly with reference to the educational theories introduced in the paper. It is decided that TAUNN could definitely be used as a concretisation aid to learning the concept of number bases, but is realistically unlikely to be useful to a learner who has had no former introduction to the concept.

Future development of the TAUNN model could advance in a variety of ways. Feedback could be adapted to include 'hints' of a more conceptual nature, for example advising a user if one of the digits they entered was not available in that base, or written explanation of the concepts involved could be included.

Another possible development route for TAUNN would be to include functionality to teach the user to perform basic calculations in different bases, for a more practical and experimental learning experience. It may also be useful to provide a more user-friendly interface, which could include analogic knowledge: for example, a binary digit as a light switch, particularly if the tool was being used by children.

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