

Teaching trigonometry using Empirical Modelling

0303417

Abstract

The trigonometric functions $\sin(x)$, $\cos(x)$ and $\tan(x)$ are relationships that exist between the angles and length of sides in a right-angled triangle. In Empirical Modelling terms, the angles in a triangle and the length of the sides are observables, and the functions that connect them are the definitions. These well-defined geometric relationships can be useful when teaching GCSE-level students about the functions, as they provide a way to visualise what can be thought of as fairly abstract functions. This paper looks at how different learning styles apply to Empirical Modelling, and presents a practical example of their use in a model to teach trigonometry.

1 Introduction

The trigonometric functions $\sin(x)$, $\cos(x)$ and $\tan(x)$ are relationships that exist between the angles and length of sides in a right-angled triangle. In Empirical Modelling terms, the angles in a triangle and the length of the sides are observables, and the functions that connect them are the definitions. These well-defined geometric relationships can be useful when teaching GCSE-level students about the functions, as they provide a way to visualise what can be thought of as fairly abstract functions. Rather than teaching students by showing them diagrams in an instructive way (already a good way of doing it), a constructive approach may allow students to gain a better understanding (Beynon).

Empirical Modelling upholds a set of principles that are in some ways similar to the different learning styles identified by Felder and Silverman (1988). This paper explores the similarities and differences between Empirical Modelling and a selection of the learning and teaching styles, with a view to creating a practical model. Schools are keen to promote the development of ICT skills alongside learning (OCR, 2007), so a model that teaches trigonometry through interaction with a computer will be useful.

The model developed to accompany this paper makes use of the Empirical Modelling Presentation Environment (Harfield, 2007). This is a GEL model that allows the creation of interactive presentations using HTML slides, an EDEN input box and a DoNaLD window. The slides can be used to explain a concept in words, and the DoNaLD window

can provide an interactive visual explanation. Some interaction can be gained through entering EDEN commands in the input box, but for the sake of usability, most of the interactions described in this paper will be modelled with mouse input to the DoNaLD window. Various changes have been made to the Empirical Modelling Presentation Environment to aid usability for non-technical people; these are described later in section 4.

2 Empirical Modelling and learning styles

Felder and Silverman give a very good analysis of learning styles observed in engineering students, and the way these differ to traditional teaching styles (1988). They are some of the most widely cited authors regarding learning styles, and the model presented in this paper is developed in light of their conclusions. It is interesting to see which of the learning styles the Empirical Modelling principles encourage.

2.1 Visual over verbal learning

Although much of Empirical Modelling is concerned with writing definitive scripts – primarily a verbal exercise, the models generated very often have a clearly visual interface. This may be because the models are based on the behavioural characteristics of a referent in the world, and we generally interact with or observe referent objects first by looking at them. A visual diagram can convey a large amount of information much more quickly to the

user than a written set of definitions can. That's not to say writing definitive scripts isn't important, the act of trying to verbally describe (in terms of EDEN observables and dependencies) a referent is a significant aid to understanding.

However, in this paper the concern is with the students who will be using rather than creating the model. Using the Empirical Modelling Presentation Environment will allow for both visual and verbal teaching styles to be applied. There is an interactive DoNaLD graphics window for visual diagrams in addition to the written slides that form the presentation. The written slides can provide words of explanation to help direct the student in their learning and understanding.

Note that while it seems correct to use the term kinesthetic learning to describe learning through interaction, this term actually refers to using the sense of touch (and smell and taste) during perception, and information processing involving physical activity while learning (Felder and Silverman, 1988). Neither of these are sufficiently described by typing or moving a mouse.

2.2 Inductive over deductive learning

The inductive learning style is the most natural for humans and probably all other creatures. It involves perceiving the world and forming theories and predictions in the mind about how it works. This is the principle behind Empirical Modelling; during the modelling process the modeller is recreating the significant behaviours of referents in the world, and in doing so are constructing theories about how the referents work. This is an important way to grasp an understanding of behaviour, by trying to implement it and learning from the mistakes and differences observed.

However, a deductive learning style is also useful in Empirical Modelling. Deductive learning stems from being made aware of the laws that govern entities (or simplifications thereof), then through an understanding of these being able to deduce possible actions and consequences. This corresponds to finding an existing model and using the definitions provided to understand what the author intended. By stating the trigonometric functions in a concise and memorable way, students are enabled to use the rules to solve a variety of problems, and see ways in which the rules relate to other things. Felder and Silverman recommend using first an inductive style of learning then a deductive one, because this is similar to the scientific method (Felder and Silverman, 1988). Using concrete examples can intrigue a student into finding out how they are linked, then

providing the theory and asking for applications will build on the understanding the student already has.

2.3 Active over reflective learning

Active learning involves participating in a problem solving activity through group discussion or individual working, whereas reflective learning involves having time to inwardly reflect on the information learned, increasing understanding and committing it to memory. In this case the choice of teaching styles named active and passive do not correspond to the active and reflective learning styles. Active teaching allows for both learning styles, but passive teaching does not give the students any chance to apply themselves either by actively discussing/problem solving or by uninterrupted thinking over the acquired knowledge. Empirical modelling is an active teaching method that allows for both learning styles. Users can be actively modelling, but there is also the need to stop and reflect on what has and will be modelled.

Felder and Silverman (1988) describe other learning and teaching styles, but in the interest of space, these three comparisons are the most important and applicable.

3 Teaching trigonometry using Empirical Modelling principles and tools

While these Empirical Modelling learning principles could be usefully applied to many subjects, in this paper just one is studied. To see how this model of learning can be applied to teach a specific subject, the relevant learning outcomes of a syllabus taught in schools must be identified to set the scope of the model.

The OCR GCSE Maths syllabus contains the following criteria regarding trigonometry:

“... plot graphs of: simple cubic functions, the reciprocal function $y = 1/x$ with $x \neq 0$, the exponential function $y = k^x$ for integer values of x and simple positive values of k , the circular functions $y = \sin x$ and $y = \cos x$, using a spreadsheet or graph plotter as well as pencil and paper” (Sequences, Functions and Graphs H2.6f, page 30) (OCR, 2007)

“... apply to the graph of $y = f(x)$ the transformations $y = f(x) + a$, $y = f(ax)$, $y = f(x + a)$, $y = af(x)$ for linear, quadratic, sine and cosine functions $f(x)$ ” (Sequences, Functions and Graphs H2.6g, page 30) (OCR, 2007)

“... understand, recall and use trigonometrical relationships in right-angled triangles, and use these to solve problems, including those involving bearings, then use these relationships in 3-D contexts, including finding the angles between a line and a plane (but **not** the angle between **two** planes or between **two** skew lines); calculate the area of a triangle using $\frac{1}{2}ab\sin C$; draw, sketch and describe the graphs of trigonometric functions for angles of any size, including transformations involving scalings in either or both the x and y directions; use the sine and cosine rules to solve 2-D and 3-D problems” (Geometrical reasoning H3.2g, page 33) (OCR, 2007)

The Edexcel GCSE Maths syllabus contains wording identical to these quotes in the following sections: Sequences, Functions and Graphs 6 f) and 6 g) (page 69) and Geometrical reasoning 2 g) (page 71) (Edexcel, 2004).

A key thing to note from these criteria is that students should understand both the relationships present in right-angled triangles and the shapes of the graphs. It is important to be able to link the two; simply relating the functions to the angles and sides doesn't produce enough understanding (Orhun). While Orhun's study doesn't use a particularly wide sample of students, the conclusion is general and can be applied here.

Following the inductive learning style before the deductive style, the material should first be presented to the students as a series of examples before explaining the theory behind it. However, this has proved difficult to apply in practice and as such the concepts and functions are given explanation as they are introduced.

Allowing students to interact with the model as they are learning is one of the key Empirical Modelling principles. This can be achieved by allowing mouse movement and clicks on the DoNaLD window to change observables. It would be good to encourage students to write EDEN definitions into the input box, as this may give more freedom to experiment. However it is also more difficult than using a mouse to interact. Different techniques are explored in the different lessons.

Using these requirements, a number of lessons can be mapped out. These lessons involve a single interactive visual DoNaLD model, but can span several HTML slides.

3.1 Lesson: radians and degrees

Different functions in EDEN and DoNaLD use either degrees or radians as the unit for measuring angles. To ensure students are able to create definitions using these functions later on, the relationship between degrees and radians should be explained. This can be demonstrated by allowing a student to change the angle between two lines, and displaying the angle as both radians and degrees. Significant angles such as 30 degrees ($\pi/6$ radians) are highlighted. Inductive learning can be used here to encourage the student to calculate the relation between degrees and radians before they are told what it is.

3.2 Lesson: identifying the sides of a triangle

The well-known geometric relations ($\tan(x) = \text{opposite} / \text{adjacent}$ etc) refer to the sides of the triangle in relation to the angle being used. For the students to be able to use the functions, they must be able to identify the hypotenuse, opposite and adjacent sides in relation to a specific angle. They can hover the mouse over the different sides of a triangle, and a label will appear indicating which side they are pointing at.

3.3 Lesson: learning the relations between side length and trigonometric functions

The next step is to learn the trigonometric functions in terms of the different sides of the triangle. At this stage it is important for the student to be able to rearrange the equation, so moving the mouse over the sides of a triangle will bring up a label showing how the equation would be rearranged to calculate the particular side.

3.4 Lesson: Finding a length given an angle and another length

Once the students have learned about the functions they can cement the knowledge by applying them in problem solving exercises, using the active learning style discussed above. This will involve calculating the length of a side, given an angle and the length of another side, by selecting the correct function and rearranging the equation. This takes the form of writing an EDEN definition into the input box and checking the evaluation of the expression.

3.5 Lesson: Drawing graph of the trigonometric functions

It is important to be able to understand how the graphs of the trigonometric functions relate to the angles and sides in a triangle (Orhun). This can be

demonstrated by using a unit circle, allowing the student to change the angle within it using the mouse (as in lesson 1 on radians and degrees) and displaying the result of the functions at that angle on the graphs. The range of the functions can be linked to the lengths of the sides of the triangle.

3.6 Lesson: Transforming graphs

Additionally, the GCSE syllabus expects students to be able to transform the trigonometric graphs using a single coefficient to multiply by or add to the angle and function (OCR, 2007). By typing in different equations, students can try and match graphs by transforming a standard graph into one that is already being displayed.

3.7 Lesson: Inverse functions

The students are able to calculate the length of a side using an angle and another side, but they should also be able to calculate an angle using two of the sides. This will be done in a similar way to finding the side length in lesson 4 above, by creating a definition using a rearranged expression involving `atan()`.

There are more concepts that could be covered, such as the sin and cos rules for calculating area, or an extension of the trigonometry functions into 3D. The lessons outlined above demonstrate the main Empirical Modelling principles used for learning – interacting with diagrams and graphs and allowing students to create their own definitions.

4 From a teacher's perspective

A teacher may want to modify the given presentation for example to emphasise a topic found particularly difficult by his or her students, or to tailor the content to a particular syllabus. It is desirable for this to be achieved without much technical knowledge (Beynon), so the task has been made easier by some changes to the structure of the Empirical Modelling Presentation Environment.

Firstly, `run.eden` automatically includes `intro.eden`, so the relevant code from `run.eden` has been copied into `initialise.eden`. This allows `initialise.eden` to be called at the beginning of many separate eden files, each defining their own presentation. Secondly, the method for including HTML slides in the presentation has been made more intuitive. Instead of hard-coding the HTML in files such as `intro.eden`, a model named `loadPresentation.eden` has been created which will load a presentation from a file

containing a comma separated, quoted list of *.html files. `loadPresentation.eden` automatically includes `initialise.eden`, and requires the EDEN variables `lessonPath`, `presentationSlideList` and `presentationModels` to be predefined. This allows a teacher editing the presentation to, for example, create `mylesson.eden` which simply sets these variables, call `cd("empeHarfield2007");` and `include("loadPresentation.eden");`. There is an increase in the number of files needed, but the complexity of the files edited by the user is decreased.

While the teacher can modify the lesson content by editing HTML files, to modify the exercises would require knowledge of EDEN. This is probably beyond the capabilities of most teachers, because EDEN (combined with DoNaLD) is not an easy language to use. It may be possible to construct a repository of exercises from which a teacher can download models to insert into the presentation as exercises. This would however introduce a problem of redundant variable names – the variable identifiers in the current exercises are already convoluted to avoid such conflicts.

5 Evaluation and further work

The model uses various forms of interaction to aid understanding. The interactions on the DoNaLD window work well, because they provide instant feedback. There were issues with the exercises that allow students to write EDEN definitions themselves. With the tan exercise to find a side length from another side length and an angle, the observable names were kept simple for users to enter them into equations (such as BC for a side length and A for an angle). However, these identifiers may have already been in use, for their definition caused problems to appear with the Empirical Modelling Presentation Environment. There was not enough time to fix these, so the exercise has been left unimplemented. This is a general issue though, providing simple variable names for users increases the likelihood that another variable of the same name is already being used.

The sin graph transformation lesson works better, in that users can type or copy to the input box an expression such as $y = \sin(x * 3) + \cos(x) * 4$; and it will be displayed on the graph. The issues that arise here are firstly that the graph will not update automatically. Introducing a constant zero dummy term into the `f<I>` function of the graph solved this initially, because setting the dummy term

to zero would update the dependency and the graph would redraw. However, to draw a user-created graph, a function to evaluate the user's expression at different values of x was required. Attempting to set a dummy variable in a procedure that was called on the update of y would cause an infinite loop, because the redrawing of the graph would itself change y . This is solved again by making the user set the value `updateGraph = 1;` through the HTML slide, but is less than ideal. The second issue is that users must remember to append semicolons to their expressions, otherwise relatively unfriendly error messages appear. This is not intuitive to anyone who hasn't programmed in a C-like language before.

Due to time constraints, not all of the lessons described in the paper were implemented. Lessons describing the sin and cos functions are not present, but would be almost identical to the lesson describing the tan function, in that a rearranged equation would be shown to calculate a side or the hypotenuse. There is no explanation of the tan graph, although it can be viewed during the sin graph transformation lesson. The first task during further work to this model would be to complete the implementation of these lessons.

The next step after this would be to present this model to maths teachers in high schools and allow them to use and evaluate it. The author of this paper has had no training in education, and there are likely to be improvements that can be made by someone more experienced at teaching maths to children. There are sure to be better ways of explaining things, and more intuitive ways of presenting the exercises. Also this would provide a usability test to gauge interest for use in schools.

There are a few security issues with the model, and with the Empirical Modelling tools in general. Currently there are no constraints on what the students may edit using the EDEN input box – they can cause havoc with the slides or the exercise models (if they can find out what the observables are), and edit the HTML of the slides directly by clicking the 'Edit slide' button. It is hoped that students would not go out of their way to disrupt the model, this would be more indicative of an attitude problem in the student rather than a problem with the model integrity.

Another usability issue that occurs is that the Empirical Modelling Presentation Environment appears to take a fairly long time to load – up to twenty seconds. This is long enough to be perceived as hanging or crashing by today's impatient students.

The model that has been produced is not something that couldn't have been made without the Empirical Modelling tools. However, it uses the Empirical Modelling principles in conjunction with ideas about different learning styles to create a learning tool with an emphasis on understanding through interaction. It provides insight into how one might go about designing a lesson plan with this kind of computer interaction in mind, and covers important parts of a trigonometry syllabus as a practical example.

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