

## Lecture Th2: Observations, Continuity and Events

### Th2.1: Background and Motivation

How should we understand the computational framework of the ADM? Seems to be appropriate only for discrete event simulation. Yet we have a need, as in the VCCS, to represent continuous variables and to consider the implications of modelling real-time processes. We also need to interpret events, such as the "station master has whistled" event in the railway station simulation that are properly conceived in terms of instantaneous changes of state.

Misleading to regard the cycles of the ADM as clocked in the manner of conventional machine cycles. In definitive programming, time is not a primitive concept. When we use a script to model the physical relationships in a Hooke's Law experiment, for example, we aren't concerned at all about *when* observations are made. In the ADM model, the transitions between machine states are associated with points of transition between states of the experiment. In reality, these are not instantaneous, but the experimenter is not concerned about what happens between observations, when perhaps the load is oscillating on the wire.

Much the same considerations apply to the railway station animation: certain activities are modelled as atomic, though in fact they take time, and might involve activities that disclose hidden potential for interference (...), or indicate how interference is resolved (e.g. physical restrictions on how many people can access a door handle beyond the scope of the specification). This points to a more appropriate interpretation of the computational states of the ADM: each state represents a particular family of conceptually instantaneous observations. In other words, what the ADM models is subject to a convention for observation.

In what sense is the discrete nature of the ADM computational model a limitation? Consideration suggests that there is a fundamental sense in which it cannot be otherwise. Processes may be continuous, but observations are discrete. As classical mathematicians discovered in formally describing continuity, quantification over sets of discrete observations is necessary. For instance:

" $f(x)$  is continuous at the point  $t$  if given  $\epsilon$ , there exists a  $\delta$  such that ..."

In other words, continuity is a statement about properties that  $f$  when we are free to choose the convention for observation. To paraphrase:

"Tell me what you want to see and I will show you how to observe it".

This points to a mode of interpretation of the ADM that is entirely different from the conventional fixed duration machine cycle. Instead, we think of the ADM as modelling discrete observations made at instants of time that can be arbitrarily close at the discretion of the designer. Notice that to exercise this discretionary power, the designer exploits the privilege to intervene in computation in an exceptionally strong sense; viz. as if to change the mode of machine execution in response to the results of computation.

A danger of circularity lurks in this informal account of how processor speed of the ADM can be adjusted at run-time. We began by disassociating the ADM from time, proposing that observation rather than time is the more appropriate primitive concept. It would be quite inappropriate to invoke time in modelling Hooke's Law. Yet we have expressed our prescription for more frequent observation in terms of "instants of time that can be arbitrarily close".

To eliminate this circularity, we instead choose to introduce time as a particular kind of experimental observation, as an experimenter necessarily does when performing an

experiment that essentially involves time (such as measuring the period of oscillation and speed of damping to equilibrium for a load on a wire). To this end, the observations modelled in the ADM include the observations of a clock, and all observations are relative to the current observed value of the clock.

Analogue variables and ' values relative to clock. Necessarily involves approximation, but also eliminates suspicion of circularity in definition. Value *now* is defined with reference to value *then*. Presumption that in all observations the clock is consulted, and can be consulted arbitrarily frequently. The faithfulness of a simulation will be subject to caveat that appropriate pattern of behaviour will be observed "if the clock step for observation is small enough". Events fit into this model as conditions expressed in terms of "x is this and y was that" variety, as in

$$x==1 \text{ and } y'==0.$$

Similar considerations apply to modelling of analogue quantities in geometry, as the lines demo illustrates. In that context, the analogue of the clock step size is the arithmetic precision. The interpretation of the lines demo presumes that the arithmetic precision is a parameter that can be adjusted on-line, in response to perceived conditions governing the proximity of singular configurations.

What has been described here in the abstract accords well with the way in which analogue variables are interpreted and implemented in the VCCS. Notice in particular that there is a distinction between the sampling rate for the integrator that represents the digitisation in the speed transducer and the step-size for the integration used to model Newton's Second Law. The first of these parameters reflects a choice in on the part of the designer, based on engineering considerations. The second is the parameter that can be chosen arbitrarily small in response to observational demands.

Issues for contemporary computer science  
from idealised model to observations  
lines demo again  
how analogue variables in the VCCS work  
semantics for analogue variables and events