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Walling Shape and Materials Usage - Plane, Buttressed, Crenelated and Wavy Walls

By T.H.Thomas, Warwick University, 2016

ABSTRACT

A straight long boundary wall should generally have a slenderness ratio of less than 14 to attain sufficient lateral strength and stiffness. Thus a 2.4m wall would need to be 17cm thick, i.e. full brick (20 cm) rather than half brick (10cm). A thinner, cheaper, half-brick wall could be used but requires buttresses placed typically 3m apart. There are however two proven alternatives to buttressing, namely employing a wavy or a crenelated wall plan, that use less material but may incur such penalties as creating rooms that are hard to furnish or boundaries that do not match plot ownership. The paper identifies 4 structural and 3 non-structural performance criteria for walls and against these compares the 4 wall-plan shapes listed in the title. Taking a 20 cm full-brick wall as datum, the relative stiffness *per brick used* for a 40cm-deep buttressed wall is about x3 and the relative lateral strength is about x1. For 40cm-deep crenelated walls, the corresponding ratios are x6 and x3. For 40cm-deep wavy walls the corresponding ratios are x13 and x4. Thus thin wavy and crenelated wall plans offer significantly better performance 'per brick' under lateral forces than do thin buttressed or thick straight walls.

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1 Introduction

This ELITH Working Paper is one of a set (WP IIB-4) examining the energy embodied in walling and how it might be reduced. WP IIB-4-1 (Walling stiffness and strength in tropical housing) developed a simplistic analysis for plane walling. This paper, WP IIB-4-3, uses that to make a comparison of three other walling plans. For purposes of comparing these different plans (shapes) we can take as a datum a stretcher-bonded ‘half-brick’ wall of thickness t_D = 100mm (Fig 1a below).

Fired-clay bricks, mortar and lean concrete blocks all have a high (per litre) energy content and carbon footprint, so there should be a strong incentive to using both only sparingly. One interesting option is to build mainly in half-brick thickness but to a wall plan that somehow gives the stiffness of a full-brick wall. The potential saving of material, firing energy and carbon footprint is up to 50%.

Fig 1a Plane wall – Half-brick thickness

Fig 1b Plane wall – Full-brick thickness, suitably bonded (*m* = 2)

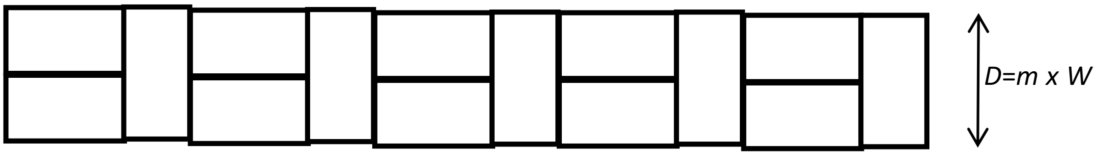


Fig 1c Buttressed wall (example: *n* = 5, *m* = 3)

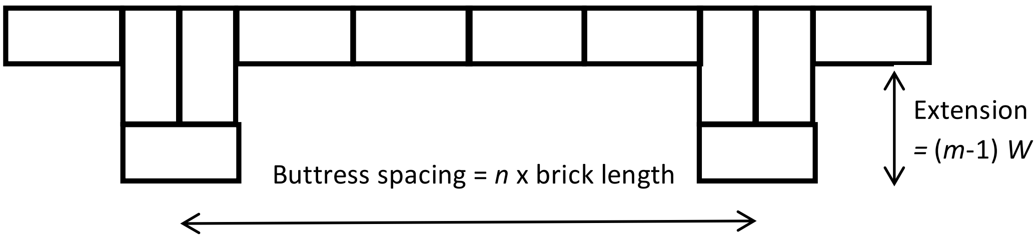


Fig 1d Crenelated wall (example: *n* = 5, *m* = 3)

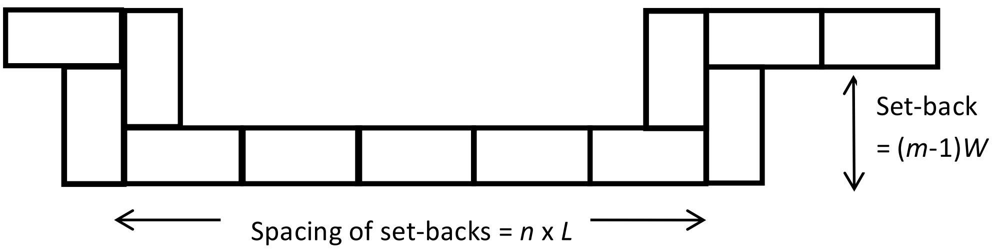
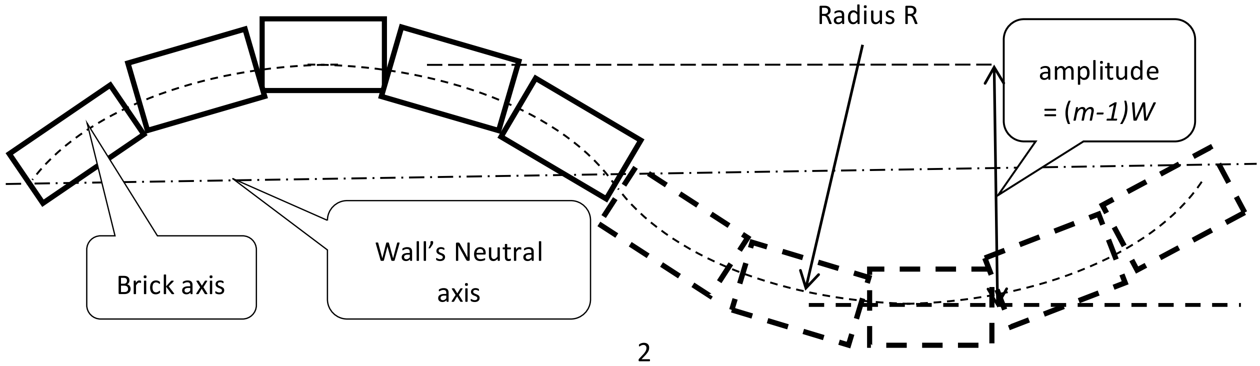


Fig 1e Serpentine wall (example: *n* = 10, *m* = 5)



However for both load-bearing house walls and for boundary walls, a full-brick (Fig 1b) rather than half-brick wall is usually employed where, provided the brick proportions are properly chosen, a variety of ‘bonds’ can be used to hold the wall strongly together. With an English bond (alternate courses of headers and stretchers without internal or external continuous vertical joints) the out-of-plane stiffness of the full brick wall is about 8 times that of the half-brick one and the lateral loading resistance is 4 times higher.

Brickwork is ancient. However bonded fired-clay bricks were not commonly used in Europe for house walling until about 1800 and the corresponding dates in Asia and Africa might be 1950 and 1990 respectively. Wattle and daub (poles and mud), dried soil blocks, matting on poles were all dominant walling techniques in the tropics until late in the 20th century. Moreover in many places the full-brick, bonded, fired-clay brick wall (and its ‘cavity wall’ variant) have given way to more complex designs or to the use of larger cementitious hollow blocks. Block walls, that are typically 150mm thick, have a stiffness intermediate between the those of 112mm half-brick and 225mm full-brick walling.

Other brick walls can be described by their mean thickness t_m expressed as a multiple u of brick thickness W. The ratio $u = t_m/t_D$ will also equal the number of bricks per course divided by the number of bricks in a datum wall of the same length. Thus for a double-thickness plane wall, U=2 and m=2.

For external house walls and for boundary walls, such double-depth brick (i.e. 200mm deep) is commonly used to achieve adequate stiffness and strength. Neglecting renders or plasters, this requires about 420 kg of brick+mortar per square meter of walling. By using a single-brick thickness (i.e. 100 mm), the mass per square meter of brick walling could be reduced to about 230 kg. Or with some more complex wall-shape for which $1<U<2$, we might achieve a lesser, but still valuable, weight saving relative to the double-depth norm.

To achieve $u < 2$, much of the wall length must be of single-brick thickness. However to achieve adequate overall stiffness and strength, the local axes of such single-brick sections need to be offset from the overall wall axis. A measure m is therefore used to describe the wall depth D as a multiple of W (i.e. $D = m \times W$). Thus (see Fig 1c) for a buttressed wall, $(m-1) \times W$ is the protrusion of the buttresses. For a crenelated wall, see Fig 1d, D the distance between the ‘front’ and ‘rear’ faces; for a wavy wall (Fig 1e) it is the maximum distance between those faces. For all three wall shapes, the wall depth is $D = m \times W$ and the mean thickness is $u \times W$. As increasing m results in greater lateral stiffness and strength, yet increasing u incurs greater cost, we are interested in ways of increasing the ‘materials productivity’ ratio m/u.

The energy consumed, and the GHGs emitted while producing house walling, depend on the quantity of material used. This in turn, for a given wall area, is roughly proportional to the mean thickness t_m of the wall. In this Working Paper we look only at the impact of wall shape on that mean thickness. It is however of little value to compare the mean thickness of walls of very different performances. Therefore we should either specify a particular performance and compare the mean thicknesses of rival designs that achieve it, or we should generate some sort of productivity figure whose numerator is a performance measure and whose denominator is wall cost or mean wall thickness.

To keep comparisons simple, we will restrict ourselves to walls of a specified height – the datum, buttressed, crenelated and sinuous walls are assumed to be the same height.

As house walling and boundary walling are typically 225mm thick, we might choose to adjust the various performance criteria to "that achieved by a 225mm-thick plane wall". This will simplify visualisation of wall properties, as $t_{b,p}$, $t_{c,p}$ and $t_{s,p}$ become the respective mean wall thicknesses of buttressed, crenelated and sinuous walls to give the same performance as a 200mm plane wall.

In a tropical setting, some aspects of performance (for example wind and noise exclusion) are little affected by wall design. In this paper we therefore concentrate on performance factors that *are* highly affected by shape or thickness, namely structural strength and stiffness. We choose not to consider here the effect of using different walling materials.

The resistance and stiffness of even plane walling is complex to calculate, for example according to Eurocode 6. For non-uniform walling, such as buttressed, crenelated and sinuous, analysis is even more complex. Therefore this paper employs highly simplified analysis, wherever possible linked to footprint area A, second moment of area I and bending-section modulus Z.

2 Four shape variants: plain, buttressed, crenelated and wavy

Plane walling (Fig 1a and 1b) is of uniform thickness (mean thickness = maximum thickness). To achieve adequate resistance to out-of-plane forces, this thickness may be quite large entailing much material. Plane walling is easy to erect and in housing produces convenient room shapes.

Buttressed walling (Fig 1c) has uneven thickness. Its effective thickness is increased by the addition of periodic buttresses in a way that – for a given stiffness or lateral strength – uses less material than a plane wall. However incorporating well-bonded buttresses requires some skill, may result in inconvenient room shapes and may consume more land area than a plain wall. A cellular floor plan (many tiny rooms) is the ultimate form of buttressing but is rarely acceptable to a house's occupants.

Crenelated walls (Fig 1d) carry the buttressing idea further with most of the wall being displaced, alternately, to one side or the other of the wall's axis. This substantially increases stiffness and resistance to out of plane forces without much increasing the mean wall thickness and hence wall cost. Again skill is required to achieve proper bonding and room shape is somewhat compromised.

Plane, buttressed and crenelated walling is normally straight, wavy serpentine walling, once known as 'crinkle-crankle walling' (Fig 1e) is not. A wavy wall uses hardly more material than a straight one of the same thickness but (provided it holds together and does not 'rack' when loaded) is many times stiffer and more force-resisting. The penalty is the inconvenient shape (in a world where almost all fittings and furniture are rectangular) and any liability to more complex failure modes. However for some application (garden walls), and some architects, the very sinuosity can be considered a virtue. Zig-zag walling can be considered a special case of serpentine walling.

3 Walling materials and methods of construction

Walling being a very ancient technology, comes in a myriad of forms. The two most-common forms of walling assembly are monolithic and block. In the former the relevant material is assembled on site as a continuum, either using shuttering or using a former such as a vertical mesh. There is some scope for light compression of the walling material by application of slow or impact forces, but any such compression is limited by the bursting strength of the shuttering. By contrast with block or brick or stone walling, the individual units are assembled without shuttering, usually by using either a jointing compound such as mortar or an arrangement of interlocks. Because each block/brick is small, before it is brought to site it can be subjected to substantial pressures or temperatures in a press or furnace, thereby cheaply improving its mechanical properties (strength, hardness, accuracy). However block walling may have almost no tensile strength at the joints between blocks. Its structural analysis commonly entails assuming a complete absence of tensile bonding unless reinforcing or post tensioning is employed.

Voids within a wall we shall regard as 'material properties' affecting material densities, strengths etc. For the purposes of comparing wall shapes, we will assume all rival shapes employ the same materials. A problem may apply with 'cavity' walls, in that cavities can easily be combined with plane walling but not with buttressing, tessellation or serpentine form. However although cavity construction was widely used in temperate countries in the 20th century to restrict damp penetration, it has since superseded by other techniques it is uncommon in the tropics and so will not be considered further in this Working Paper.

Walls can be non-homogenous, for example having two leaves made of different materials or having surface renders/plasters. Even though they are usually thin such renders can contribute significantly to wall performance. Again in this Working Paper such refinements will be ignored.

The main purposes of walling are (i) as a climate-excluding curtain or (ii) as a combination of curtain and structural support. Curtain walling (i) only has to carry its own rather light weight and withstand local forces including impacts; most structural loads in such buildings are carried by a mesh of beams and columns between which the walling is stretched. Curtain walling is usually of brick/block or of factory-made panel, or of glass. By contrast, 'structural' or 'load-bearing' walling (ii) carries not only its own weight (including the weight of the walling of higher floors) but also the transferred weight of suspended floors and roofing. It also resists the horizontal forces due to its connection to those more horizontal elements. A further horizontal force on load-bearing walls is that due to wind-loading, seismic accelerations and impacts.

4 Single-storey, two-storey and multi-storey walling

Very high buildings comprise a structural frame and a covering of curtain walling, hanging from that frame. Historically load-bearing walling (with no frame) was used up to about six stories, however the exact analysis of such high walls was not possible and they had to be built with large but indeterminate safety factors. This increased their cost and provoked the early 20th century change to framed construction for all high-rise buildings. More recently the analysis of multi-storey load-bearing brick walls has been advanced [Eurocode 6 1996, Hendry 1997] and there is a slight revival of interest in them. For the purposes of this paper we will consider only 1, 2 and 4 storey housing with load-bearing walls. Single-storey represents the great bulk of rural housing in the tropics. Two-storey construction is now common in towns and is also observable in stilt form in rural SE Asia. Four storey housing is usually in the form of apartments, one or more to each floor. ELITH Working Paper IIB-3 compares the properties of housing of these three heights.

5 Criteria for comparison

As well as cost (for a given material, this is strongly correlated with mean thickness t_m), we are interested for house-walling in

Table 1 Criteria for assessing wall performance

	Criterion
(i)	Land-area (footprint) occupied by the wall and the external part of its foundations
(ii)	The convenience to the house's occupants, for example of resultant room shapes
(iii)	Vertical load-carrying capacity
(iv)	Horizontal load-carrying capacity, including failure modes
(v)	Seismic capacity, natural frequency and damping
(vi)	Lateral stiffness

Other criteria like appearance, thermal insulation, durability, cleanliness etc are more affected by choice of wall material than by choice of wall shape, so they lie outside the scope of this Working Paper.

Footprint & Convenience (i & ii) : The first criterion above, *footprint per unit length of wall* equals at least the mean wall thickness t_m. For certain purposes we might want to increase this measure by a factor of about 2 (for boundary walls) and 1.5 for external house walls) to account for that part of the wall's foundation width not underneath a floor. In the case of buttressed, crenelated and serpentine walls, there is an additional land-take to which it is difficult to assign a cost.

In the case of a wavy boundary wall, the wall creates bays that are alternately on the 'inside' and the 'outside' of the wall. The areas of these bays might be planted and so not be 'lost' to the garden area. Or only the inside bays are planted and the outside bays are regarded as sacrificed land. There is a similar issue with the space between buttresses/piers on boundary walls. That between inwards facing buttresses is usable in various ways but that between outwards facing buttresses might have to be regarded as lost because buttresses must be kept within the plot boundary.

For house walls, the interior bays (serpentine) or recesses (buttressed or crenelated) in rooms will certainly be used but may be seen as inconveniently shaped and therefore of lower value per square meter.

Discussing the 'inconvenience penalty' of different wall shapes is continued in sections 6 to 9 below. The criteria (iii) to (vi) in Table 1 are structural and all are usually more relevant to load-bearing walls than to curtain walls.

Vertical Strength (iii) Crushing is not the common mode of wall failure: the *vertical load-carrying capacity* depends on (per unit length of wall) the Euler bucking resistance $F = \pi^2 E\, I / H_{eff}^2$, which for any given eccentricity and constraints is proportional to the material stiffness (E) and the (per unit length) second moment of area I. The effective height H_{eff} for a wall of height H cantilevered from its base and laterally constrained at its top, equals 0.71 H, or if not so laterally constrained equals 2 H. So in comparing wall shapes we need to compare their respective values of I (e.g. I_p, I_b, I_c, I_s). These values are tabulated below for a range of wall configurations. The Euler formula above does not apply to a wall buckling under its self-weight, which is however rarely the dominant failure mechanism.

Horizontal load-carrying capacity (iv), for example the maximum acceptable wind pressure acting on the whole wall face or top-edge lateral force (per unit of length) to overturn the wall, is determined by the onset of tensile cracking in the wall-face to which the load is applied. Load capacity depends on compensation by wall weight and on resistance due to the vertical tension strength of wall's material.

The first factor, wall weight, causes a compressive pre-stress of $\sigma_{comp} = \rho g H$ (where H is the wall height above the level chosen for analysis, g is gravity, ρ is wall density). H can be conveniently chosen as total wall height, so that tensile separation is assumed to occur at the bottom of the wall. If the brickwork is assumed to have no tensile strength, then ultimate failure will occur when the tensile stress in a wall face due to out-of-plane forces equals the pre-stress due to gravity. The maximum tensile stress will be the overturning moment M (per unit length) at the chosen level divided by the modulus Z per unit length ($Z = I/y_{max}$). So the sustainable moment is $M_g = \sigma_{comp} Z$ However if the wall material *does* have some tensile strength σ_{tens} then the moment that can be resisted is increased by $M_t = \sigma_{tens} Z$, giving $M_{failure} = (\sigma_{tens}+\sigma_{comp}) Z$. In either case, the comparative ability to resist lateral forces of different wall shapes is proportional to their respective values of per unit modulus Z (namely Z_p, Z_b, Z_c, Z_s,)

Conventional wall analysis often represents the effect of gravity by a vertical pressure p_g, so that at any chosen height h within the wall, the moment that can be carried can be written

$M = p_g Z$ where $p_g = \rho\, g\, (H - h)$ for a wall with 100% brick-to-brick contact.

If however the brick-to-brick contact area A_c is less than the brick's plan area A, then the 'gravity' equivalent pressure is higher: $p_g = (A/A_c)\, \rho\, g\, (H - h)$.

or where the wall also has some tensile strength σ_{tens} and/or is subject to post-tension of $\sigma_{post\text{-}tens}$

$M = (p_g + \sigma_{tens} + \sigma_{post\text{-}tens}) Z$ where only p_g varies with position h up the wall.]

Earthquake resistance (v) is a complex phenomenon that depends on wall stiffness, internal damping and other design parameters. It is therefore too extensive a topic to be covered by this Paper. However we note that it is generally lateral acceleration that damages walling during tremors and the effect of such accelerations is similar to wind loading. Thus seismic performance depends upon the ability of the wall to resist lateral forces (just argued to depend upon Z) and on the wall's mass, since horizontal seismic accelerations are converted into lateral forces via this mass. Mass, for a given material and wall height, will be proportional to mean wall thickness t_m. Thus resistance to seismic failure is proportional to Z/t_m.

Lateral stiffness (vi) is, like Euler load, proportional to $E\, I$ or (for constant E) just to the per unit 2nd moment of are I. For mortared walls, lateral stiffness is usually adequate to prevent any malfunction e.g. cracking of internal plaster. However for unmortared walls (of interlocking blocks), stiffness is much lower. This may result in walls having too low a natural frequency and hence resonance problems during 'quakes.

6 Plain walls

Despite the bricks in a plane wall normally having no interlocks (although sometimes 'frogs' are indented to their top surfaces), the mortar has sufficient shear bonding to the bricks to hold the wall together. This adhesion is enhanced by overlap between bricks on successive courses and the avoidance of continuous perp lines. For single thickness walls a stretcher bond gives 50% overlaps. For double thickness a variety of bonds have been developed to give at least 25% overlap within each leaf and to tie the two leaves together. Single-thickness (e.g. 100mm) plane brick walls are not normally used for external house walling or for boundary walling. By contrast single-thickness is normal for block walls (typically 150mm thick) and mortar adhesion may be assisted by interlocking.

Doubling the thickness of a wall raises its crushing strength by a factor x2, its Euler collapse load and lateral stiffness by factor x8 (both being dependent on 2nd moment I), and its lateral strength by factor x4.

Vertical loading is due to self-weight and the weight of supported upper floors and roof.

Lateral loading arises from:

wind forces, whose pressure centroid is half way up the wall and whose moment is greatest at the wall base, having value $M_{w} = pressure.H^2/2$ per unit length;

horizontal seismic acceleration forces, whose centre of action is half way up the wall and whose moment above the wall base has the value $M_e = acceleration.\rho g t_m\ H^2/2$ per unit length of wall, where t_m is the mean wall thickness (again hinging is equally probable at all heights);

horizontal line forces transmitted from ceilings (span floors) and roof. Hinging/overturning due to any of these forces is equally probable at all heights, as the restoring moment due to wall weight at any course is proportional to the wall height above that course.

7 Buttressed walls

A buttressed wall is shown in Fig 1c: the particular example depends upon the buttress depth *m-1* (expressed as a multiple of brick width *W*) and the buttress spacing *n* (expressed as a multiple of brick length *L*). Buttresses are also called piers, except that piers, like columns, can be inserted symmetrically in the wall line and not just to one side. 'Pier + Panel' is a further special case where only the (reinforced) pier has full foundations.

Provided a wall behaves as a laterally rigid continuum, then the measures of greatest interest to the structural designer are its (mean) unit second moment of area I and its modulus Z. By 'unit' we mean per unit length of walling and by 'mean' that this is averaged along the length of the wall. Modulus is not averaged along the wall, as it depends upon the *maximum* distance (y_{max}) on the tension side between the surface of the wall and the wall's neutral axis. Since I rises with the third power of thickness, a wall with deep buttresses has a much larger stiffness than one with small buttresses. And of course making buttresses more frequent also makes the wall stiffer.

Let I_0 be the *unit* 2nd moment of area (units are m^3) of a wall with buttresses spaced infinitely far apart. Let I_b be the unit 2nd moment of area of a brick wall with one buttress (a single brick wide) every *n* bricks of its length. Let *m*-1 be the depth of the buttress, measured in brick thicknesses perpendicular to the wall's face. Thus for a plane wall, *m* = 0 and mean thickness = t_o. When *m* is increased or *n* is decreased, the stiffness measure I_b/I_0 increases strongly, the strength measure Z/Z_0

increases moderately and the cost measure t_b/t_0 increases weakly. In the tables 2 to 5 the stiffness and strength measures re also normalised to the number of bricks used.

Table 2 Properties of buttressed walling

Wall depth/$W = m$	2		3		4		5	
Spacing n in brick lengths	5	10	5	10	5	10	5	10
Stiffness & Euler ratio I_c / I_0	3.2	2.2	10.3	6.3	24.4	14.8	47.1	28.8
Strength ratio Z/Z_0	1.2	0.8	2.5	1.4	4.4	2.4	7.0	3.8
Cost ratio $U_b = t_b/t_0$	1.2	1.1	1.4	1.2	1.6	1.3	1.8	1.4
Stiffness ratio per brick $(I_b/ I_0) / U_b$	2.7	2.0	7.4	5.3	15.3	11.4	26.2	20.6
Strength ratio per brick (also seismic ratio) $(Z_c /Z_0) / U_b$	1.0	0.7	1.8	1.2	2.8	1.9	3.9	2.7

So for the example shaded above, namely adding a 3-brick buttress every 10 bricks, will increase I and hence stiffness about 15 fold, Z and hence lateral strength (assuming force applied on the buttressed side) about 2.4 fold, yet the number of bricks needed increases by only 30%. This buttress spacing (10 brick lengths) is approximately equal to the likely wall height H for single-storey housing.

However wall analysis normally takes account of the position of not buttresses but 'returns' (well-bonded perpendicular support walls) assumed to be infinitely strong and rigid. Yet with such returns spaced closely (spacing equals approx. the wall height), the returns increase the lateral pressure load the wall's weight can resist only by a factor of about 3 [Hendry, Fig 7.6]. Since a buttress has rather low stiffness, it is structurally worth less than a return. Therefore even a very deep buttress is unlikely to increase lateral strength more than 3-fold.

A buttress has to bond with the rest of the wall. The simplest bond (assuming a single-brick main wall) is to have a double header on alternate courses, or diagonally spaced single headers on every course. This is not a strong joint in tension (where wall-to-buttress forces are transmitted by brick-to-mortar-to-brick shear). If ¾ bricks are available, other bonds including those with double-thickness walls are possible.

Boundary walls often fail by excessive leaning, due to very slow subsidence under their foundations, rather than collapse under unusually high loading. (The bell tower of Pisa Cathedral is a famous example, where leaning developed over eight centuries.) If buttresses are to resist this form of failure their foundations must be substantial, rigid and if possible extend beyond their ends. One method of extending the support leverage (for any given buttress volume) is to employ sloping or triangular buttresses. However as buttresses are usually short, they have poor resistance to racking caused by sliding at course joints. Such sliding /shearing is reduced by having much weight on the joints – which points towards using full-height buttresses.

Buttresses are thought to 'intrude' and are therefore usually placed on the outside of a house wall. Placed internally they slightly reduce the flexibility of rooms, especially if their depth exceeds 0.3 meters (3 bricks). Placed externally, and with foundations, they directly occupy an extra land area of approximately twice their plan area. However if the house wall is also on the plot boundary then it may have to be moved inwards by the buttress depth, thereby losing considerably more land area.

8 Crenellated walls

Crenulation, see Fig 1d, is a means of increasing a wall's effective thickness without using much extra material. The contribution to the wall's *I*, by any individual brick, rises with approximately the square of the displacement of that brick's axis from the wall's axis. So a crenelated wall with large setbacks has a much larger stiffness than one with a small set back. We define 'set-back' as the distance from the axis of its front row to the axis of its back row (see Fig 1c) and express it as a multiple *m-1* of brick thickness; $D = m \times W$ is therefore the depth of the wall.

Table 3 Properties of crenellated walling

Wall depth *m*	2		3		4		5	
Spacing *n* of set backs(brick lengths)	5	10	5	10	5	10	5	10
Stiffness & Euler ratio I_c / I_0	4.8	4.4	15.8	14.4	35.2	31.6	64.0	56.6
Strength ratio Z_c/Z_0	2.4	2.0	5.3	4.8	8.8	7.9	12.8	11.3
Cost ratio $U_c = t_c/t_0$	1.2	1.1	1.4	1.2	1.6	1.3	1.8	1.4
Stiffness ratio per brick $(I_c / I_0) / U_c$	4.0	4.0	11.3	12.0	22.0	24.3	35.6	40.4
Strength ratio per brick (also seismic ratio) $(Z_c / Z_0) / U_c$	2.0	2.0	3.8	4.0	5.5	6.1	7.1	8.1

Again taking *m* = 4 i.e. (a setback-depth of 3W) and *n* = 10 (i.e. a setback-spacing of 10L) as our example, we now have a very large increase in stiffness and a quite large increase in strength for only a modest (30%) increase in brick use. Moreover the symmetrical nature of a crenelated wall means the forward and reverse properties are the same as each other.

Unlike with buttresses, there is no need for a particularly stiff foundation under any part of the wall. However the inconvenience of the wall shape – its impact on room usage and furnishing - is greater than for buttressing. Indeed in countries that used to have coal fireplaces in every room, their disuse has led to the expensive demolition of 'chimney breasts', suggesting that residents found their presence too intrusive. If the crenulations are 'inwards' from the external wall-line of a typical house, and there is one crenulation per room, then each room area is reduced by typically 3%. For boundary walling there is little area loss, since the wall's direct footprint is less than that of a straight wall. However the individual rectangular bays on both the inside and outside may have little utility and come to be counted as part of the footprint.

Construction of a stretcher-bond crenelated wall is straightforward, using classic corner bonds. The possibility of racking is however significant. The connection between the two leaves of a crenelated wall can also be a reinforced brick pier, for example a 300mm x 300mm pier connecting two 100mm wythes.

9 Wavy (serpentine) walls

Serpentine walls are uncommon, although a number (under the name 'crinkle-crankle wall') that were built over 200 years ago still survive in England, the Netherlands and USA. They achieve effective depth via their curvature, not via the addition of extra bricks. However the angle between the brick's local axis and the wall's axis does not usually exceed 30^0, so their along-the-wall length and hence brick-count is commonly only a few percent more than their length along the wall's axis. If normal straight bricks are used, it is desirable that the angle between successive bricks does not exceed about 9^0, otherwise the wall will appear too knobbly with bricks overhanging those below them by over 2mm. A sinusoidal wall plan is probably most visually satisfying. However a serpentine wall constructed of alternate arcs each of constant radius *R* will result in a lower *maximum* curvature than a sinusoidal wall of the same depth and wavelength. Using such constant-curvature arcs is normal practice.

The American Brick Institute's technical notes (1982 & 1999) discuss the proportions of serpentine walls that have shown very long-term durability. These are generally very deep, with depths not less than half of wall heights.

Fig 3 Serpentine boundary wall in seismic zone (Fort Portal, Uganda 2015) 100mm-thick

Analysis of the per unit 2^{nd} moment of area *I* and the modulus Z of serpentine walls is difficult, unless we restrict ourselves to cycle lengths ('wavelengths') corresponding to integer numbers of bricks per half wave.

As for the other wall types, we define *m* as the number of brick widths that make up the wall's overall depth *D* and *n* the number of bricks to make up two of the wall's arcs (i.e. a complete cycle).

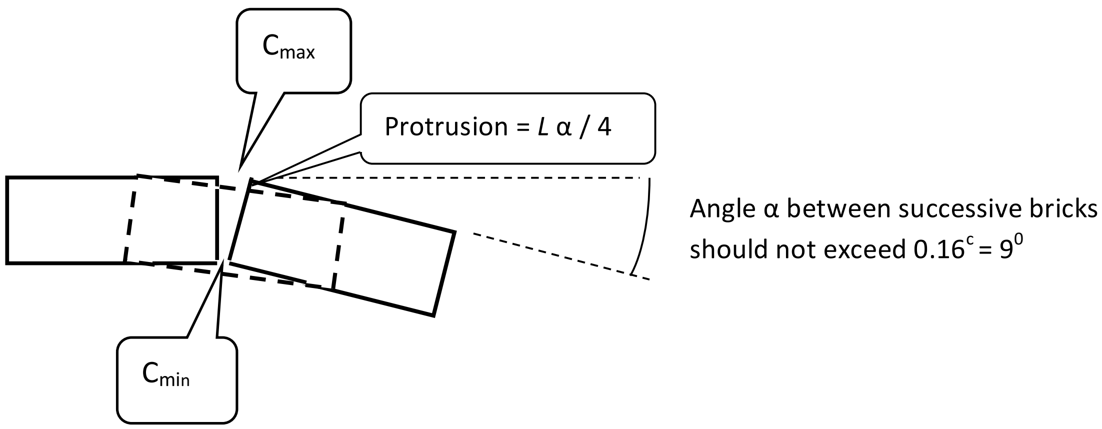
Table 4 Properties of wavy walling

						Recommended
Radius of arc / t	13	40	10	20	6	15
Arc-angle/degrees	44	26	74	52	120	74
Max angle to wall axis	22	13	37	26	60	37
Wall depth/W = m	3		5		7	
Bricks / cycle = n	10	18	12	18	12	18
Wavelength/bricklength	10	18	11.2	17.5	10	16.7
Stiffness & Euler ratio = I_s / I_0	7.1	7.5	25.8	26.2	57.6	57.3
Strength ratio Z_s/Z_0	2.4	2.5	5.2	5.3	8.2	8.2
Cost ratio $u_s = t_s/t_0$	1.03	1.01	1.07	1.03	1.21	1.07
Stiffness ratio per brick = $(I_s /I_0) /u_s$	6.9	7.4	24.1	25.4	47.6	53.6
Strength ratio per brick $(Z_s /Z_0) /u_s$	2.4	2.5	4.9	5.1	6.8	7.7
Angle between successive bricks	8.8^0	2.9^0	12.3^0*	5.7^0	20.0^0*	8.3^0
Depth/wavelength	0.15	0.09	0.21	0.14	0.35	0.22

* unacceptably large

As the angle between successive bricks in a course should not exceed about 9^o which requires that brick-lengths per half wave (n) would normally be about 9, giving typically a 4m wavelength.

Figure 4 Angle between successive bricks



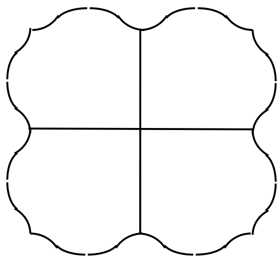
Assuming a mean horizontal spacing between bricks of C_{mean} = 10 mm, and a minimum spacing C_{min} of 2 mm, requires C_{max} = 18 mm. Hence for bricks of thickness W = 100 mm, the maximum mortar wedge angle should not exceed $(C_{max}- C_{min})/W$ = 0.16 rad = 9^0. This gives an outwards 'protrusion' of about 8mm between the joint between two bricks and the brick in the course below. Such protrusion is rather large, giving the wall a knobbly appearance.

Normal shear friction between courses may not be sufficient to hold the wall together, so some horizontal reinforcement in a mortar course near the top of the wall is desirable.

Experiments with a section, similar to the dimensions 'recommended' in Table 4, of the wavy wall shown in Fig 3, indicated a lateral stiffness about 10 times that of an adjacent straight wall of the same height and thickness. This is less than the factor of around 57 shown in the theory. Further experiments will be performed.

The brick count of a serpentine wall is low – often only a few percent more than a single-wythe plane wall. Indeed the main attraction of the wavy wall shape is that it allows a large increase in its effective depth (*m*) to be made without significantly increasing its use of materials. So serpentine walls are usually built deeper than crenelated walls; e.g. for the case *m*=7, shown highlit in Table 3 above, the distance from front face to back face would be 70cm (if bricks are 10cm wide). If, for a boundary wall, the whole serpentine must lie *inside* the plot boundary, then there is significant extra land-take, about 50% of which is effectively donated to the neighbouring plot. If however the wall's axis lies *along* the plot boundary, then both this and the neighbouring plot experience alternate intrusion and retreat (bays).

Example of a simple house plan with serpentine exterior walls and straight internal walls.



10 Numerical comparisons of the 4 wall plans

We can now compare (Table 5) the 4 shape variants – plane, buttressed, crenelated and serpentine.

Table 5 Comparison of wall shapes ($m = D/W = 3$ shown shaded).

Wall shape	Plain			Buttressed every 5 bricks (n = 5)			Crenulated every 5 bricks (n = 5)			Wavy wavelength (n = 18)		
Wall-depth *m*	1	**2**	3	2	3	5	2	3	5	3	5	7
Cost ratio $u_s = t_s/t_0$	1	**2**	3	1.2	1.4	1.8	1.2	1.4^2	1.8	1.01	1.03	1.07
Stiffness ratio (I/I_0)	1	**8**	27	3.2	10.3^+	47.1	4.8^+	15.8^+	64.0	7.5^+	26.2	57.3
Strength ratio (Z/Z_0)	1	**4***	9	1.2	2.5*	7.0*	2.4*	5.3*	12.8	2.5*	5.3*	8.2
$(I_s/I_0)/u_s$	1	**4**	9	2.7	7.4	26.2	4.0	11.3	35.6	7.4	25.4	53.6
$(Z_s/Z_0)/u_s$	1	**2**	3	1.0	1.8	3.9	2.0	3.8	7.1	2.5	5.1	7.7

Notes: Stiffness-ratio/cost-ratio = $(Z_s/Z_0)/u_s$ is also the seismic acceleration tolerance ratio
Wavy wall wavelength is long (n = 18), else the brick-to-brick angle would be too high
For plain walls *m*=2 is commonly used – as shown by bold properties

To simplify this comparison let us first look at the case m = 3, i.e. where the wall depth D, front to back, is 3 times a brick thickness. In the table, the data columns for this case are high-lit.

In terms of cost, for a given length of wall and wall depth m = 3, the order is:
wavy (best), crenelated, buttressed, plane (much the worst).

In terms of performance as measured by 2nd moment I and section modulus K, the order is:
plane / crenelated (good), buttressed / wavy (poor).

If we combine performance and cost by expressing performance per brick used, we get:
crenelated (best), plane, wavy, buttressed (worst).

If however we had taken a more typical 2-brick (m=2) plain wall as our datum, then the 'performance per brick' test would have ranked the options:

crenelated (best), wavy, plane, buttressed (worst)

A different and more realistic approach is to first define a performance and then assess the materials cost of meeting it, using each of the four wall shapes. Thus choosing (for convenience) a resistance to overturning forces equal to that of a double-thickness (m = 2) plane wall, for which Z/Z_0 = 4, where Z_0 is the modulus of our datum single-wythe plain wall. We get, approximately, by interpolation (of candidates marked * in Table 5), the options of:

a buttressed wall, with m = 4, n = 5, costing 20% less than the plane wall

a crenelated wall, with m = 3, n = 5, costing 30% less

a wavy wall, with m = 4, n = 18, costing nearly 50% less

Applying the same approach to getting a *stiffness* (and also quake resistance and Euler buckling load) equal to that of the double (m=2) plane wall, we could use (candidates with I/I_0 = approx 8, as marked $^+$ in Table 5):

a buttressed wall, with m = 3, n = 5, costing 30% less than the plane wall;

a crenelated wall, with m = 2.5, n = 5, costing 35% less;

a wavy wall, m = 3, n = 18, costing 50% less.

In general therefore, buttressing is not a very efficient use of brick, whereas both crenulation and employing wavy walling offer substantial savings. The main price to pay for this saving is the possible inconvenience of having stepped or curving surfaces in a room or losing a little plot area next to a boundary wall.

10 Conclusions

Crenelated and wavy walls use up to 50% less bricks than plane walls to achieve the same resistance to buckling, same stiffness and same resistance to lateral forces. Buttressed walls are intermediate in terms of brick economy. Indeed these designs can be used with a single brick thickness (e.g. 100 mm) to get satisfactory performance in both housing and boundary walls; by contrast with plane walling a double thickness (e.g. 200 mm) is normally required..

Both crenelated and wavy designs are suitable for increasing strength and stiffness by making the wall deeper yet without using many more bricks. The shape of these materially-efficient wall plans may conflict with the rectangular norm of most modern buildings. Conversely they offer scope for architectural distinctiveness.

The simplified analysis used in this paper assumes that during loading, the walls hold together and act as a whole. Racking due to slippage between courses has therefore not been considered. These assumptions may prove unfounded in the case of very deep crenelated or serpentine walls – e.g. m values exceeding say 5.

11 Bibliography

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Photos of wavy walling:
http://www.geograph.org.uk/gallery/crinkle_crankle_or_serpentine_walls_15374, acc'd Dec 2016

Appendix
Calculations for Constant Radius Wavy Walls

Assume wall is a sequence of 'facing' arcs each of radius ***R,*** arc angle ***2θ***, and thickness ***t***, *where* — ***R = λt***

2nd moment of wavy wall is **μ** times that of a straight wall of thickness ***t*** and of the same 'axial' length; . — $I = \mu\, Io$

However as the curved wall is slightly longer (for the same axial length), we have a brick usage multiplier of — **σ = θ/sin(θ)**

Depth, front-face to back-face of wall , ***D*** = *t* + 2**R**(1-cos(θ)); datum wall depth = ***t***; *relative* depth is ***m = D/t*** — ***m*** **= 1 + 2*λ*(1-cos(θ))** where **λ = *R/t***

Relative stiffness **μ** = (θ/sin(θ))*{1+6λ^2(1 - 3(sin(θ)*cos(θ)/θ) + 2cos^2(θ)} — **μ = σ*(1+6λ^2*e)**

where **e = (1 - 3(sin(θ)*cos(θ)/θ) + 2cos^2(θ)**

As a close approximation — **μ = μ' = 1+1.59*(m-1)^2**

Relative stiffness per brick used is — **μ / σ = 1+6λ^2*e**

wall thickness	radius	R/t	half arc angle			I / Io relative stiffness per meter	relative brick usage	relative stiffness per brick	Wall depth		relative strength	Wave length	depth/ wavelth		
t	***R***	**λ**	**θ**	**θ**	***e***	**μ**	**σ**	**μ / σ**	***D***	***m***	***μ / m***	***W***	***D / W***		**μ'/μ**
m	m		rads	degree					m	*d / t*					
0.1	0.6	6	1.05	60	0.2620	**57.6**	1.21	47.6	0.70	7.0	8.19	2.08	0.34		0.979
0.1	1	10	1.571	90	1.0004	**601.2**	1.57	382.7	2.10	21.0	28.62	4.00	0.53	semicircs	0.943
0.1	**1**	**10**	**0.785**	**45**	**0.0900**	55.0	**1.11**	**49.5**	**0.69**	**6.9**	**8.02**	**2.83**	**0.24**	1/4 circles	0.991
0.1	**1**	**10**	**0.523**	**30**	**0.0189**	**12.4**	**1.05**	**11.8**	**0.37**	**3.7**	**3.36**	**2.00**	**0.18**	1/6 circs	1.000
0.1	5	50	0.1	6	0.0000	**1.4**	1.00	1.4	0.15	1.5	0.93	2.00	0.08		1.002
0.1	**5**	**50**	**0.2**	**11**	**0.0004**	**7.4**	**1.01**	**7.3**	**0.30**	**3.0**	**2.46**	**3.97**	**0.08**		1.005
0.1	5	50	0.284	16	0.0017	**26.6**	1.01	26.3	0.50	5.0	5.32	5.60	0.09		1.004
0.1	5	50	0.3	17	0.0021	**32.8**	1.02	32.4	0.55	5.5	6.01	5.91	0.09		1.004
0.1	5	50	0.4	23	0.0066	**100.3**	1.03	97.7	0.89	8.9	11.28	7.79	0.11		1.002
0.1	5	50	0.6	34	0.0323	**484.9**	1.06	456.3	1.85	18.5	26.26	11.29	0.16		0.998
0.1	2.5	25	0.1	6	0.0000	**1.1**	1.00	1.1	0.12	1.2	0.88	1.00	0.13		1.001
0.1	2.5	25	0.2	11	0.0004	**2.6**	1.01	2.6	0.20	2.0	1.30	1.99	0.10		1.003
0.1	2.5	25	0.3	17	0.0021	**9.0**	1.02	8.8	0.32	3.2	2.77	2.96	0.11		1.004
0.1	**2.5**	**25**	**0.4**	**23**	**0.0066**	**25.8**	**1.03**	**25.1**	**0.49**	**4.9**	**5.22**	**3.89**	**0.13**		1.002
0.1	2.5	25	0.523	30	0.0189	**72.0**	1.05	68.8	0.77	7.7	9.37	4.99	0.15	1/6 circs	1.000
0.1	2.5	25	0.6	34	0.0323	**122.0**	1.06	114.8	0.97	9.7	12.53	5.65	0.17		0.998