## 2016 Past Paper -Question 1

(a) The FSR is the difference between the resonant peaks, i.e when

## $2 k n L=2 m \pi$

Writing this in frequency terms

$$
\begin{aligned}
& 2\left(\frac{2 \pi}{\lambda_{m}}\right) n L=2 m \pi \\
& 2\left(\frac{2 \pi f_{m}}{c}\right) n L=2 m \pi \\
& f_{m}=m \frac{c}{2 n L} \\
& f_{m+1}=(m+1) \frac{c}{2 n L} \\
& \text { So } f_{m+1}-f_{m}=\frac{c}{2 n L}
\end{aligned}
$$

(b) The mirrors are identical with amplitude transmission coefficient $t$ and reflection coefficient $r$

... and so on....
So the amplitude transfer function is geometric progression

$$
t_{F}(\theta)=t^{2} \exp [-j \theta] \sum_{n=0}^{\infty}(r)^{2 n} \exp [-j 2 n \theta]=\frac{t^{2} e^{-j \theta}}{1-r^{2} e^{-j 2 \theta}}
$$

For an ideal mirror $t^{2}=1-r^{2}$, and so we can also substitute then intensity reflection coefficient $R=r^{2}$

$$
t_{F}(\theta)=\frac{(1-R) e^{-j \theta}}{1-R e^{-j 2 \theta}}
$$

The transmission coefficient is the modulus squared of this expression

$$
T_{F}(\theta)=\frac{(1-R) e^{-j \theta}}{1-R e^{-j 2 \theta}} \times \frac{(1-R) e^{j \theta}}{1-R e^{j 2 \theta}}=\frac{(1-R)^{2}}{1+R^{2}-2 R \cos (2 \theta)}
$$

(c) The FWHM, $\Delta \theta$, is defined as the point when

$$
\frac{(1-R)^{2}}{1+R^{2}-2 R \cos (2\{\Delta \theta / 2\})}=\frac{1}{2}
$$

Assume that we can expand $\cos (\Delta \theta) \approx 1-(\Delta \theta)^{2} / 2$

$$
\begin{aligned}
& \frac{(1-R)^{2}}{1+R^{2}-2 R\left\{1-(\Delta \theta)^{2} / 2\right\}}=\frac{1}{2} \\
& \frac{(1-R)^{2}}{(1-R)^{2}+R(\Delta \theta)^{2}}=\frac{1}{2} \\
& R(\Delta \theta)^{2}=(1-R)^{2} \\
& \Delta \theta=\frac{(1-R)}{\sqrt{R}}
\end{aligned}
$$

(d) The FSR should be $\geq N \Delta f_{c h}$, so in the limit

$$
\begin{gathered}
\frac{c}{2 n L}=N \Delta f_{c h} \\
N=\frac{c}{2 n L \Delta f_{c h}} \\
N=\frac{3 \times 10^{8}}{2 \times 1 \times 50 \times 10^{-6} \times 50 \times 10^{9}}=60 \text { channels }
\end{gathered}
$$

## 2016 Past Paper -Question 2

(a) The core network provides transport between exchanges and is further divided into long haul and metropolitan networks for network planning. The access network supplies services from the exchange to the customer.
(b) Passive Optical Network (PON): This sends all traffic through and optical line termination (OLT), which contains all the expensive components. The network is a passive splitting tree to optical network units (ONUs) that are the end of the optical path. Customers are connected via copper at network terminations (NTs).


SDH Ring: A point to point circuit is provisioned using add drop multiplexers (ADMs). The connection of rings is accomplished through access nodes (ANs). Traffic from the ADMs may travel over copper or fibre to the NTs. The architecture provides resilience as the ring will offer an alternative path if a fibre is broken.

(c) To resolve port contention in packet switching, packets are stored prior to being forwarded to the appropriate place. There are no random access optical buffers currently available making this a current difficulty for optical networks. The only option is to have optical fibres emulating buffers by delaying packets for a fixed time.
(d) The star topology needed is:


Each node comprises an optical transmitter and an optical receiver. The $8 \times 8$ couple is made from $2 \times 2$ couplers as shown below


In general, the splitting power loss for each couple is 3 dB , there are three couplers in any path and so there will be a loss of 9 dB in total.

The optical transmitters are identical with output powers of $0 \mathrm{dBm}(1 \mathrm{~mW})$ each. A 3dB loss means that each launches 0.5 mW into its port. The output from a port is thus
$P_{\text {out }}=\frac{0.5}{8}-0.0625 m W$ or -12.04 dBm .
(e) short haul: for example, short optical data links to connect computers and terminals within the same building or between two buildings. The low loss and the wide bandwidth of optical fibres are not of primary importance for such data links. Fibres are used mainly because of their other advantages, such as immunity to electromagnetic interference.
long haul: for example, undersea lightwave systems are used for high-speed transmission across continents with a link length of several thousands of kilometres. Low losses and a large bandwidth of optical fibres are important factors in the design of transoceanic systems from the standpoint of reducing the overall operating cost.

When the link length exceeds a certain value dependent on the operating wavelength, it is necessary to compensate for fibre losses

A regenerator is a receiver-transmitter pair that detects the incoming optical signal, recovers the electrical bit stream, and then converts it back into optical form by modulating an optical source. A regenerator performs three functions-reamplification, reshaping and retiming (the 3Rs).

Fibre losses can also be compensated by using optical amplifiers, which amplify the optical bit stream directly without requiring conversion of the signal to the electric domain. Optical amplifiers are especially valuable for wavelength-division multiplexed (WDM) lightwave systems as they can amplify many channels simultaneously.

In the absence of a commercial all-optical regenerator, most terrestrial systems use a combination of the two techniques shown.

## 2016 Past Paper -Question 3

(a). The fundamental mechanism in photodetection is optical absorption, and the pn diode may be represented in abstract form as below:


The reverse biased $p-n$ junction has depletion region free of carriers where a large electric field opposes $n \rightarrow p$ flow of electrons and $p \rightarrow n$ flow of holes.

If the energy of incident photons, $h f$, is greater than the semiconductor bandgap energy then light falling on the junction creates electron hole pairs as the photons are absorbed

The electric field sweeps out the electrons and holes producing a current
Using a lightly doped or undoped intrinsic region between the $p$ and the $n$ layers to make a pin diode increases the depletion region width and hence the efficiency of the diode.
(b) The quantum efficiency, $\eta$, represents how many electrons are produced per photon

$$
\eta=\text { electron generation rate / photon incidence rate. }
$$

Since the number of electrons per second is the current divided by the electronic charge and the number of photons per second is the power divided by the energy per photon we can say

$$
\eta=\frac{I_{p} / q}{P_{o} / h f} \quad \text { rearranging } \quad I_{p}=\frac{\eta q}{h f} P_{o}
$$

(c) Pictorially, the absorption appears as below


Power that is absorbed and creates a photocurrent is the difference between the power entering the depletion region and that leaving

$$
P_{a}=\left(1-R_{f}\right) P_{o}\left\{\exp \left(-\alpha x_{1}\right)-\exp \left(-\alpha x_{2}\right)\right\}
$$

The quantum efficiency can thus be defined as
$\eta=\frac{P_{a}}{P_{0}}=\left(1-R_{f}\right)\left[\exp \left(-\alpha x_{1}\right)-\exp \left(-\alpha x_{2}\right)\right]$
$\eta=\left(1-R_{f}\right) \exp \left(-\alpha x_{1}\right)\left[1-\exp \left(-\alpha\left[x_{2}-x_{1}\right]\right)\right]$

To optimise this $\quad x_{1} \rightarrow 0 \quad$ (small p-layer) and $\quad x_{2} \rightarrow \infty \quad$ (maximum depletion width).
(d) The difference $x_{2}-x_{1}$ is the $i$-layer thickness; for 633 nm , using the equation given,

$$
\begin{aligned}
& \alpha_{633}=6.9 \times 10^{7} \times \exp (-633 / 130)=5.3 \times 10^{5} \mathrm{~m}^{-1} \\
& \eta_{633}=0.7 \exp \left(-5.3 \times 10^{5} \times 10^{-6}\right)\left\{1-\exp \left(-5.3 \times 10^{5} \times 25 \times 10^{-6}\right)\right\}=0.412 \\
& I_{633}=\frac{0.412 \times 1.6 \times 10^{-19} \times 10^{-6} \times 633 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}=209 \quad \mathrm{nA}
\end{aligned}
$$

Similarly, at $850 \mathrm{~nm} \alpha_{850}=6.9 \times 10^{7} \times \exp (-850 / 130) \approx 10^{5} \mathrm{~m}^{-1 ;}$
$\eta_{850}=0.7 \exp \left(-10^{5} \times 10^{-6}\right)\left\{-\exp \left(-10^{5} \times 25 \times 10^{-6}\right)\right\}=0.581$
$I_{850}=\frac{0.581 \times 1.6 \times 10^{-19} \times 10^{-6} \times 850 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}=395$

## 2016 Past Paper -Question 4

(a) Monochromatic light travels at the phase velocity, which is the product between frequency and wavelength. For a group of wavelengths, we need the velocity of points of constant amplitude for the whole group, the group velocity given by the rate of change of propagation constant with angular frequency.
(b) The group index is the apparent refractive index of an optical fibre and hence found, for propagation constant $\beta$ and angular frequency $\omega$, from

$$
N_{g e}=\frac{c}{v_{g}}=c \frac{d \beta}{d \omega}
$$

The normalised propagation constant brings $\beta$ into the range 0 to 1 by use of the free space propagation constant $k$ and the core $\left(n_{1}\right)$ and cladding $\left(n_{2}\right)$ refractive indices thus

$$
b=\frac{(\beta / k)^{2}-n_{2}^{2}}{n_{1}^{2}-n_{2}^{2}}
$$

(c) The expression for the normalised propagation constant may be rewritten in terms of the relative refractive index

$$
\Delta=n_{1}^{2}-n_{2}^{2} / 2 n_{1}^{2}
$$

$$
\begin{aligned}
b=\frac{(\beta / k)^{2}-n_{2}^{2}}{2 n_{1}^{2} \Delta} & \\
& \Rightarrow \beta=k\left[n_{2}^{2}+2 b n_{1}^{2} \Delta\right]^{\frac{1}{2}}=n_{2} k\left[1+2 b \frac{n_{1}^{2}}{n_{2}^{2}} \Delta\right]^{\frac{1}{2}}
\end{aligned}
$$

for small $\Delta$, and use binomial expansion

$$
n_{1}^{2} / n_{2}^{2} \approx 1
$$

$$
\beta \approx k n_{2}[1+b \Delta]
$$

(d) Starting from
$\tau_{g}=\frac{1}{c} \frac{d \beta}{d k}$
and using $\beta \approx k n_{2}[1+b \Delta]$

$$
\begin{aligned}
& \tau_{g}=\frac{1}{c} \frac{d \beta}{d k}=\frac{1}{c}\left\{n_{2}+k \frac{d n_{2}}{d k}[1+b \Delta]+n_{2} \Delta \frac{d(k b)}{d k}+k n_{2} b \frac{d \Delta}{d k}\right\} \\
& \approx \frac{1}{c}\left\{n_{2}+k \frac{d n_{2}}{d k}+n_{2} \Delta \frac{d(k b)}{d k}\right\}
\end{aligned}
$$

since $V=k a(N A)$

$$
\begin{aligned}
& \frac{d(k b)}{d k}=\frac{d\left(\frac{V b}{a(N A)}\right)}{d V} \frac{d V}{d k}=\frac{d(V b)}{d V} \frac{1}{a(N A)} a(N A)=\frac{d(V b)}{d V} \\
& \tau_{g} \approx \frac{1}{c}\left\{n_{2}+k \frac{d n_{2}}{d k}+n_{2} \Delta \frac{d(V b)}{d V}\right\}
\end{aligned}
$$

(e) To be just single moded $V=2.405$

$$
\frac{d(b V)}{d V} \approx 1.3-\frac{1}{2.405^{2}}=1.127
$$

$$
\Delta=\frac{1.5-1.495}{1.5}=0.0033
$$

$\tau_{g}=\frac{1}{3 \times 10^{8}}\{1.495+0.01+1.495 \times 0.0033 \times 1.127\}=5.03 \mathrm{~nm} / \mathrm{m}$ or $5.03 \mu \mathrm{~s} / \mathrm{km}$

