STABILITY ANALYSIS OF PLANE FRAMES OF FIBRE REINFORCED POLYMER HAVING SEMI-RIGID JOINTS AND SHEAR-FLEXIBLE MEMBERS

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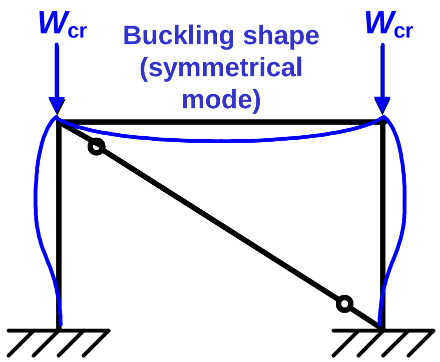
THE UNIVERSITY OF WARWICK

Stability of Shear-Flexible Frames

Objective:

To determine the elastic critical buckling load for plane frame structures possessing shear flexibility and semi-rigid joints.



Braced frame with rigid joints

WARWICK PP slide show is available from Personal Web-page.

Classes of Shear-Flexible Structures

1. Structures with batten-braced members (antennae).

2. Double-layered grid-shell structures.

3. Structures of FRP materials:

Solid or Sandwich constructed members having “high” moduli ratio *(E/G)*

- 2.6 for conventional isotropic materials (shear-rigid)
- 6 to 80 for FRP materials (increasingly shear-flexible)

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Non-linear Frame Analysis

Matrix stiffness method:

Linear elastic

Modified slope–deflection equations with stability functions:

- shear-rigid
- two different shear-flexible formulations

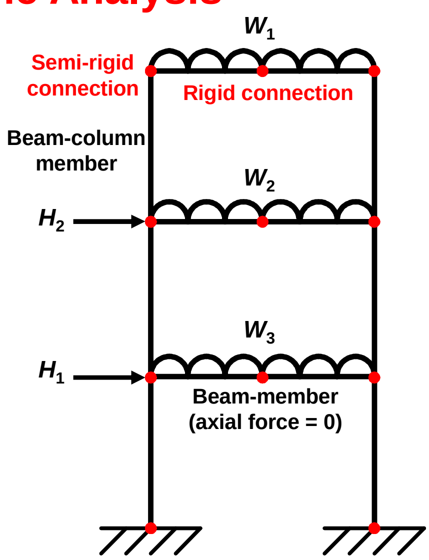
Shear-deformable elements

Semi-rigid connections (by piecewise M-ϕ curve)

Second-order P-Δ effects

Stability analysis (for W_{cr}).

No geometric imperfections modelled.



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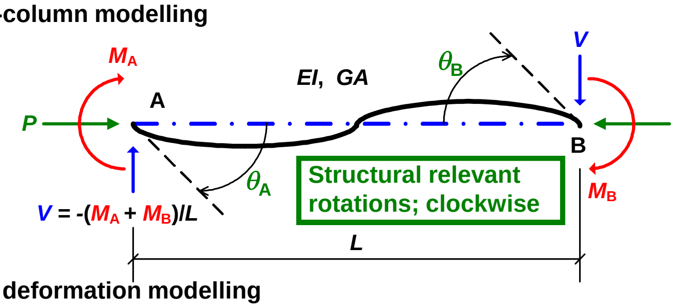
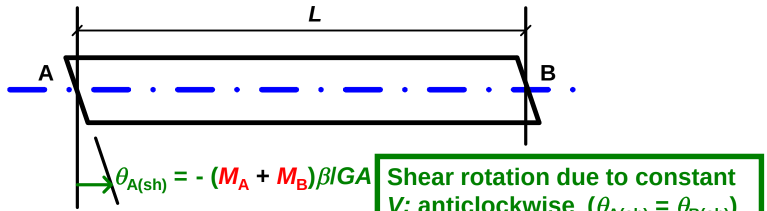
Shear-Flexible Beam-Column Element

Beam-column modelling



Shear deformation modelling



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Stability Functions

Shear-stiff stability functions

$\phi_1 = \alpha \cot \alpha$, where non-dimensional load parameters α is from

$$\rho = \frac{4\alpha^2}{\pi^2} = \frac{P}{P_E} \quad \text{, giving} \quad \alpha = \frac{\pi}{2}\sqrt{\rho} = \frac{\pi}{2}\sqrt{\frac{P}{P_E}}$$

P_E is the Euler critical buckling load for this member as a pin-ended concentric loaded shear-stiff column.

No-sway equations

$$M_A = \frac{EI}{L}(s\theta_A + sc\theta_B) \qquad M_B = \frac{EI}{L}(sc\theta_A + s\theta_B)$$

Shear-stiff stability functions (For P +ve; that is compressive)

stiffness function $s = \dfrac{\alpha^2 + \phi_1(1-\phi_1)}{(1-\phi_1)}$ carry-over function $c = \dfrac{\alpha^2 - \phi_1(1-\phi_1)}{\alpha^2 + \phi_1(1-\phi_1)}$

Typo in paper!

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Stability Functions

Shear-flexible stability functions

Need to modify ϕ_1 and α, and introduce the shear flexibility parameter

$$u = \frac{\beta P_E}{GA} = \frac{\beta \pi^2 EI}{L^2 GA}$$

Stability functions now depend on member properties

No-sway equations
$$M_A = \frac{EI}{L}\left(\bar{s}\theta_A + \bar{s}\bar{c}\theta_B\right) \qquad M_B = \frac{EI}{L}\left(\bar{s}\bar{c}\theta_A + \bar{s}\theta_B\right)$$

$$\bar{\alpha} = \frac{\pi}{2}\sqrt{\bar{\rho}} \qquad \bar{\rho} = \frac{\rho}{1 - u\rho} \qquad \bar{\phi}_1 = \bar{\alpha}\cot\bar{\alpha}$$

Shear-flexible stability functions (conventional – free θ_A and $\theta_{A(sh)}$)

$$\bar{s} = \frac{(1-u\rho)\bar{\alpha}^2 + \bar{\phi}_1\left(1-(1-u\rho)\bar{\phi}_1\right)}{\left(1-(1-u\rho)\bar{\phi}_1\right)} \qquad \bar{c} = \frac{(1-u\rho)\bar{\alpha}^2 - \bar{\phi}_1\left(1-(1-u\rho)\bar{\phi}_1\right)}{(1-u\rho)\bar{\alpha}^2 + \bar{\phi}_1\left(1-(1-u\rho)\bar{\phi}_1\right)}$$

Shear-flexible stability functions (Mottram and Aberle – for compatibility of rotation at joints where beam and beam-column members connect)

$$\bar{s}_{zero} = \frac{(1-u\rho)\left(\bar{\alpha}^2 + \bar{\phi}_1(1-\bar{\phi}_1)\right)}{(1-\bar{\phi}_1)} \qquad \bar{c}_{zero} = \frac{\bar{\alpha}^2 - \bar{\phi}_1(1-\bar{\phi}_1)}{\bar{\alpha}^2 + \bar{\phi}_1(1-\bar{\phi}_1)}$$

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$\bar{s}_{zero}$ and $\bar{c}_{zero}$ are of 'same' format as s and c!

Analysis for Critical Buckling Load

- Global stiffness matrix [*K*] is function of load because of geometric non-linearity.
- This is used as stability criterion.
- Positive determinant to [*K*] implies a stable state.
- Negative determinant implies unstable state.
- Zero determinant implies state of neutral equilibrium and critical buckling load.
- Computational result given as a normalised load factor.

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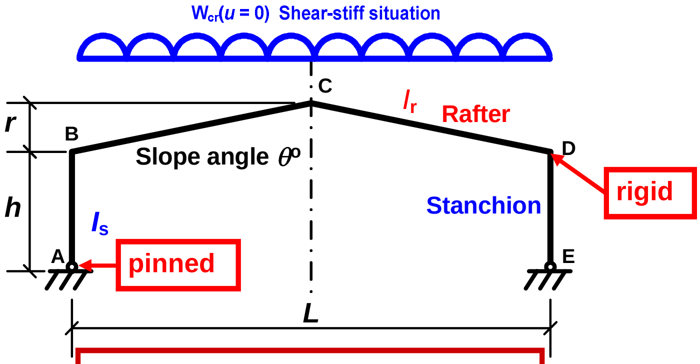
Example: Portal Frame with Sloping Rafters



Parameters:

Slope angle θ is 10°, 20° and 30°.

L/h = 4.28; r/L = 0.868 (10°); I_s/I_r = 1.128.

Frame is adequately braced against all other forms of instability.

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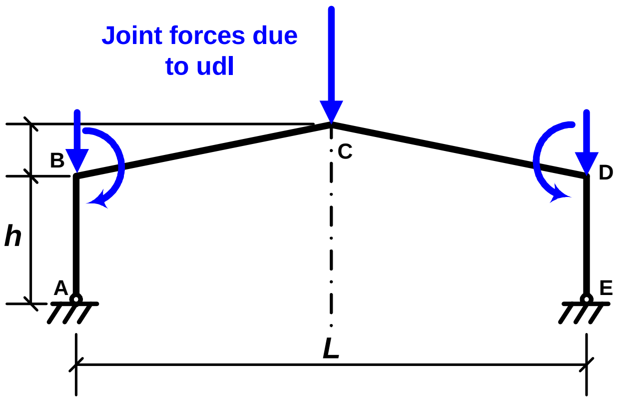
Example: Portal Frame with Sloping Rafters



Analysis: Shear flexibility parameter u increases from 0 to 1.5.

Same value of u for both stanchions and rafters.

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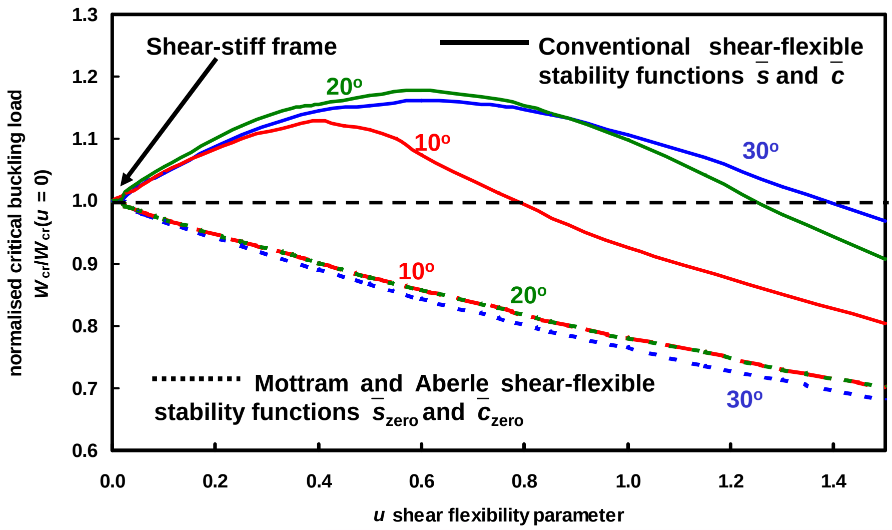
Example: Portal Frame with Sloping Rafters



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Failure is Anti-symmetrical mode; mistake in paper to say it is symmetrical

Example: Portal Frame with Sloping Rafters

Current wisdom says that shear-flexibility will reduce the instability load of continuous frames.

Do the new results indicate that the weakness in the formulation of the conventional shear-flexible stability functions is the reason for an INCREASE in buckling resistance?

POSTSCRIPT (July 2007): The same characteristic shaped curves in the plot in previous slide have now been obtained by the author solving the portal frame problem using the stability function approach. (Theory for this method is from the 1963 PhD thesis by K. I. Majid at University of Manchester).

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Frame Analysis and Codes of Practice

Guidance on the choice between first and second-order global analysis is given for STEEL structures in Clause 5.2.1 of BS EN 1993-1-1:2005. 1st-order analysis may be used provided that the effects of deformations on internal member forces and moments and on structural behaviour are negligible. This may be assumed to be the case provided that Equ. (5.1) in 5.2.1(3) is satisfied

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 10 \quad \text{for elastic analysis} \qquad (5.1)$$

- α_{cr} is the factor by which the design loading would have to be increased to cause elastic instability in a global mode,
- F_{Ed} is the design loading on the structure, *c.f.* W in portal frame example
- F_{cr} is the elastic critical buckling load for global instability mode based on initial elastic stiffnesses. *c.f.* $W_{cr(u)}$ in portal frame example

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Code of Practice for Pultruded FRP Structures

We can use shear-flexible frame analysis to establish α_{cr} for structures when shear deformation cannot be neglected.

Pultruded FRP Structures (braced and no-sway)




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Code of Practice for Pultruded FRP Structures

New Project – assistance is required (we need test data for code calibration)

"*Standard for Load Resistance Factor Design (LRFD) of Pultruded Fiber-Reinforced Polymer (FRP) Structures*", sponsored by ASCE and ACMA.

Three years, starting June 07. Limited funds; none for new physical tests.

Eight chapter drafters contributing for the "glory of it".

CHAPTERS:

1. GENERAL PROVISIONS;
2. DESIGN RESISTANCE;
3. TENSION MEMBERS;
4. COMPRESSION MEMBERS AND BEARING;
5. MEMBERS IN BENDING SHEAR;
6. MEMBERS UNDER COMBINED LOADS;
7. PLATES (Girders);
8. JOINTS AND CONNECTIONS.

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Conclusions

- We have a code that successfully predicts the elastic critical buckling load of plane frames having semi-rigid joints and shear-flexible members.
- New results, using a portal frame example with inclined rafters, suggests that the incompatibility condition in the formulation of the conventional shear-flexible stability functions $\bar{s}$ and $\bar{c}$ might be a limitation to their application.
- By using the Mottram and Aberle stability functions $\bar{s}_{zero}$ and $\bar{c}_{zero}$, formulated to give rotational compatibility, the buckling resistance is found to reduce with increasing shear-flexibility.
- Physical testing is required to further our understanding.
- Our analysis tool can be used to establish the limit to α_{cr} for 1st-order analysis to be used in the design process of elastic frames of Pultruded FRP sections (solid and batten-braced construction).

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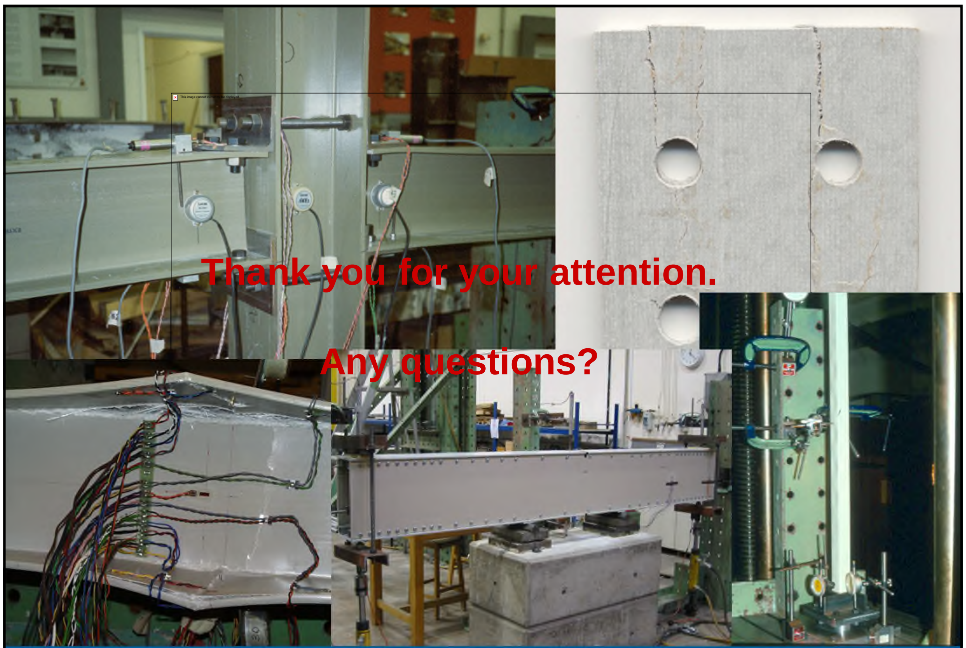

Thank you for your attention.

Any questions?

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