STABILITY ANALYSIS FOR PITCHED PORTAL FRAMES OF FIBRE REINFORCED POLYMER

by

J. Toby Mottram DSc FIStructE

School of Engineering

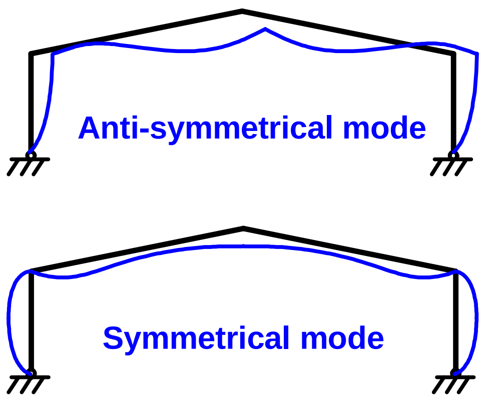
4^{th} International Conference on
FRP Composites in Civil Engineering, CICE 08

THE UNIVERSITY OF WARWICK

Stability of Shear-Flexible Frames

Objective:

To determine the elastic critical buckling load for plane frame structures of shear flexible members.



Pitched portal frame with rigid apex and haunches, and pinned bases

Classes of Shear-Flexible Structures

1. Structures with batten-braced members (antennae).

2. Double-layered grid-shell structures.

3. Structures of FRP materials:

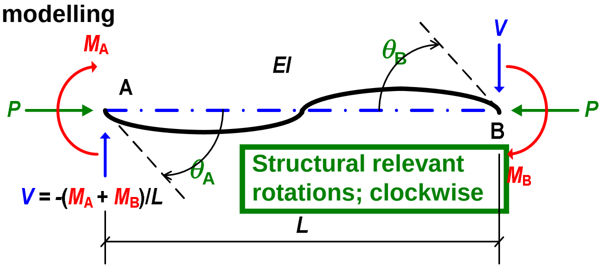
Solid or Sandwich constructed members having “high” *E*/*G* moduli ratio

- 2.6 for conventional isotropic materials (shear-rigid)
- 6 to 200 for FRP materials (increasingly shear-flexible)

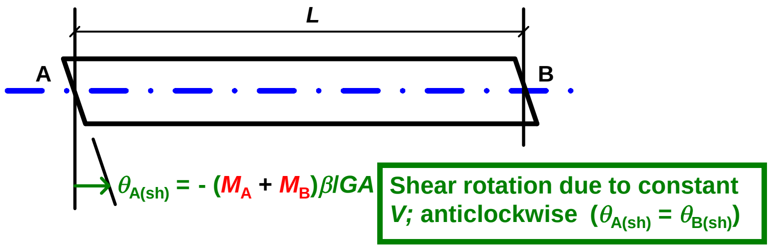
WARWICK

Shear-Flexible Beam-Column Element

Beam-column modelling



Shear deformation modelling



WARWICK

Stability Functions

CICE08-05

Shear-stiff stability functions

$\phi_1 = \alpha \cot \alpha$, where non-dimensional load parameters α is from

$$\rho = \frac{4\alpha^2}{\pi^2} = \frac{P}{P_E} \quad \text{, giving} \quad \alpha = \frac{\pi}{2}\sqrt{\rho} = \frac{\pi}{2}\sqrt{\frac{P}{P_E}}$$

P_E is the Euler critical buckling load for this member as a pin-ended concentric loaded shear-stiff column.

No-sway equations

$$M_A = \frac{EI}{L}(s\theta_A + sc\theta_B) \qquad M_B = \frac{EI}{L}(sc\theta_A + s\theta_B)$$

Shear-stiff stability functions

(For P +ve; that is compressive)

stiffness function $s = \dfrac{\alpha^2 + \phi_1(1-\phi_1)}{(1-\phi_1)}$ carry-over function $c = \dfrac{\alpha^2 - \phi_1(1-\phi_1)}{\alpha^2 + \phi_1(1-\phi_1)}$

WARWICK If P is –ve, for TENSION, expressions for s and c change.

Stability Functions

CICE08-06

Shear-flexible stability functions

Need to modify ϕ_1 and α, and introduce the shear flexibility parameter

$$u = \frac{\beta P_E}{GA} = \frac{\beta \pi^2 EI}{L^2 GA}$$

Stability functions now depend on member properties

No-sway equations

$$M_A = \frac{EI}{L}(\bar{s}\theta_A + \bar{s}\bar{c}\theta_B) \qquad M_B = \frac{EI}{L}(\bar{s}\bar{c}\theta_A + \bar{s}\theta_B)$$

$$\bar{\alpha} = \frac{\pi}{2}\sqrt{\bar{\rho}} \qquad \bar{\rho} = \frac{\rho}{1-u\rho} \qquad \bar{\phi}_1 = \bar{\alpha}\cot\bar{\alpha}$$

Shear-flexible stability functions (conventional – free θ_A and $\theta_{A(sh)}$)

$$\bar{s} = \frac{(1-u\rho)\bar{\alpha}^2 + \bar{\phi}_1(1-(1-u\rho)\bar{\phi}_1)}{(1-(1-u\rho)\bar{\phi}_1)} \qquad \bar{c} = \frac{(1-u\rho)\bar{\alpha}^2 - \bar{\phi}_1(1-(1-u\rho)\bar{\phi}_1)}{\bar{\alpha}^2 + \bar{\phi}_1(1-(1-u\rho)\bar{\phi}_1)}$$

Shear-flexible stability functions (Mottram and Aberle – for compatibility of rotation at joints where beam and beam-column members connect)

$$\bar{s}_{zero} = \frac{(1-u\rho)(\bar{\alpha}^2 + \bar{\phi}_1(1-\bar{\phi}_1))}{(1-\bar{\phi}_1)} \qquad \bar{c}_{zero} = \frac{\bar{\alpha}^2 - \bar{\phi}_1(1-\bar{\phi}_1)}{\bar{\alpha}^2 + \bar{\phi}_1(1-\bar{\phi}_1)}$$

WARWICK $\bar{s}_{zero}$ and $\bar{c}_{zero}$ are of 'same' format as s and c!

Warwick University Frame Analysis

CICE08-07

Matrix stiffness method:

Linear elastic

Modified slope–deflection equations with stability functions:

- **shear-rigid**
- **two different shear-flexible formulations** (slide CICE08-06)

Shear-deformable elements

Semi-rigid connections (by piecewise *M*-ϕ curve)

Second-order *P*-Δ effects

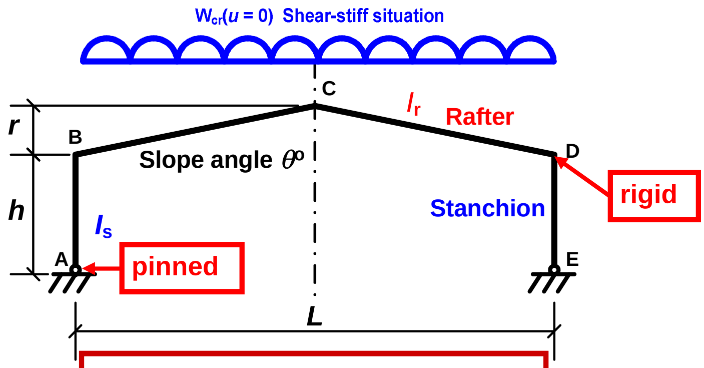
Stability analysis.

No geometric imperfections modelled.

WARWICK

Pitched Portal Frame with udl

CICE08-08



Parameters:

Slope angle θ is 10°.

L/h = 4.28; r/L = 0.868 (10°); I_s/I_r = 1.128.

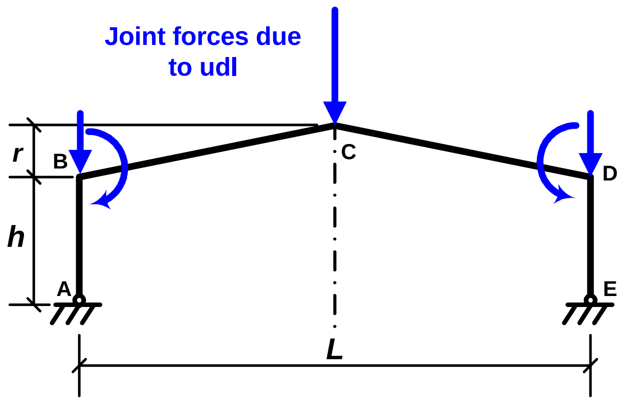
Frame is adequately braced against all other forms of instability.

WARWICK

Source: Mottram, J. T. 2007. Proc. 3rd Inter. Conf. on Advanced Composites in Construction (ACIC07).

Pitched Portal Frame with udl

CICE08-09



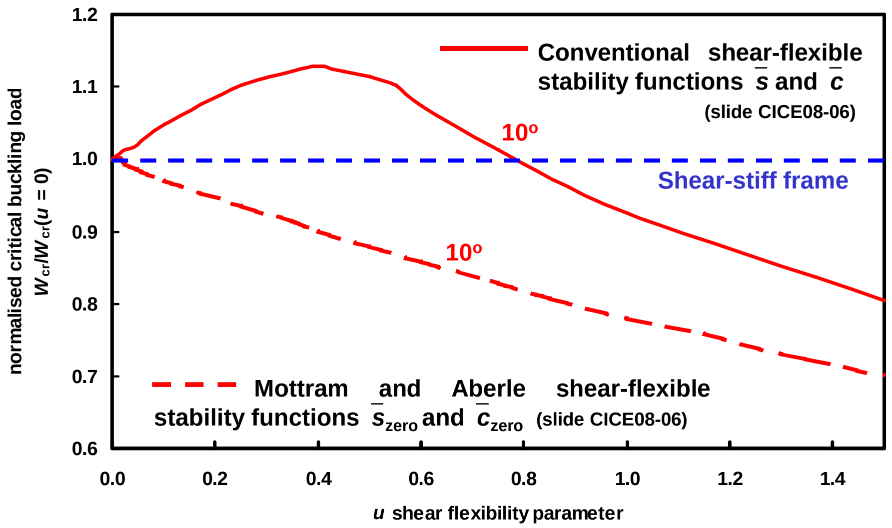
Analysis: Shear flexibility parameter *u* increases from 0 to 1.5.

Same value of *u* for both stanchions and rafters.

WARWICK

Pitched Portal Frame with udl

CICE08-10



WARWICK

CICE08-11

Buckling Resistance and Frame Shear Flexibility

Current wisdom says that shear-flexibility will reduce the instability load of continuous frames.

Does the ACIC07 contribution indicate that a weakness in the formulation of the conventional shear-flexible stability functions is the reason for an INCREASE in buckling resistance, or is it due to a coding problem?

WARWICK

CICE08-12

Majid’s Closed-form Expressions

Virtual energy method and relevant set of “fictitious” horizontal forces to perturb either symmetrical of anti-symmetrical modes. Solutions given for udl loading (as in ACIC 07) and for vertical point load at apex C. Modified so that rafter and stanchion members have different second moment of areas and “constant” elastic rafter compression force is calculated by a Klienlogel formula.

Majid and udl and anti-symmetrical mode

$$0 = 2(1-\phi_{1,s}) - \frac{2\cos\theta\ s_r(1-c_r^2)(1-c_s)}{q\,s_s(1-c_s^2)+2\cos\theta\,s_r(1-c_r^2)}$$

with $\rho_s = \frac{W_{cr}}{2P_{E,s}}$ $\qquad q = \frac{L}{h}$ $\qquad k = \frac{r}{h}$

$$\rho_r = \frac{\rho_s}{2}\left(\frac{I_s}{I_r}\right)\left(\frac{L}{2\cos\theta\,h}\right)^2\left[\frac{q(3+5(1+k))\cos\theta}{8((I_r 2\cos\theta\,h)/(I_s L)+1+(1+k)+(1+k)^2)}+\sin\theta\right]$$

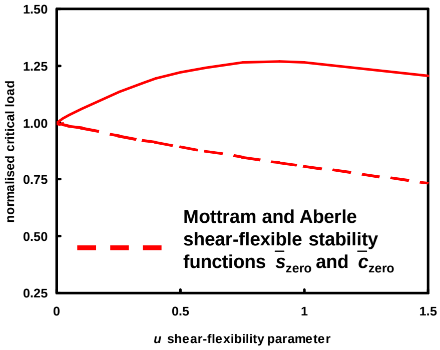
WARWICK

Source: Majid, K. I. *Elastic Plastic Structural Analysis*, PhD thesis, University of Manchester, 1963.

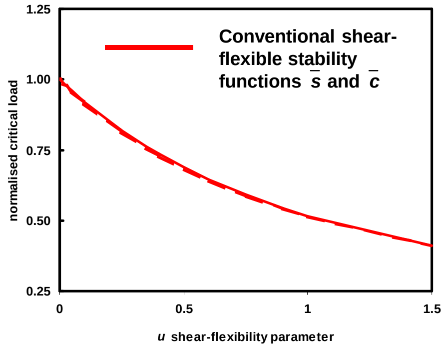
CICE08-13

Majid’s – UDL loading (see also ACIC07)

Anti-symmetrical mode



Symmetrical mode



Same variations in resistance with the anti-symmetrical mode.

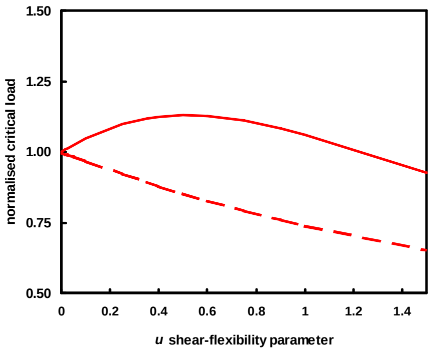
Choice of shear flexibility functions does not change buckling resistance with the symmetrical mode (shear stiff critical load is 3.75 times higher than anti-symmetrical)

WARWICK

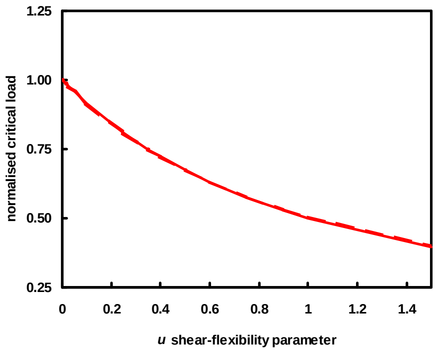
CICE08-15

Majid’s – Apex Point Loading

Anti-symmetrical mode



Symmetrical mode



This example does not provide uncertainty on what is the ‘constant’ rafter compression force.

Findings from uld loading example (slide CICE08-14) are supported.

WARWICK

Shear stiff critical load for symmetrical mode is 2.71 times higher than for the anti-symmetrical mode.

Conclusions

CICE08-16

- Two different analytical methods to determine the elastic critical buckling load of pitched portal frame problems have shown that the incompatibility condition in the formulation of the conventional shear-flexible stability functions $\bar{s}$ and $\bar{c}$ might be a limitation to their application when the mode of instability is anti-symmetrical.
- By using the Mottram and Aberle stability functions $\bar{s}_{zero}$ and $\bar{c}_{zero}$, formulated to give rotational compatibility, buckling resistance is found to continuously reduce with increasing member shear-flexibility.
- An increase in buckling resistance with member shear flexibility is not to be expected from finite element simulations using 'stick' elements formulated using Timoshenko beam theory.
- Physical testing is required to further our understanding.

WARWICK

Thank you for your attention.

Any questions?

Conferences for 2009, held in Edinburgh, Scotland.

17th Inter. Conf. on Composite Materials (ICCM17) 27-31 July. Deadline for abstracts is 31st Oct. 2008 (Topic: *Advanced Composite Materials in Construction* (Urs Meier, Toby Mottram, Geoffrey Turvey))

4th Inter. Conf. on Advanced Composites in Construction (ACIC 09) 1-3 September. (Abstracts to Claire Whysall at: info@acic-conference.com Deadline is 3rd Nov. 2008)

WARWICK Email: J.T.Mottram@warwick.ac.uk