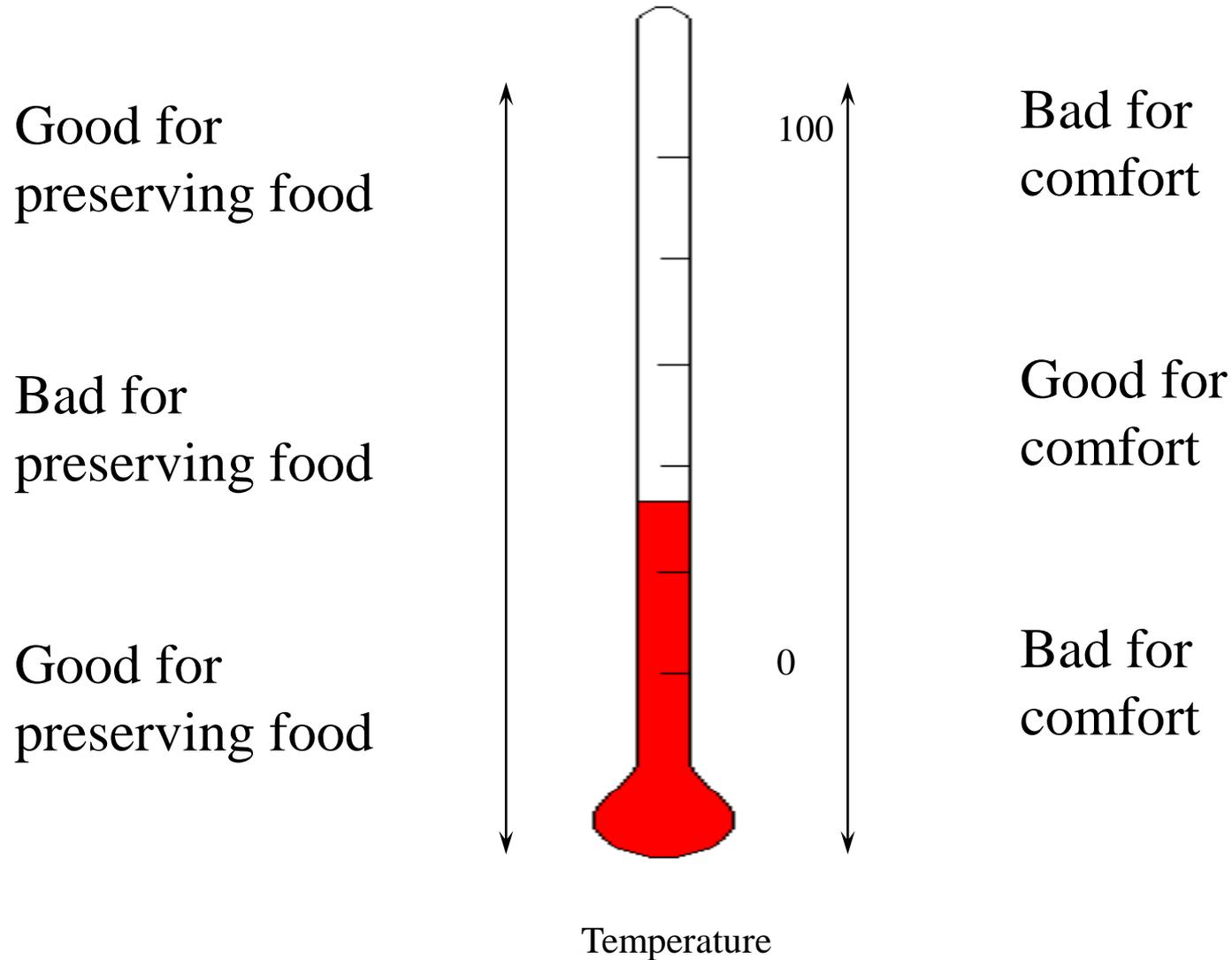


# The Analytic Hierarchy Process (AHP) for Decision Making

---

Decision Making involves setting priorities and the AHP is the methodology for doing that.

# Nonmonotonic Relative Nature of Absolute Scales



# Making a Decision

Widget B is cheaper than Widget A

Widget A is better than Widget B

Which Widget would you choose?

# Basic Decision Problem

Criteria:      Low Cost > Operating Cost > Style

|               |   |   |   |
|---------------|---|---|---|
| Car:          | A | B | B |
|               | V | V | V |
| Alternatives: | B | A | A |

Suppose the criteria are preferred in the order shown and the cars are preferred as shown for each criterion. Which car should be chosen? It is desirable to know the strengths of preferences for tradeoffs.

# Analytic Hierarchy Process (AHP)

- The Analytic Hierarchy Process (AHP) is a *multi-dimensional, multi-level* and *multifactorial* decision-making method based on the idea that it is possible to prioritize elements by:
  - grouping them into meaningful categories and sub-categories (*hierarchy*);
  - performing *pairwise comparisons*;
  - defining a coherent framework of *quantitative and qualitative* knowledge;
  - measuring *intangible* domains.
- This hierarchical approach allows:
  - the construction of a consistent framework for step-by-step decision-making,
  - breaking a complex problem into many small less-complex ones that decision-makers can more easily deal with.

*Hierarchic  
Thinking*



**GOAL**

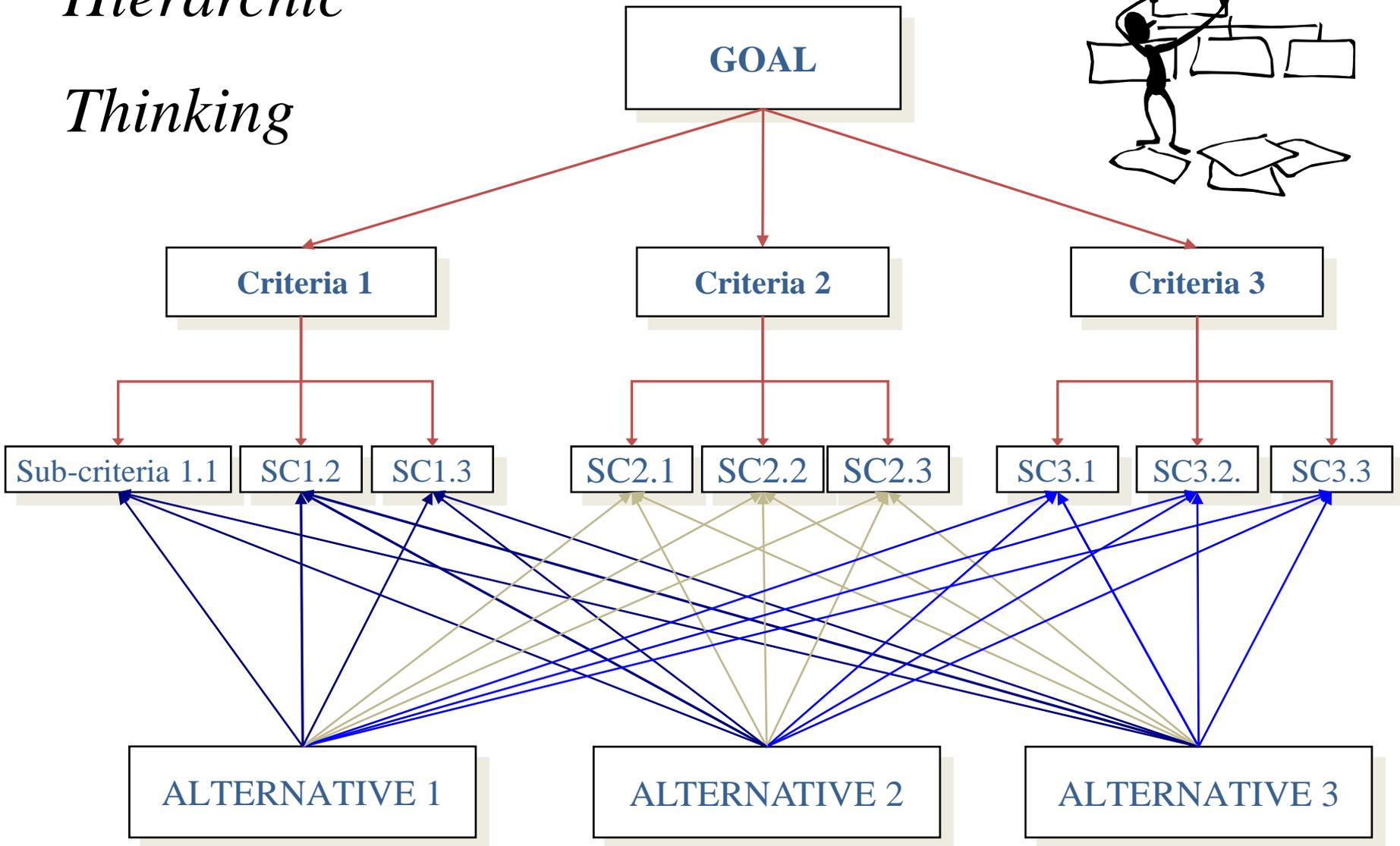
**CRITERIA**



**ALTERNATIVES**



# Hierarchic Thinking



# Relative Measurement

## The Process of Prioritization

### **Relative measurements:**

judgment expressed on pair of elements with respect to a common property they share.

A **pair of elements**, in a level of the hierarchy, are compared with respect to **parent** elements to which they relate in the level above.

# Relative Measurement (cont.)

If, for example, we are comparing two apples according to size we ask:

- Which apple is bigger?
- How much bigger is the larger than the smaller apple?  
*Use the smaller as the unit and estimate how many more times bigger is the larger one.*
- The apples must be relatively close (homogeneous) if we hope to make an accurate estimate.

# Relative Measurement (cont.)

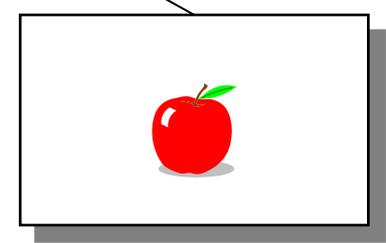
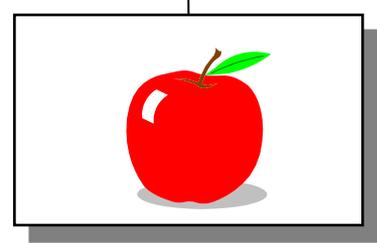
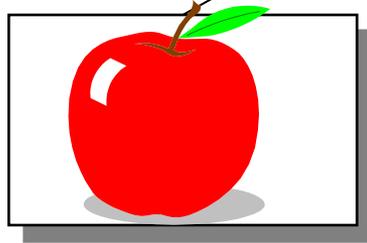
## Algorithm:

- The smaller apple is assumed as pivot value [unit]
- Judgment are assumed to be reciprocal [if  $A=3B \Rightarrow B=1/3A$ ]
- If the objects under comparison are not homogeneous, we organize them in homogeneous categories (i.e. almost the same size)
- Max 7-9 elements in each category



**GOAL**

**Dimension**

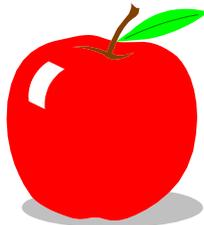


**CRITERIA**

**ALTERNATIVES**

# Comparison Matrix

Given: Three apples of different sizes.



**Apple A**



**Apple B**

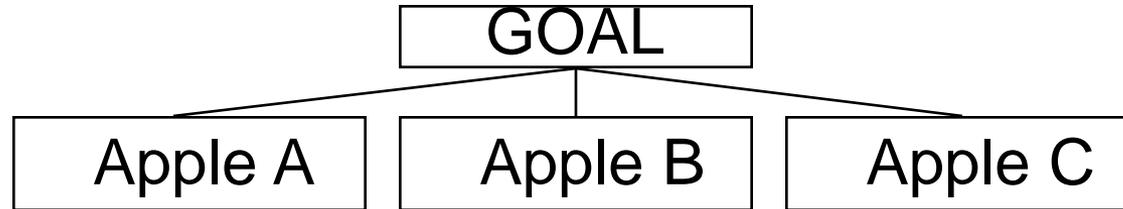


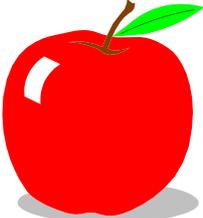
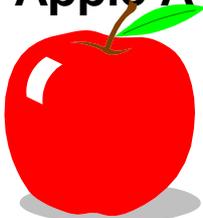
**Apple C**

We assess their Relative sizes by forming ratios

| <b>Size Comparison</b> | <b>Apple A</b> | <b>Apple B</b> | <b>Apple C</b> |
|------------------------|----------------|----------------|----------------|
| <b>Apple A</b>         | $S_1/S_1$      | $S_1/S_2$      | $S_1/S_3$      |
| <b>Apple B</b>         | $S_2/S_1$      | $S_2/S_2$      | $S_2/S_3$      |
| <b>Apple C</b>         | $S_3/S_1$      | $S_3/S_2$      | $S_3/S_3$      |

# Pairwise Comparisons



| Size Comparison  |         | Apple A   | Apple B  | Apple C   | Resulting Priority Eigenvector | Relative Size of Apple |
|--|---------|---|--|---|--------------------------------|------------------------|
|   | Apple A |  |  |  |                                |                        |
|   | Apple B | <b>1/2</b>  | <b>1</b>   | <b>3</b>  | <b>3/10</b>                    | <b>B</b>               |
|  | Apple C | <b>1/6</b>  | <b>1/3</b>   | <b>1</b>  | <b>1/10</b>                    | <b>C</b>               |

When the judgments are consistent, as they are here, any normalized column gives the priorities.

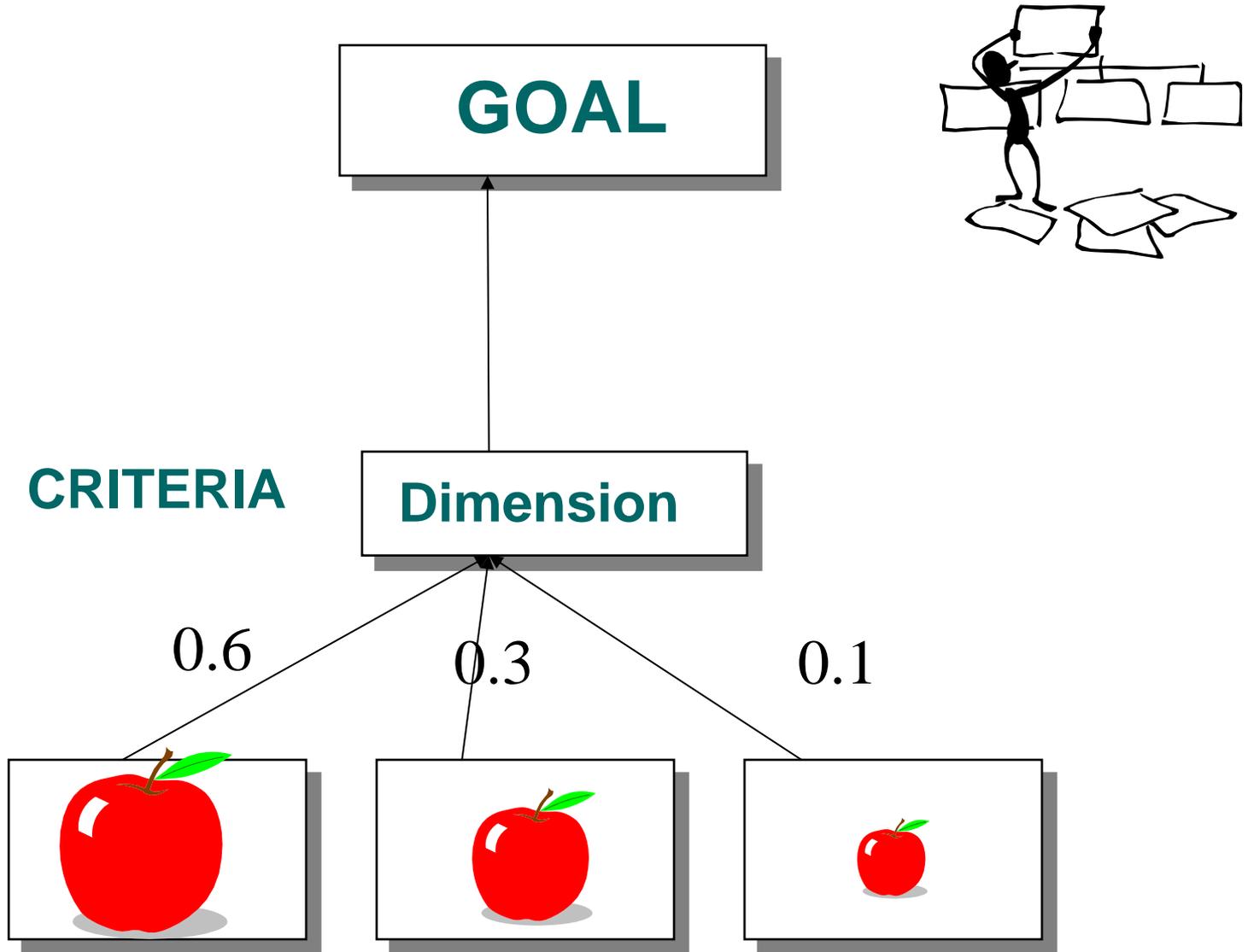
$$\left\{ \begin{array}{l} A = 2B \\ A = 6C \\ B = 3C \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} A = 6C \\ B = 3C \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} A = 6C \\ B = 3C \\ C = 1 \end{array} \right.$$

Normalizing to the max (6):

$$\left\{ \begin{array}{l} A = 1 \\ B = 0.5 \\ C = 1.67 \end{array} \right.$$

Normalizing to the tot  
(6+3+1=10):

$$\left\{ \begin{array}{l} A = 0.6 \\ B = 0.3 \\ C = 0.1 \end{array} \right.$$



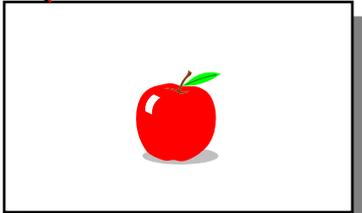
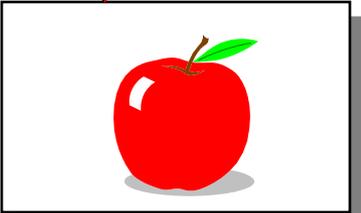
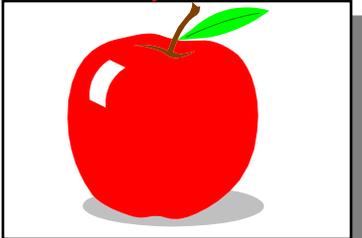
**GOAL**  
**(big and cheap)**



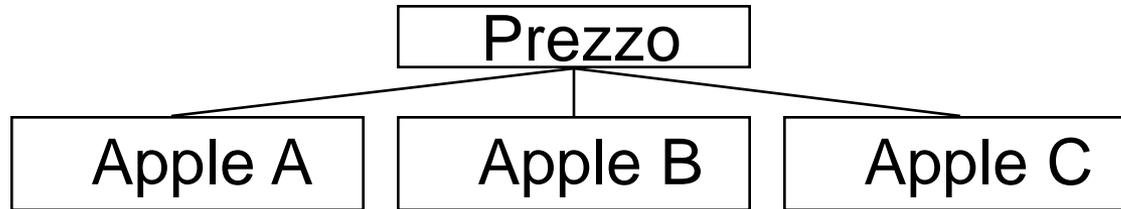
**CRITERIA**

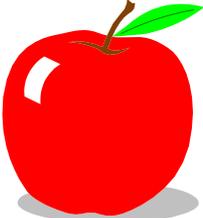
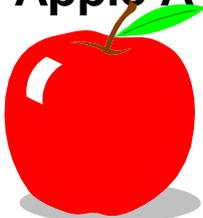
**Dimension**

**Lower Cost**



# Pairwise Comparisons



| Size Comparison  | Apple A   | Apple B  | Apple C   |                                |                        |
|--|---|--|---|--------------------------------|------------------------|
| <br><br> |  |  |  | Resulting Priority Eigenvector | Relative Size of Apple |
| Apple A  | 1   | 1/3  | 1/6   | 1/10                           | A                      |
| Apple B  | 3   | 1  | 1/2   | 3/10                           | B                      |
| Apple C  | 6   | 2  | 1   | 6/10                           | C                      |

When the judgments are consistent, as they are here, any normalized column gives the priorities.

# La mela ottima (+ grande; - cara)



CRITERIA

dimensione

prezzo

0.6

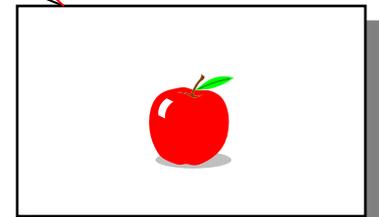
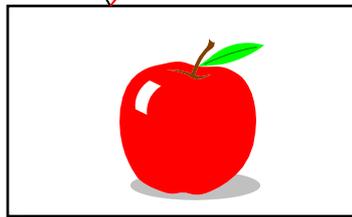
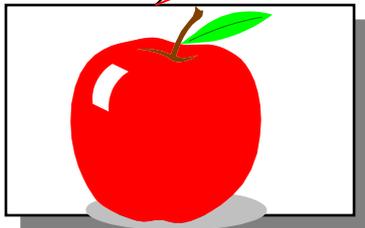
0.1

0.3

0.3

0.1

0.6



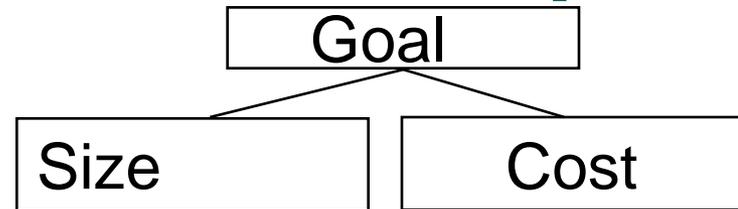
Dr. Leandro Pecchia

L.Pecchia@warwick.ac.uk

ALTERNATIVES

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# Pairwise Comparisons



|             | <b>Size</b> | <b>Cost</b> | <b>Priority<br/>Eigenvector</b> | <b>Normalized<br/>(al tot)</b> |
|-------------|-------------|-------------|---------------------------------|--------------------------------|
| <b>Size</b> | <b>1</b>    | <b>2</b>    | <b>0.89</b>                     | <b>0,67</b>                    |
| <b>Cost</b> | <b>0.5</b>  | <b>1</b>    | <b>0.45</b>                     | <b>0.33</b>                    |

# GOAL (Size; Cost)



## CRITERIA

dimensione

prezzo

0.67

0.33

0.6

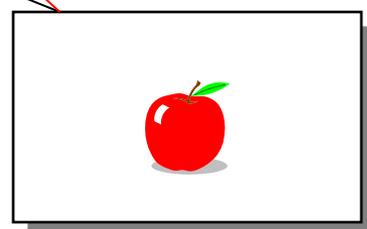
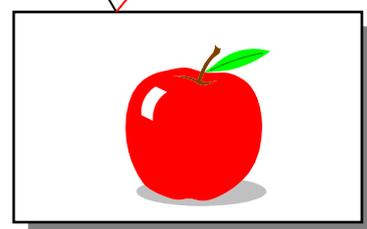
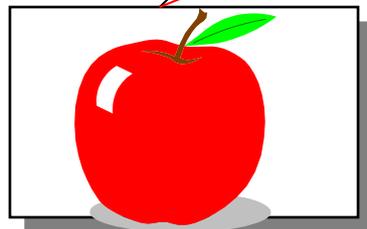
0.1

0.3

0.3

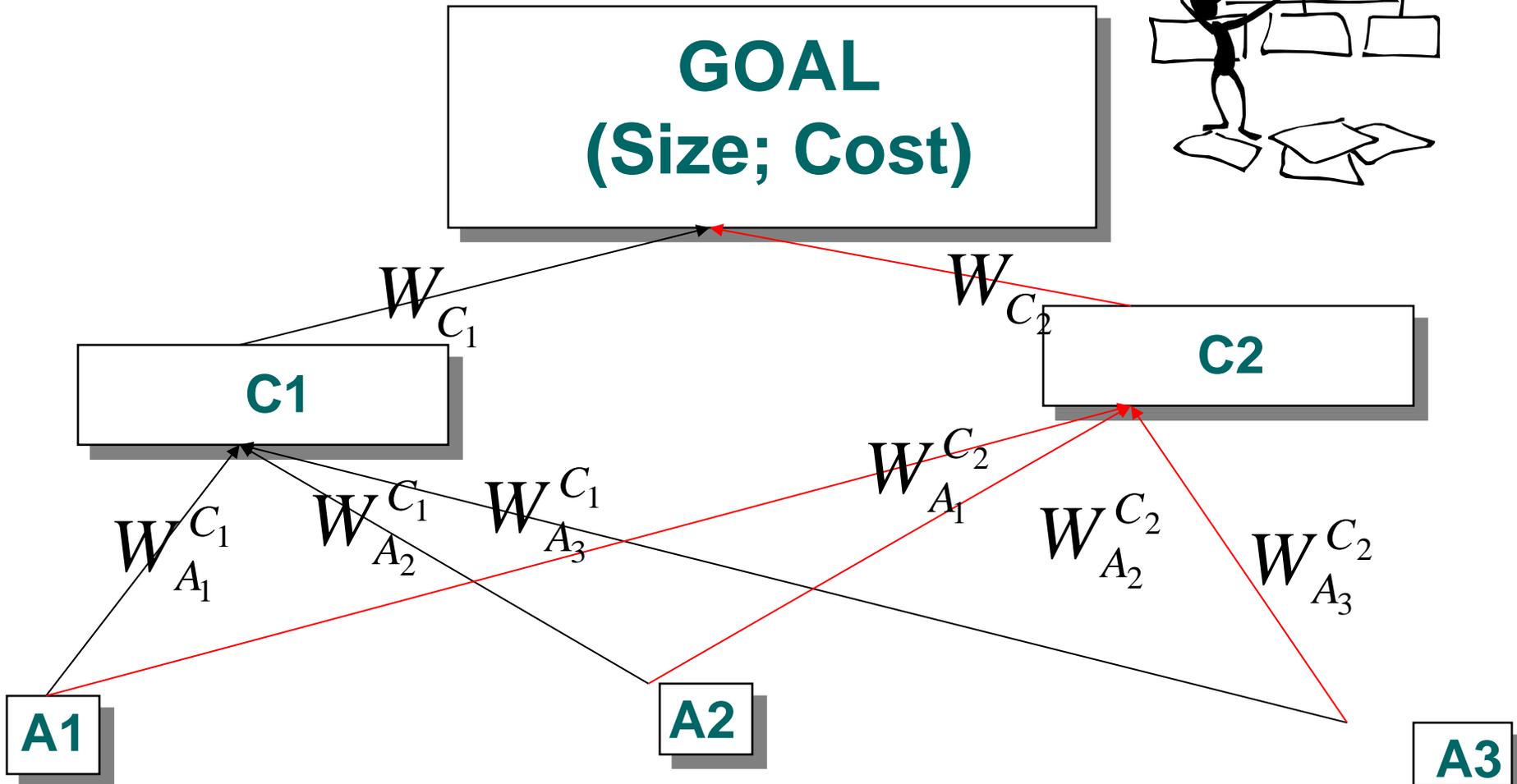
0.1

0.6



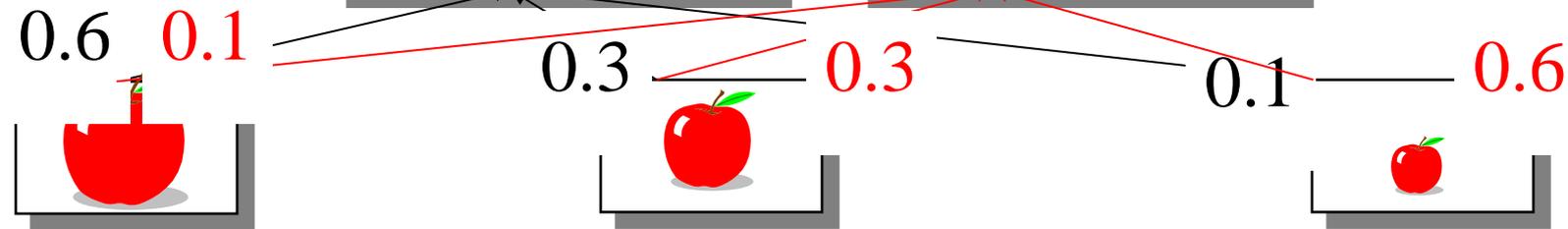
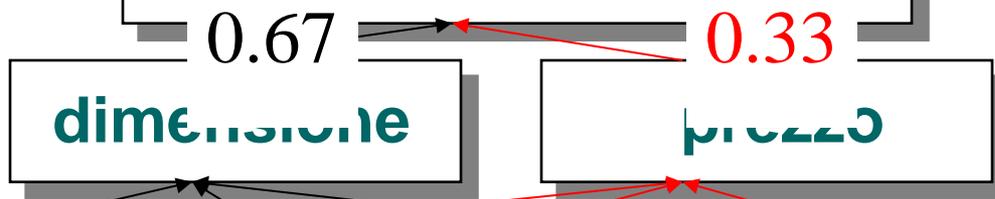
## ALTERNATIVES

**GOAL**  
**(Size; Cost)**



$$P_{A_1} = \sum_{i=1}^{n_{c_j}=2} W_{C_j} * W_{A_1}^{C_i} = (W_{C_1}; W_{C_2}) * \begin{pmatrix} W_{A_1}^{C_1} \\ W_{A_1}^{C_2} \end{pmatrix} = \vec{W}_c * \vec{W}_{A_1}^C$$

# GOAL (Size; Cost)



$$P_{A1} = (0,67;0,33) * \begin{pmatrix} 0,6 \\ 0,1 \end{pmatrix} = 0,435$$

$$P_{A2} = (0,67;0,33) * \begin{pmatrix} 0,3 \\ 0,3 \end{pmatrix} = 0,3$$

$$P_{A3} = (0,67;0,33) * \begin{pmatrix} 0,1 \\ 0,6 \end{pmatrix} = 0,265$$

**GOAL**  
**(Size; Cost)**



$$W_{C_1} * W_{A_1}^{C_1} + W_{C_2} * W_{A_1}^{C_2}$$

**A1**

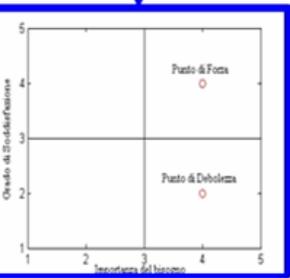
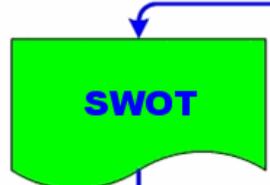
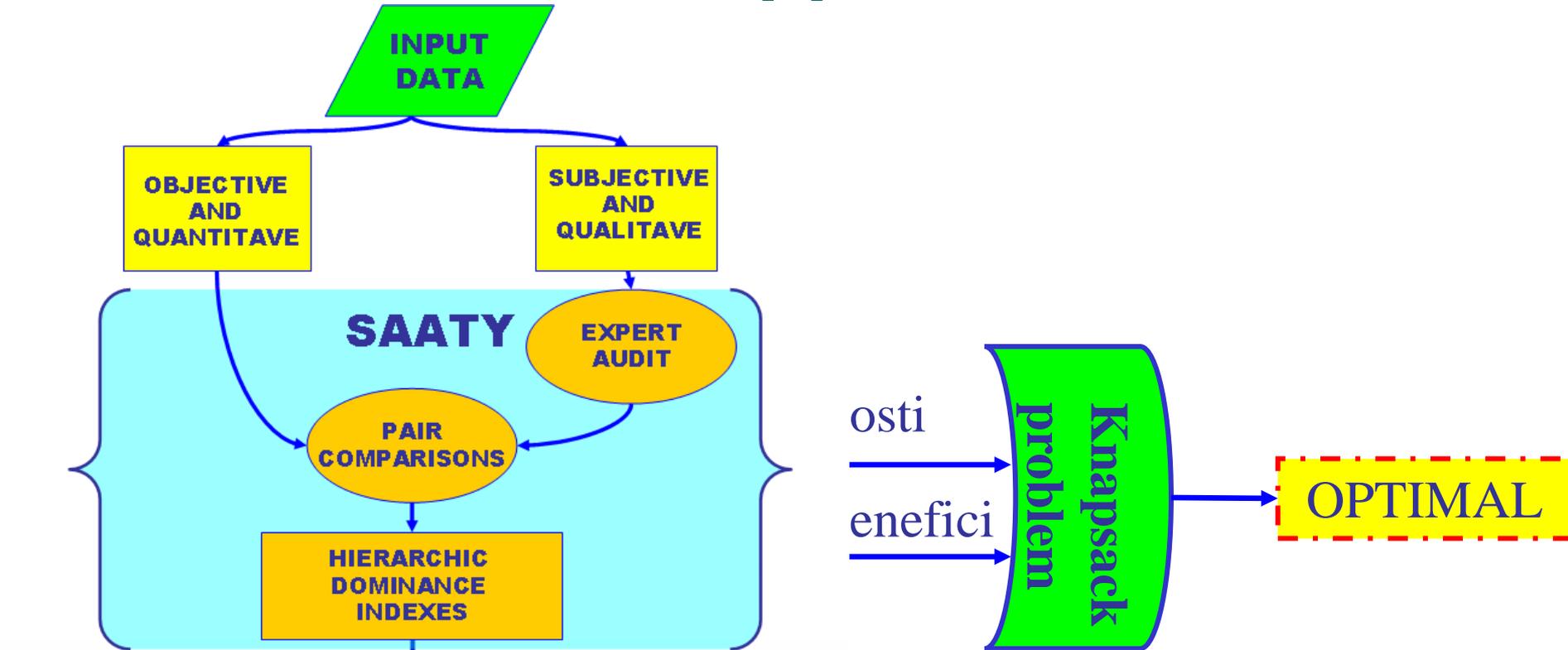
$$W_{C_1} * W_{A_3}^{C_1} + W_{C_2} * W_{A_3}^{C_2}$$

**A3**

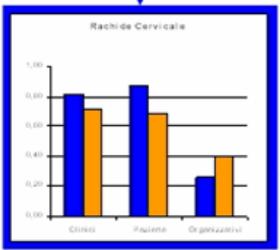
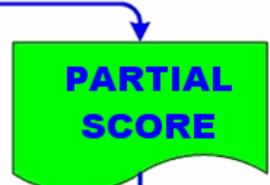
$$W_{C_1} * W_{A_2}^{C_1} + W_{C_2} * W_{A_2}^{C_2}$$

**A2**

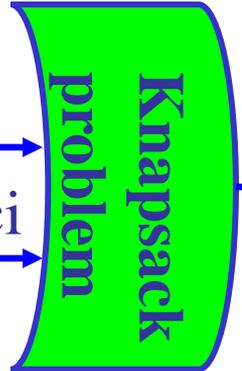
# Possibile modello Applicativo



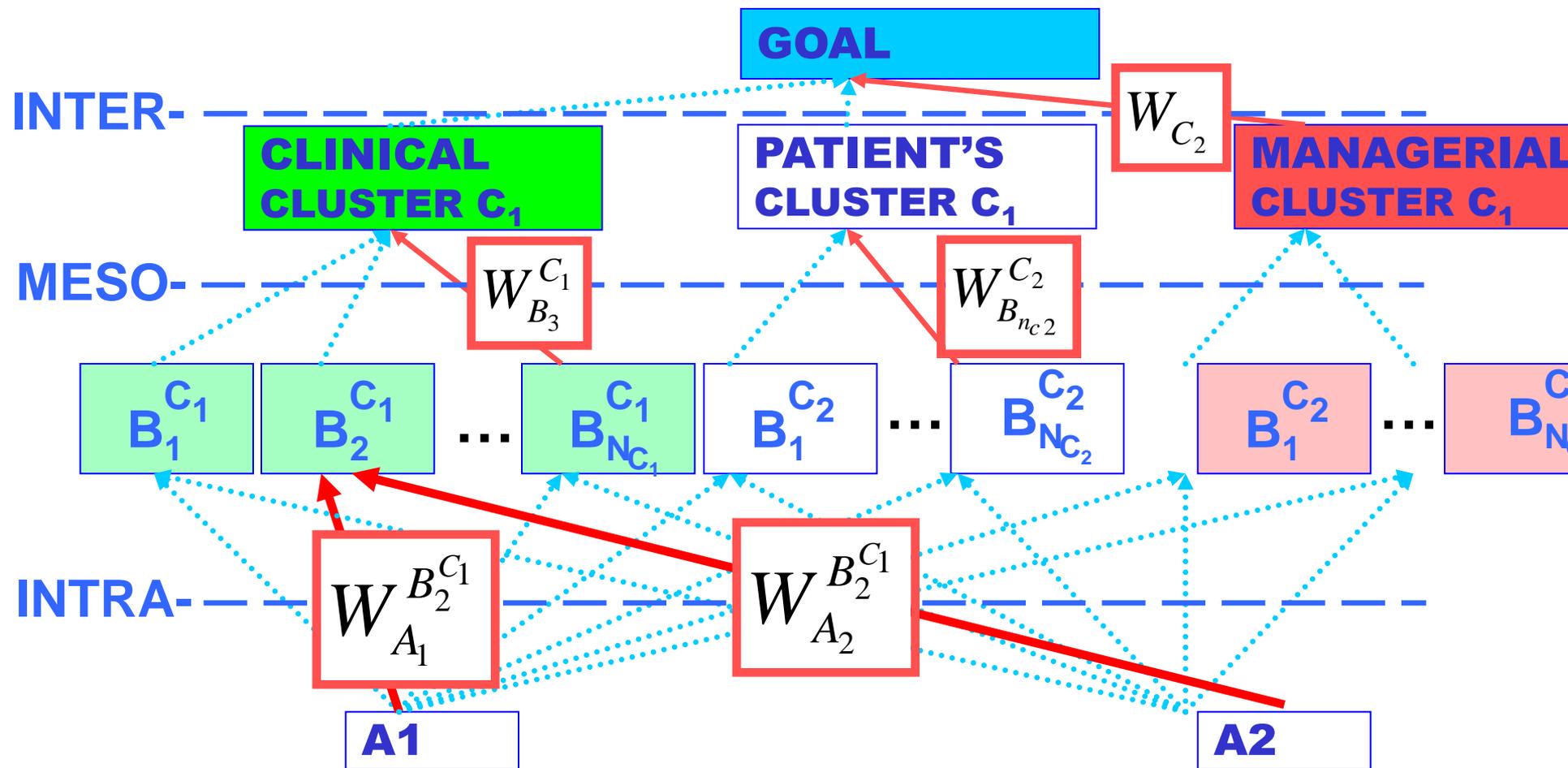
| Rachide Cervicale | Somafix          | Cage             |
|-------------------|------------------|------------------|
| IVT               | 0,638            | 0,555            |
| Rachide Lombare   | Barre in Nitinol | Barre in Titanio |
| IVT               | 0,592            | 0,572            |



costi  
benefici



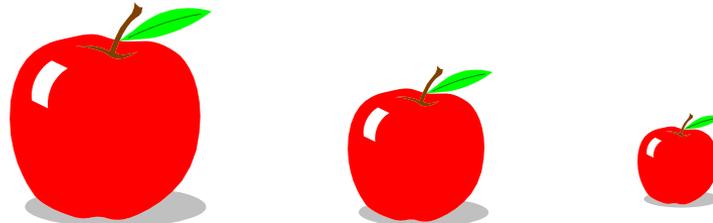
OPTIMAL



$$TotScore \quad A_1 = \sum_{j=1}^m W_{C_j} * \sum_{i=1}^{n_{c_j}} W_{B_i}^{C_j} * W_{A_1}^{B_i^{C_j}}$$

# Consistency (cont.)

- Consistency itself is a necessary condition for a better understanding of relations in the world but it is not sufficient. For example we could judge all three of the apples to be the same size and we would be perfectly consistent, but very wrong.
- We also need to improve our validity by using redundant information.
- It is fortunate that the mind is not programmed to be always consistent. Otherwise, it could not integrate new information by changing old relations.



# Consistency

Apple B is 3 times larger than Apple C.

We can obtain this value directly from the comparisons of Apple A with Apples B & C as  $6/2 = 3$ .

$$\begin{cases} A = 2B \\ A = 6C \end{cases} \Rightarrow B = \frac{6}{2} C = 3C$$

# Consistency

Alternatively, we could ask a third question (redundant) and use this info to measure the responder consistency.

But if we were to use judgment we may have guessed  $B = 4C$  (and not 3).

In that case we would have been inconsistent.

# Consistency

Now guessing it as 4 is not as bad as guessing it as 5 or more.

The farther we are from the true value the more inconsistent we are.

The AHP provides a theory for checking the inconsistency throughout the matrix and *allowing a certain level of overall inconsistency but not more.*

# Consistency (cont.)

- We can say the the matrix  $M$ ...

$$\underline{\underline{M}} = \begin{pmatrix} a_1^1 & a_2^1 & a_3^1 \\ a_1^2 & a_2^2 & a_3^2 \\ a_1^3 & a_2^3 & a_3^3 \end{pmatrix}$$

...is consistent  $\Leftrightarrow \{a_j^i = a_1^i * a_j^{i-1}\}$  for any  $i, j$

# Consistency (cont.)

- It is possible to show that:

$$\underline{\underline{M}} = \begin{pmatrix} 1 & a_2^1 & a_3^1 \\ 1/a_2^1 & 1 & a_3^2 \\ 1/a_1^3 & 1/a_2^3 & 1 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 1 & a_2^1 & a_3^1 \\ 1/a_2^1 & 1 & a_3^1/a_2^1 \\ 1/a_1^3 & a_2^1/a_3^1 & 1 \end{pmatrix}$$

$$\left\{ a_3^2 = a_1^2 * a_3^1 \right\}$$

- then, this matrix will have:
  - one and only one eigenvalue  $\lambda$
  - $\lambda$  will be equal to  $n$  (number of rows and columns):

$$Mx = \lambda x = nx \Leftrightarrow \exists! \lambda = n$$

# In-Consistency

The theorem of eigenvalues perturbations suggests that:

If  $\lambda$  is an eigenvalue of  $M$ , than for small perturbation  $\varepsilon > 0$ , it exist an eigenvalue  $\lambda(\varepsilon)$  of the matrix  $A(\varepsilon)$  which can be written as Taylor power expansion:

$$\lambda(\varepsilon) = \lambda + \varepsilon \lambda^{(1)} + \varepsilon^2 \lambda^{(2)} + \dots$$

and corresponding right and left eigenvectors  $w(\varepsilon)$  and  $v(\varepsilon)$  such that:

$$w(\varepsilon) = w + \varepsilon w^{(1)} + \varepsilon^2 w^{(2)} + \dots$$

$$v(\varepsilon) = v + \varepsilon v^{(1)} + \varepsilon^2 v^{(2)} + \dots$$

and:

$$\lim_{\varepsilon \rightarrow 0} \lambda(\varepsilon) = \lambda$$

$$\sum_{j=1}^n a_{ij} w_j = \lambda_{\max} w_i \quad \sum_{i=1}^n w_i = 1$$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad \exists \lambda / Ax = \lambda x \Leftrightarrow \lambda \text{ is eigenvalue}$$

$$\det(A - \lambda I) = p(\lambda) = 0$$

$$|A - \lambda| = \begin{bmatrix} -\lambda & 1 \\ -1 & -2 - \lambda \end{bmatrix} = (\lambda + 1)^2 = 0 \Leftrightarrow \begin{cases} \lambda = -1 \\ \text{ma}(\lambda) = 2 \end{cases}$$

$$(A - \lambda_1 I) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{bmatrix} -\lambda_1 & 1 \\ -1 & -2 - \lambda_1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$V_\lambda = \langle \mathbf{1} \quad -\mathbf{1} \rangle$$

# In-Consistency

How to measure the in-consistency:

1. *Consistency index:*

$$C.I. = \frac{\lambda_{\max} - n}{n - 1}$$

2. *Random Consistency Index (R.I.):*

|      |   |   |     |     |      |      |      |      |      |      |
|------|---|---|-----|-----|------|------|------|------|------|------|
| n    | 1 | 2 | 3   | 4   | 5    | 6    | 7    | 8    | 9    | 10   |
| R.I. | 0 | 0 | .52 | .89 | 1.11 | 1.25 | 1.35 | 1.40 | 1.45 | 1.49 |

3. *Consistency Ratio (C.R.):*

$$C.R. = \frac{C.I.}{R.I.}$$

4. If  $C.R. \leq 0.1$  consistent.

# In-Consistency

The R.I. was calculated by Forman (1990):

1. Computing all the reciprocal matrices containing values 1-9;

$$\left[ n = 3 \Leftrightarrow 3^9 = 19.683 \right] \quad \left[ n = 4 \Leftrightarrow 4^9 = 262.144 \right]$$

2. Calculating the average of all these.

Alternativel, DeSchutter's Congecture:

$$R.I. = 1.98 \left[ 1 - \frac{n-1}{n(n-1)/2} \right]$$

When the judgments are consistent, we have two ways to get the answer:

1. By adding any column and dividing each entry by the total, that is by **normalizing** the column, any column gives the same result. A quick test of consistency is if all the columns give the same answer.
2. By adding the rows and normalizing the result.

When the judgments are inconsistent we have two ways to get the answer:

1. An approximate way: By normalizing each column, forming the row sums and then normalizing the result.
2. The exact way: By raising the matrix to powers and normalizing its row sums

# Comparison of Intangibles

The same procedure as we use for size can be used to compare things with intangible properties. For example, we could also compare the apples for:

- TASTE
- AROMA
- RIPENESS

# The Analytic Hierarchy Process (AHP) is the Method of Prioritization

- AHP captures priorities from paired comparison judgments of the elements of the decision with respect to each of their parent criteria.
- Paired comparison judgments can be arranged in a matrix.
- Priorities are derived from the matrix as its principal eigenvector, which defines an absolute scale. Thus, the eigenvector is an intrinsic concept of a correct prioritization process. It also allows for the measurement of inconsistency in judgment.
- Priorities derived this way satisfy the property of an absolute scale.

# Decision Making

We need to prioritize both tangible and intangible criteria:

- ◆ In most decisions, intangibles such as
  - political factors and
  - social factorstake precedence over tangibles such as
  - economic factors and
  - technical factors
- ◆ It is not the precision of measurement on a particular factor that determines the validity of a decision, but the importance we attach to the factors involved.
- ◆ How do we assign importance to all the factors and synthesize this diverse information to make the best decision?

# Verbal Expressions for Making Pairwise Comparison Judgments

Equal importance

Moderate importance of one over another

Strong or essential importance

Very strong or demonstrated importance

Extreme importance

# Fundamental Scale of Absolute Numbers Corresponding to Verbal Comparisons

- 1 Equal importance
- 3 Moderate importance of one over another
- 5 Strong or essential importance
- 7 Very strong or demonstrated importance
- 9 Extreme importance
- 2,4,6,8 Intermediate values

Use Reciprocals for Inverse Comparisons

# Extending the 1-9 Scale to 1- $\infty$

- The 1-9 AHP scale does not limit us if we know how to use clustering of similar objects in each group and use the largest element in a group as the smallest one in the next one. It serves as a pivot to connect the two.
- We then compare the elements in each group on the 1-9 scale get the priorities, then divide by the weight of the pivot in that group and multiply by its weight from the previous group. We can then combine all the groups measurements as in the following example comparing a very small cherry tomato with a very large watermelon.



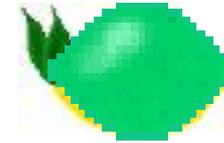
.07

Unripe Cherry Tomato



.28

Small Green Tomato



.65

Lime

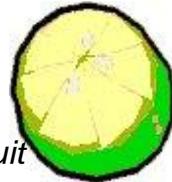


.08

Lime

$$\frac{.08}{.08} = 1$$

$$1 \times .65 = .65$$

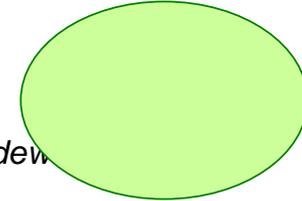


.22

Grapefruit

$$\frac{.22}{.08} = 2.75$$

$$2.75 \times .65 = 1.79$$

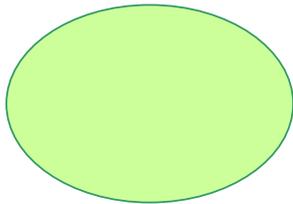


.70

Honeydew

$$\frac{.70}{.08} = 8.75$$

$$8.75 \times .65 = 5.69$$

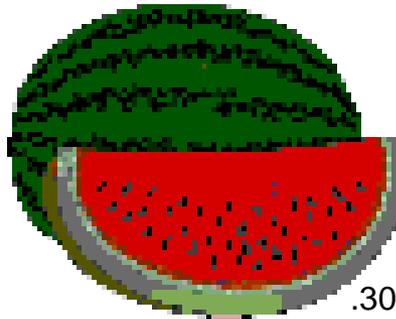


.10

Honeydew

$$\frac{.10}{.10} = 1$$

$$1 \times 5.69 = 5.69$$

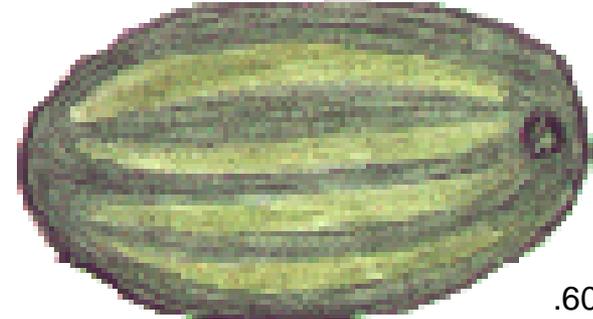


.30

Sugar Baby Watermelon

$$\frac{.30}{.10} = 3$$

$$3 \times 5.69 = 17.07$$



.60

Oblong Watermelon

$$\frac{.60}{.10} = 6$$

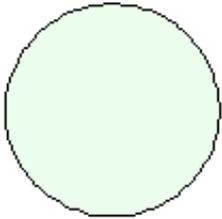
$$6 \times 5.69 = 34.14$$

This means that  $34.14 / .07 = 487.7$  unripe cherry tomatoes are equal to the oblong watermelon

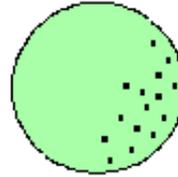
# Clustering & Comparison

## Color

How intensely more green is X than Y relative to its size?



Honeydew



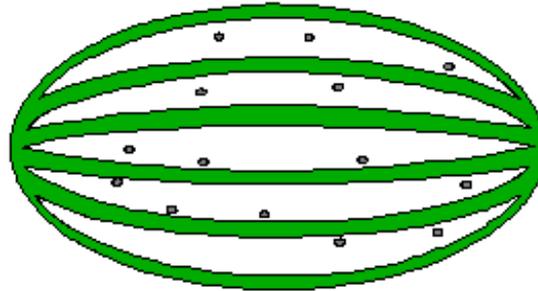
Unripe Grapefruit



Unripe Cherry Tomato



Unripe Cherry Tomato



Oblong Watermelon



Small Green Tomato



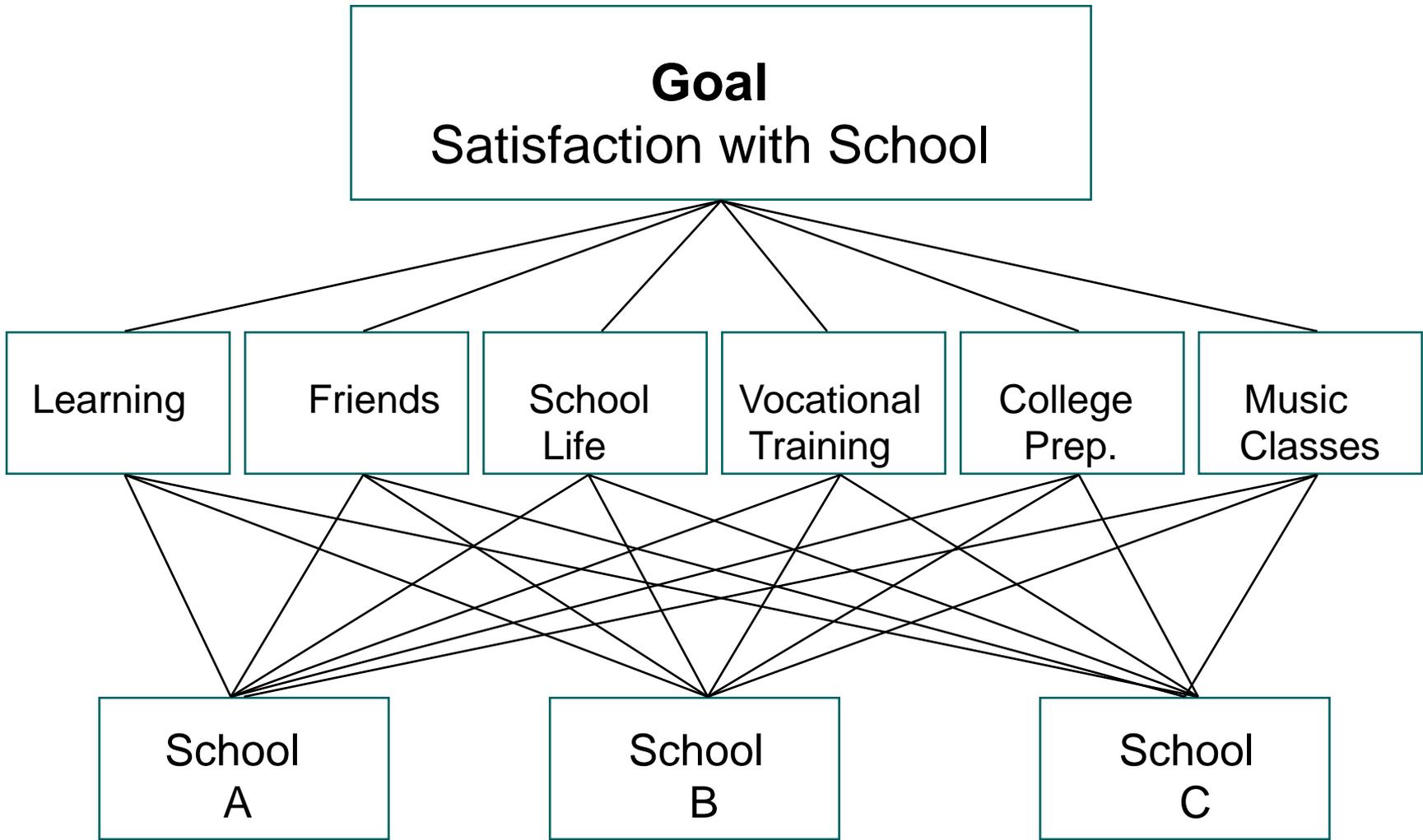
Small Green Tomato



Sugar Baby Watermelon



Large Lime



# School Selection

|                  | L   | F   | SL | VT  | CP  | MC  | Weights |
|------------------|-----|-----|----|-----|-----|-----|---------|
| Learning         | 1   | 4   | 3  | 1   | 3   | 4   | .32     |
| Friends          | 1/4 | 1   | 7  | 3   | 1/5 | 1   | .14     |
| School Life      | 1/3 | 1/7 | 1  | 1/5 | 1/5 | 1/6 | .03     |
| Vocational Trng. | 1   | 1/3 | 5  | 1   | 1   | 1/3 | .13     |
| College Prep.    | 1/3 | 5   | 5  | 1   | 1   | 3   | .24     |
| Music Classes    | 1/4 | 1   | 6  | 3   | 1/3 | 1   | .14     |

## Comparison of Schools with Respect to the Six Characteristics

|   | <b>Learning</b> |     |     | Priorities |
|---|-----------------|-----|-----|------------|
|   | A               | B   | C   |            |
| A | 1               | 1/3 | 1/2 | .16        |
| B | 3               | 1   | 3   | .59        |
| C | 2               | 1/3 | 1   | .25        |

|   | <b>Friends</b> |   |   | Priorities |
|---|----------------|---|---|------------|
|   | A              | B | C |            |
| A | 1              | 1 | 1 | .33        |
| B | 1              | 1 | 1 | .33        |
| C | 1              | 1 | 1 | .33        |

|   | <b>School Life</b> |   |     | Priorities |
|---|--------------------|---|-----|------------|
|   | A                  | B | C   |            |
| A | 1                  | 5 | 1   | .45        |
| B | 1/5                | 1 | 1/5 | .09        |
| C | 1                  | 5 | 1   | .46        |

|   | <b>Vocational Trng.</b> |   |     | Priorities |
|---|-------------------------|---|-----|------------|
|   | A                       | B | C   |            |
| A | 1                       | 9 | 7   | .77        |
| B | 1/9                     | 1 | 1/5 | .05        |
| C | 1/7                     | 5 | 1   | .17        |

|   | <b>College Prep.</b> |     |   | Priorities |
|---|----------------------|-----|---|------------|
|   | A                    | B   | C |            |
| A | 1                    | 1/2 | 1 | .25        |
| B | 2                    | 1   | 2 | .50        |
| C | 1                    | 1/2 | 1 | .25        |

|   | <b>Music Classes</b> |   |     | Priorities |
|---|----------------------|---|-----|------------|
|   | A                    | B | C   |            |
| A | 1                    | 6 | 4   | .69        |
| B | 1/6                  | 1 | 1/3 | .09        |
| C | 1/4                  | 3 | 1   | .22        |

# Composition and Synthesis

## Impacts of School on Criteria

|   | .32<br>L | .14<br>F | .03<br>SL | .13<br>VT | .24<br>CP | .14<br>MC | <b>Composite<br/>Impact of<br/>Schools</b> |
|---|----------|----------|-----------|-----------|-----------|-----------|--|
| A | .16      | .33      | .45       | .77       | .25       | .69       | .37  |
| B | .59      | .33      | .09       | .05       | .50       | .09       | .38  |
| C | .25      | .33      | .46       | .17       | .25       | .22       | .25  |

# The School Example Revisited Composition & Synthesis: Impacts of Schools on Criteria

## Distributive Mode

(Normalization: Dividing each entry by the total in its column)

|   | .32<br>L | .14<br>F | .03<br>SL | .13<br>VT | .24<br>CP | .14<br>MC | Composite<br>Impact of<br>Schools |
|---|----------|----------|-----------|-----------|-----------|-----------|-----------------------------------|
| A | .16      | .33      | .45       | .77       | .25       | .69       | .37                               |
| B | .59      | .33      | .09       | .05       | .50       | .09       | .38                               |
| C | .25      | .33      | .46       | .17       | .25       | .22       | .25                               |

The Distributive mode is useful when the uniqueness of an alternative affects its rank. The number of copies of each alternative also affects the share each receives in allocating a resource. In planning, the scenarios considered must be comprehensive and hence their priorities depend on how many there are. This mode is essential for ranking criteria and sub-criteria, and when there is dependence.

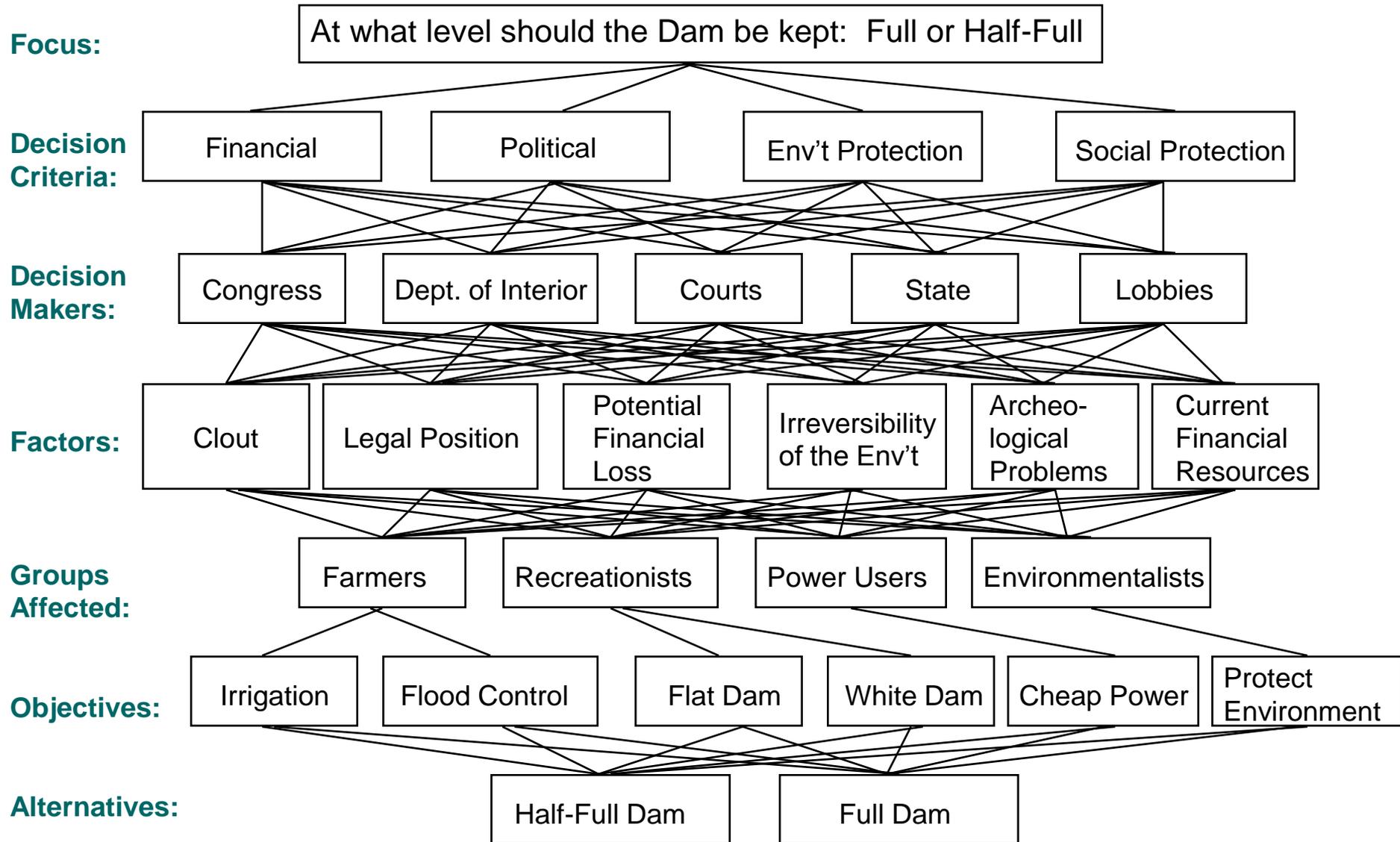
## Ideal Mode

(Dividing each entry by the maximum value in its column)

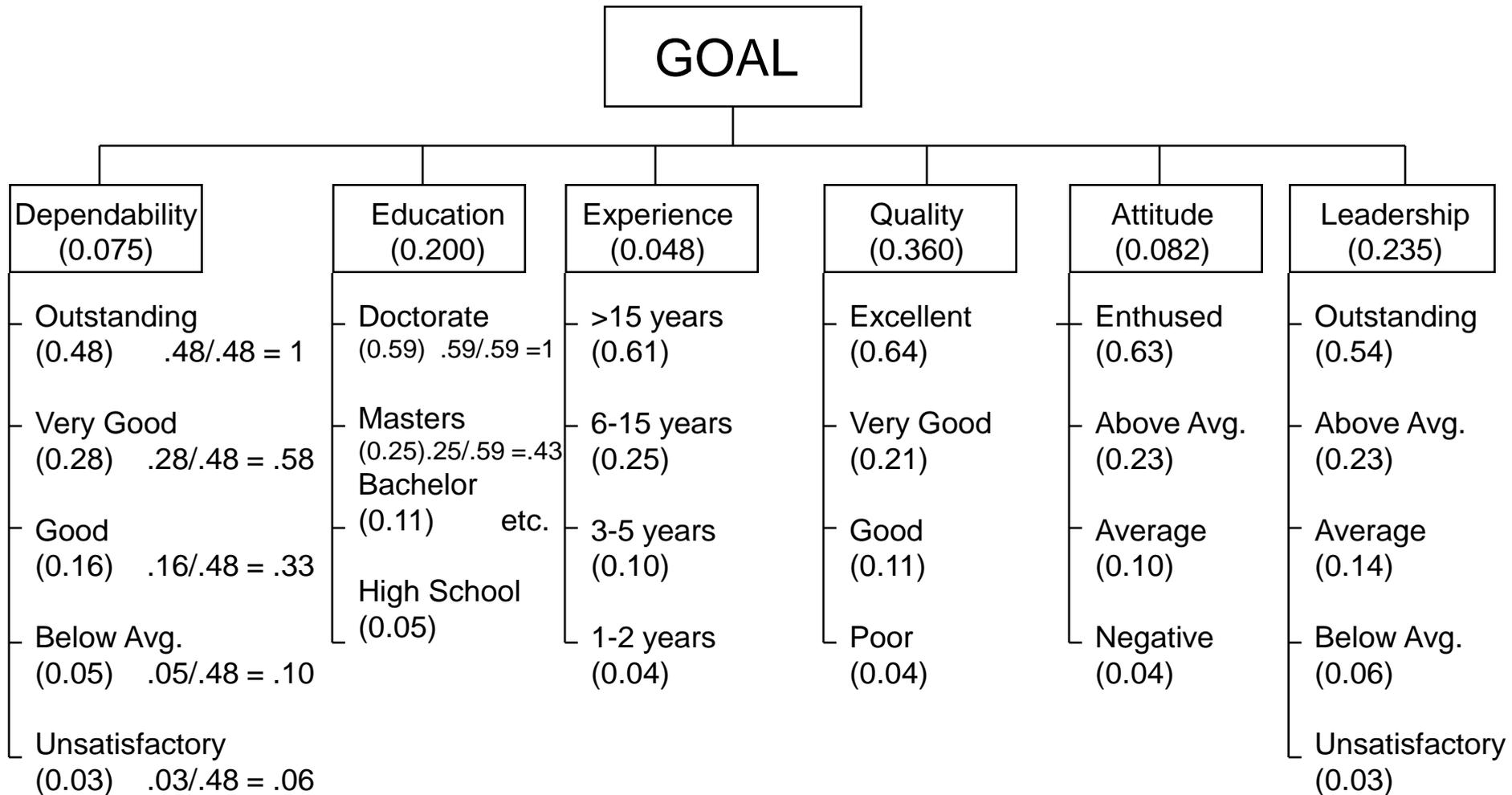
|   | .32<br>L | .14<br>F | .03<br>SL | .13<br>VT | .24<br>CP | .14<br>MC | Composite<br>Impact of<br>Schools | Normal-<br>ized |
|---|----------|----------|-----------|-----------|-----------|-----------|-----------------------------------|-----------------|
| A | .27      | 1        | .98       | 1         | .50       | 1         | .65                               | .34             |
| B | 1        | 1        | .20       | .07       | .50       | .13       | .73                               | .39             |
| C | .42      | 1        | 1         | .22       | .50       | .32       | .50                               | .27             |

The Ideal mode is useful in choosing a best alternative regardless of how many other similar alternatives there are.

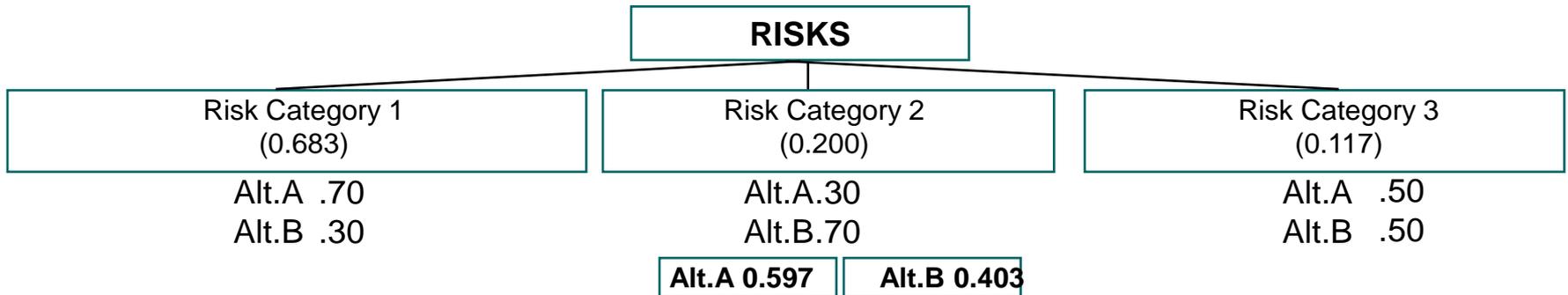
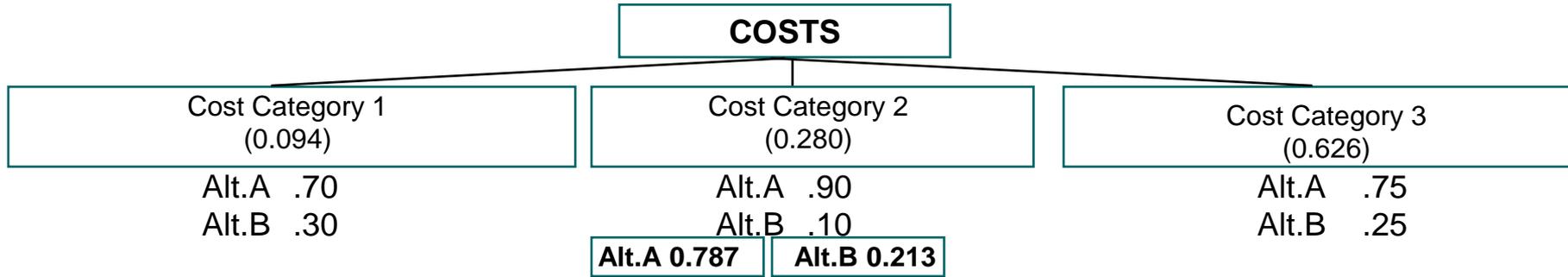
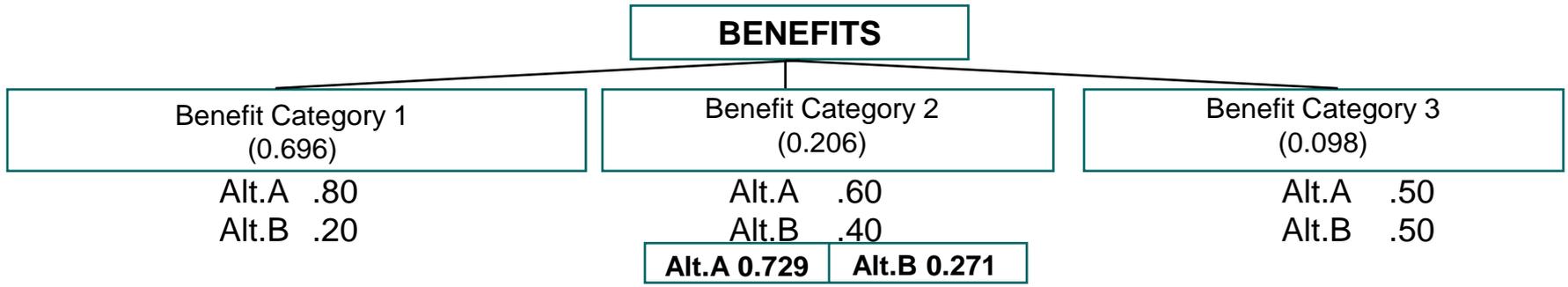
# A Complete Hierarchy to Level of Objectives



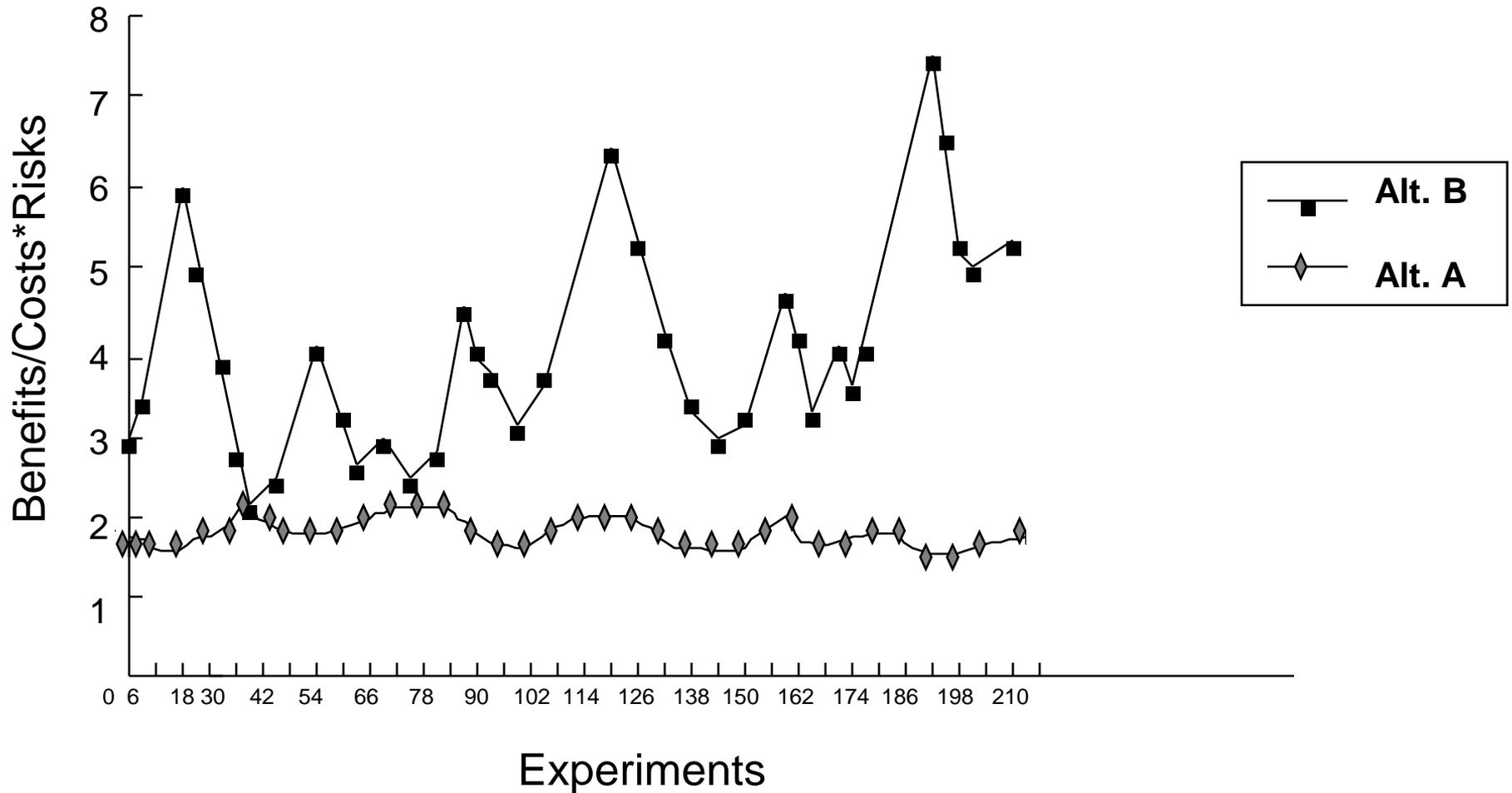
# Evaluating Employees for Raises



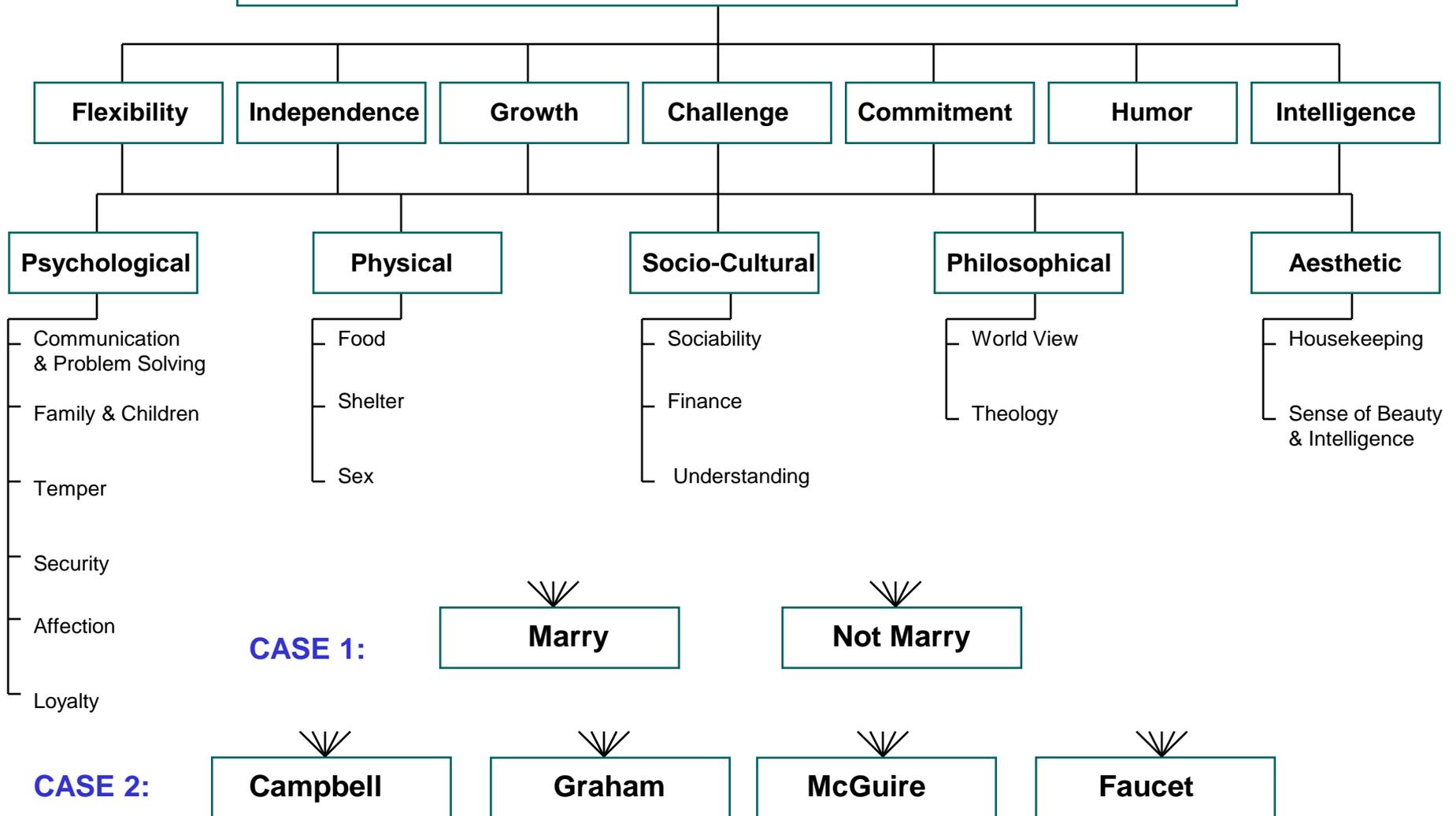
# B.O.R.C.



**Result:**  $\frac{\text{Benefits}}{\text{Costs} \times \text{Risks}}$  ;    **Alt.A**  $\frac{.729}{.787 \times .597} = 1.55$     **Alt.B**  $\frac{.271}{.213 \times .403} = \boxed{3.16}$

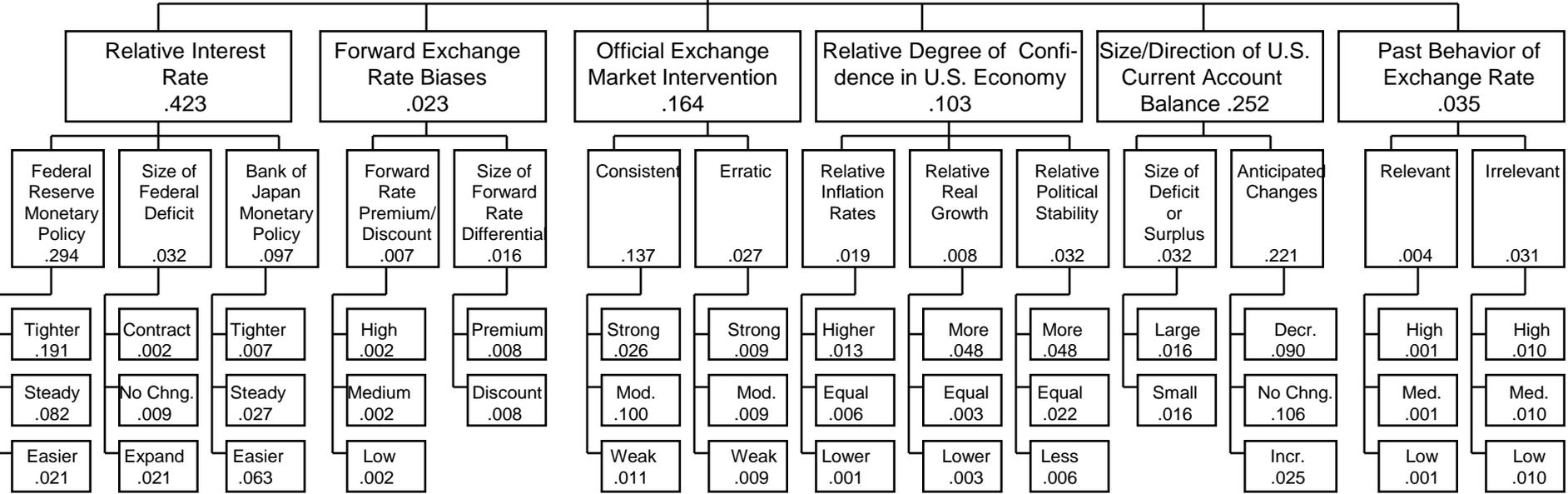


# Whom to Marry - A Compatible Spouse





**Value of Yen/Dollar Exchange : Rate in 90 Days**

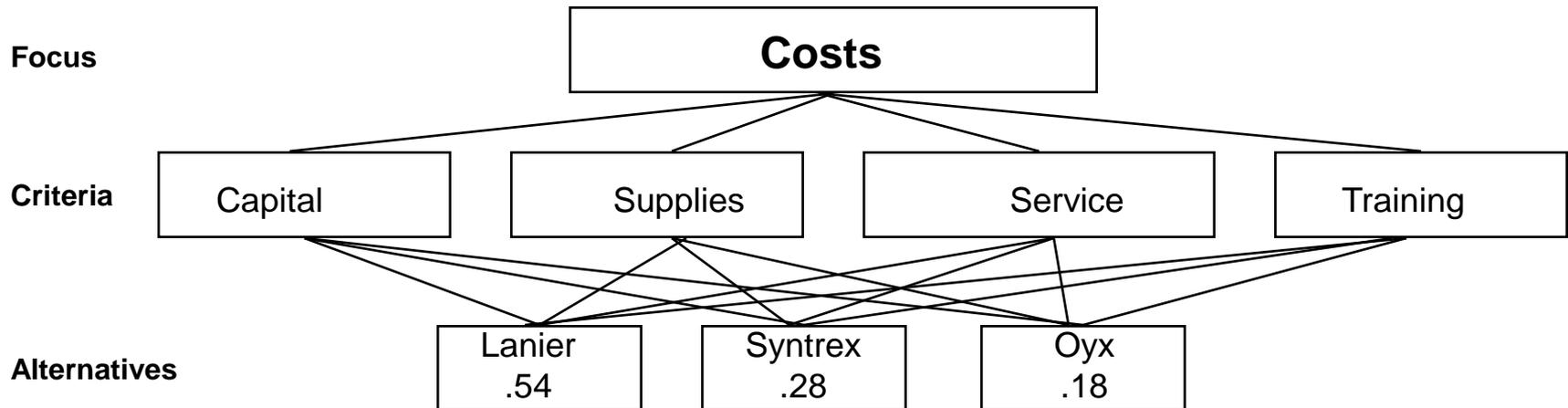
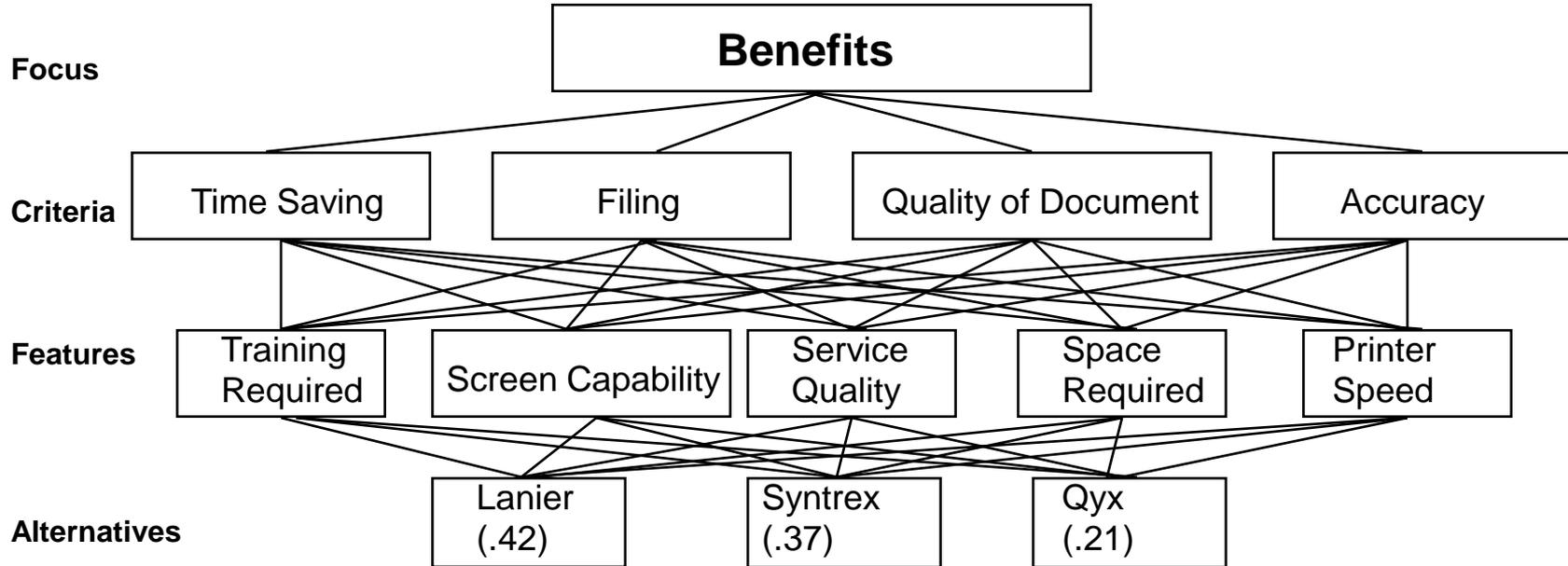


**Probable Impact of Each Fourth Level Factor**

|                            |                               |                        |                                |                             |
|----------------------------|-------------------------------|------------------------|--------------------------------|-----------------------------|
| 119.99<br>and below        | 119.99-<br>134.11             | 134.11-<br>148.23      | 148.23-<br>162.35              | 162.35<br>and above         |
| Sharp<br>Decline<br>0.1330 | Moderate<br>Decline<br>0.2940 | No<br>Change<br>0.2640 | Moderate<br>Increase<br>0.2280 | Sharp<br>Increase<br>0.0820 |

Expected Value is 139.90 yen/\$

# Best Word Processing Equipment



# Best Word Processing Equipment Cont.

## Benefit/Cost Preference Ratios

Lanier

$$\frac{.42}{.54} = 0.78$$

Syntrex

$$\frac{.37}{.28} = 1.32$$

Qyx

$$\frac{.21}{.18} = 1.17$$



Best Alternative

# Group Decision Making and the Geometric Mean

Suppose two people compare two apples and provide the judgments for the larger over the smaller, 4 and 3 respectively. So the judgments about the smaller relative to the larger are  $1/4$  and  $1/3$ .

## Arithmetic mean

$$4 + 3 = 7$$

$$1/7 \neq 1/4 + 1/3 = 7/12$$

## Geometric mean

$$\sqrt{4 \times 3} = 3.46$$

$$1/\sqrt{4 \times 3} = \sqrt{1/4 \times 1/3} = 1/\sqrt{4 \times 3} = 1/3.46$$

That the Geometric Mean is the unique way to combine group judgments is a theorem in mathematics.

