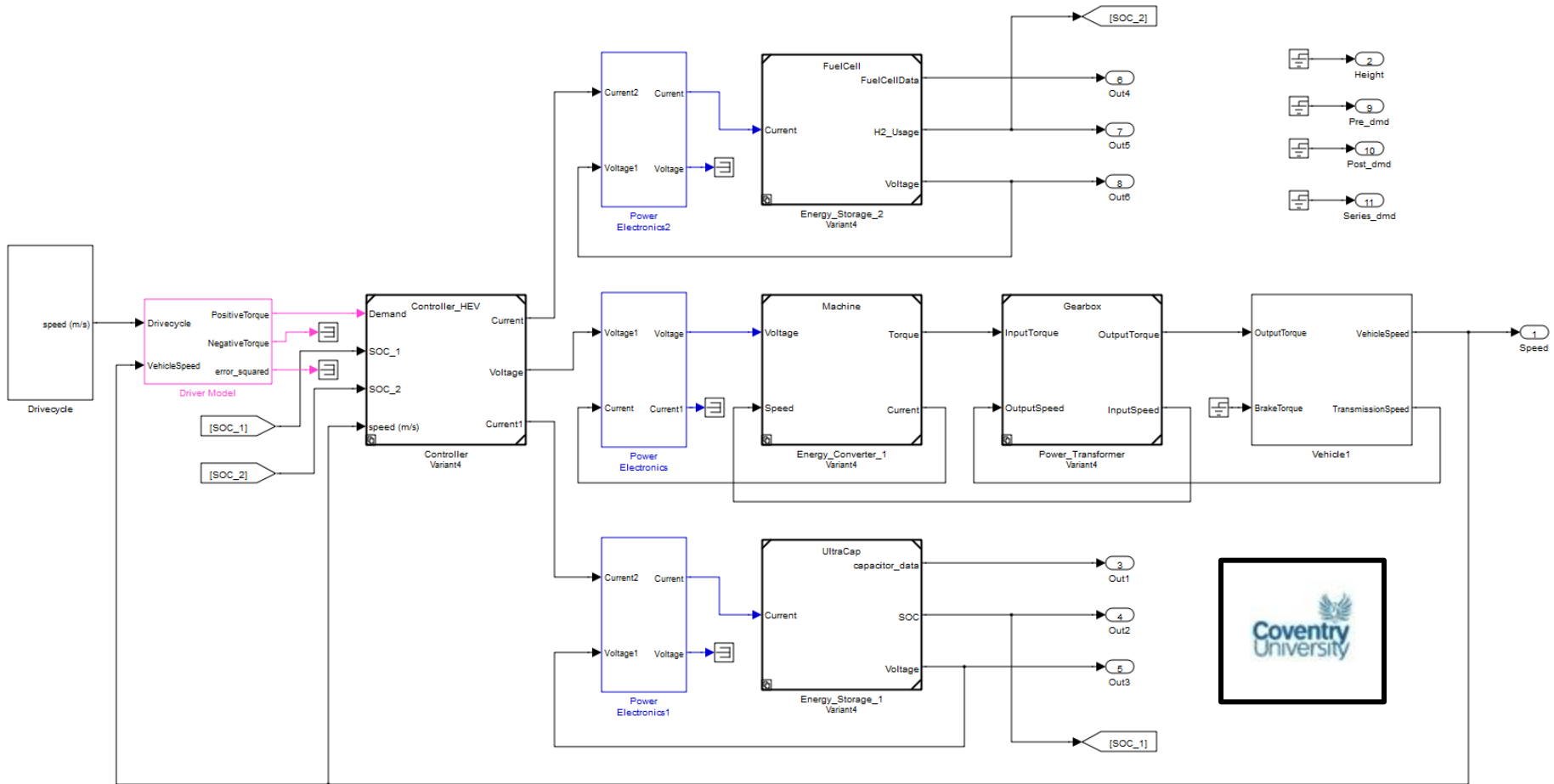


MODEL ORDER REDUCTION

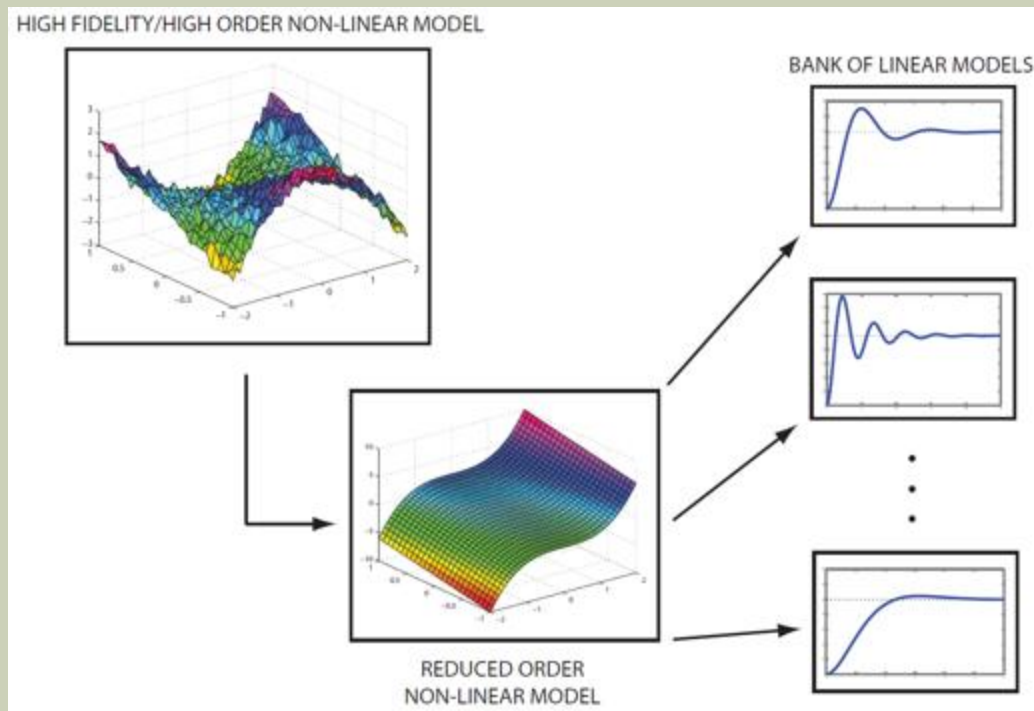


OUTLINE

- Need for model order reduction (MOR)
- Methodology for MOR
- Classical approaches
 - Supercapacitor
- Data driven approaches
 - Design of experiment
 - Linear approach: Batteries
 - Nonlinear SISO / MISO approach: Power converter
 - Nonlinear MIMO approach: Fuel cell
- Summary of findings

MODEL ORDER REDUCTION OVERVIEW

- MOR is required to reduce computational complexity of model yet retain sufficient accuracy for a specific purpose, i.e. control, diagnosis, prognosis



Model order reduction procedure to obtain banks of linear models

METHODOLOGY

- Models for purpose:

- Control
- Diagnosis
- Prognosis

- Approaches for MOR:

- Classical

- Based on mathematical manipulation of system equations

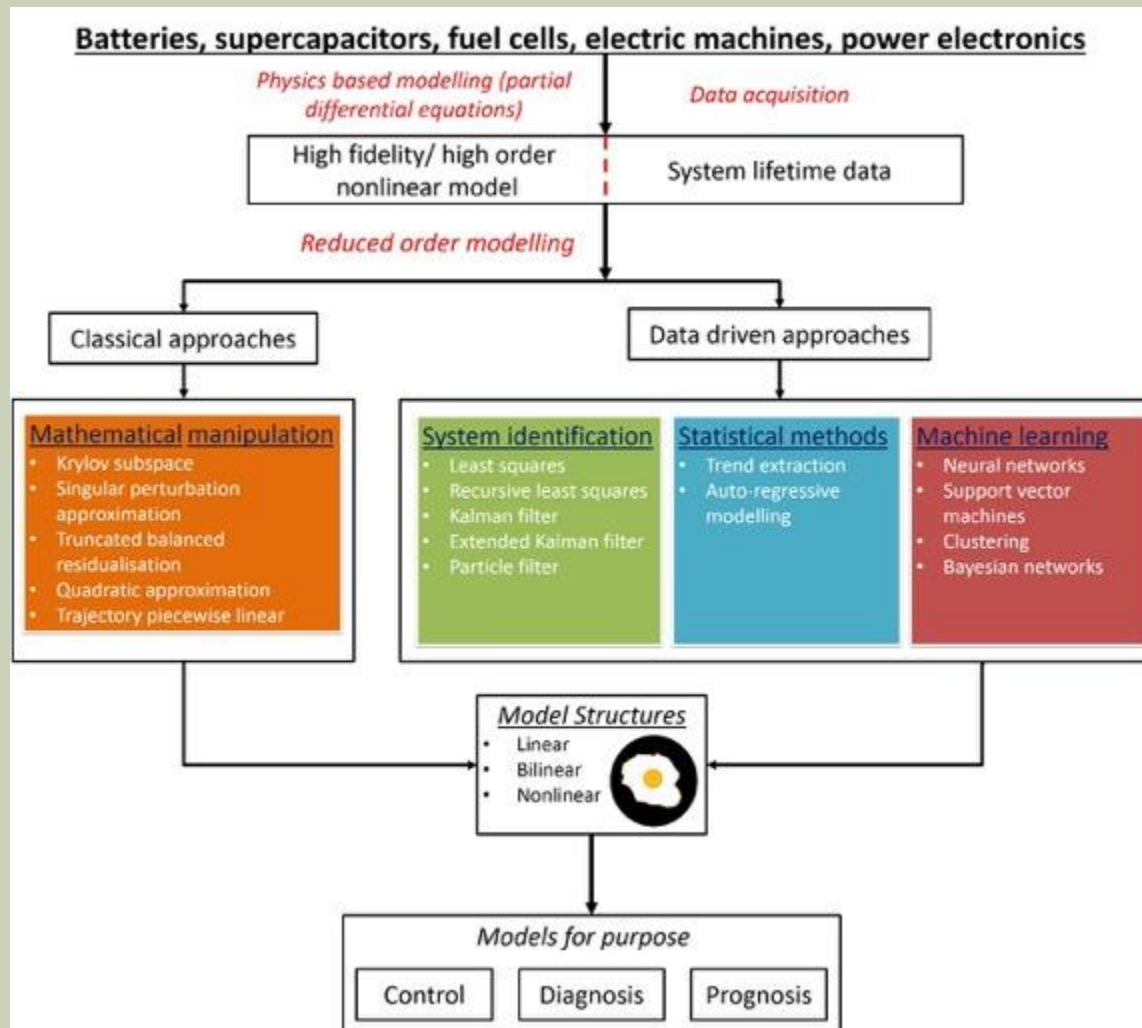
- Data-driven

- Models derived from data collected from complex models or hardware
- Include System Identification and Machine Learning methods

- All approaches **retain dominant modes** whilst **discarding modes with low contribution** to system dynamics

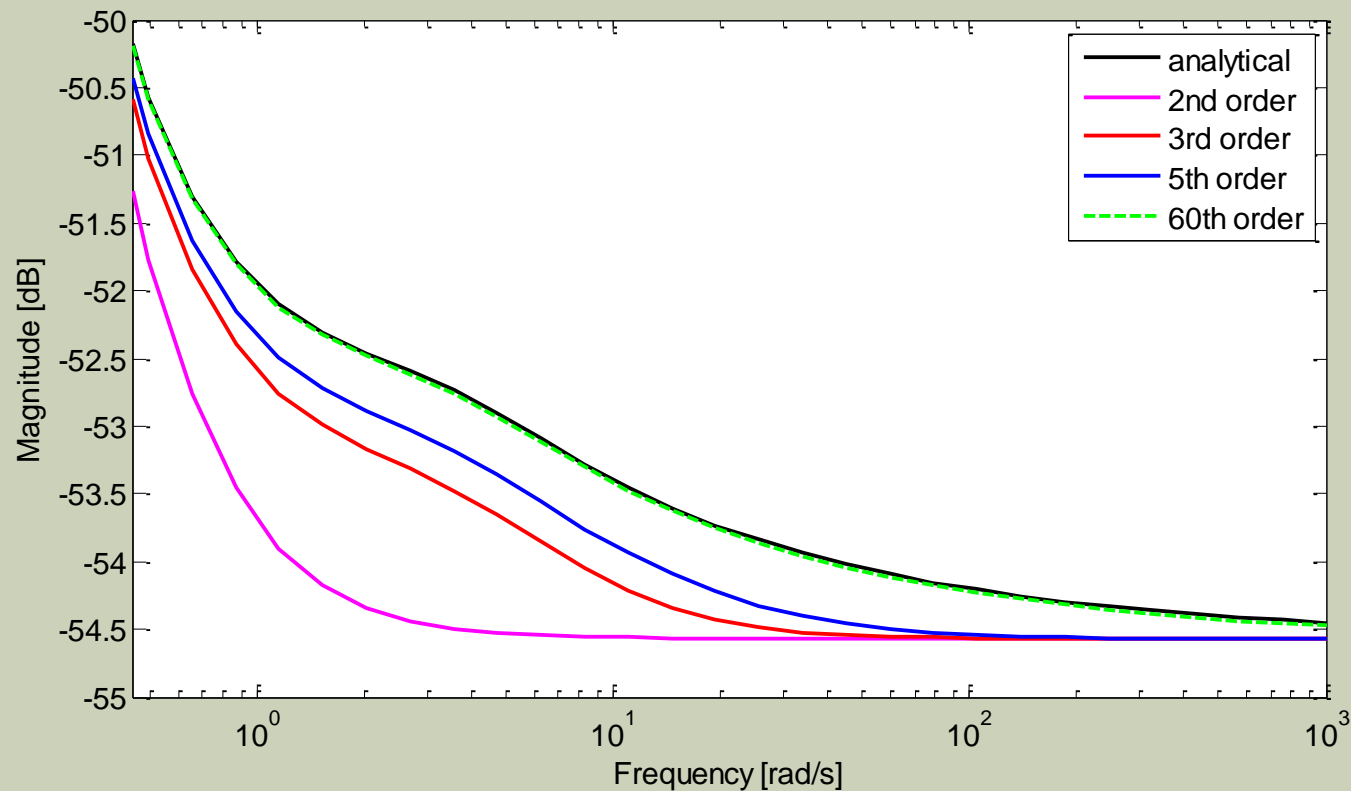
	control	diagnosis	prognosis
classical			
data-driven			

METHODOLOGY



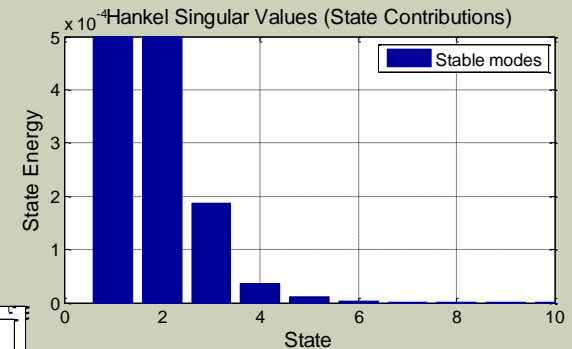
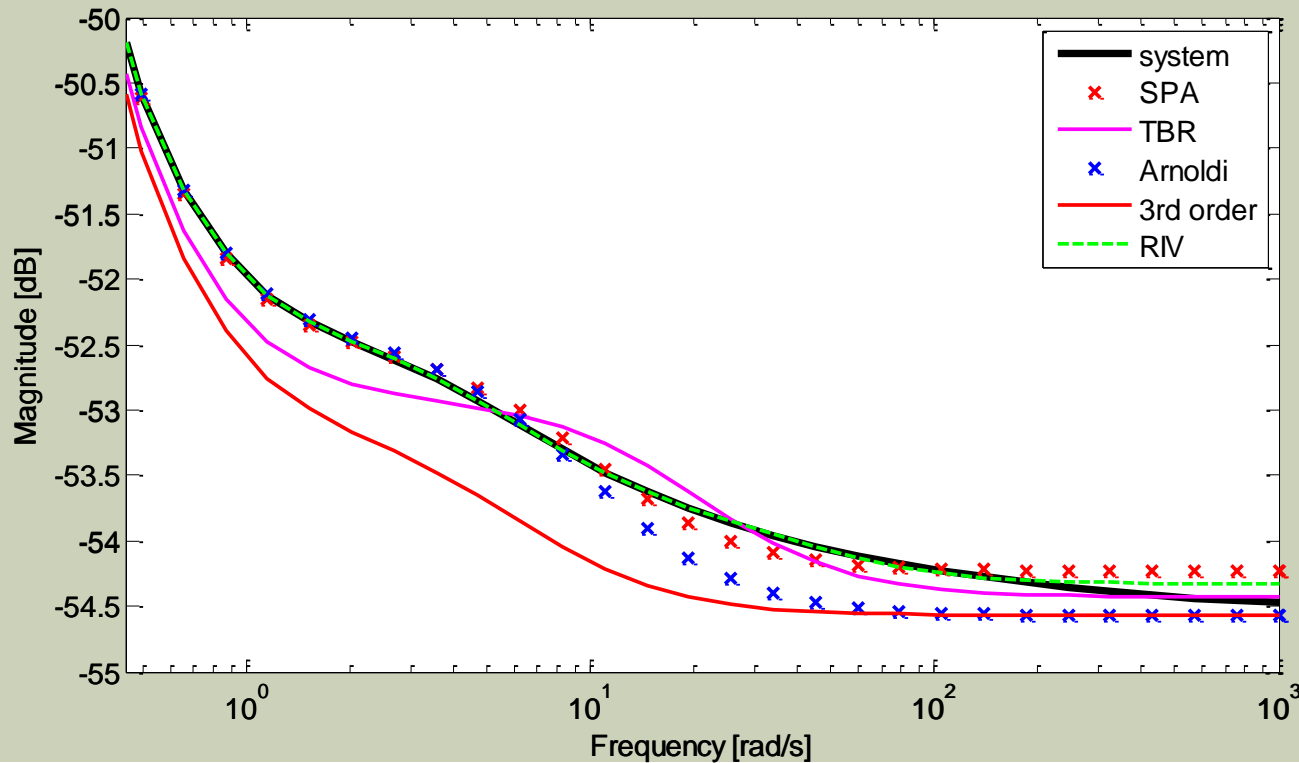
CASE STUDY: SUPERCAPACITOR

- Single input single output (SISO) nonlinear model
- 60th order model used as baseline for model order reduction
- Truncation without model order reduction methods



CASE STUDY: SUPERCAPACITOR

- Comparison of 3rd order model variants obtained via selected reduced order modelling techniques



SPA – Singular perturbation approximation

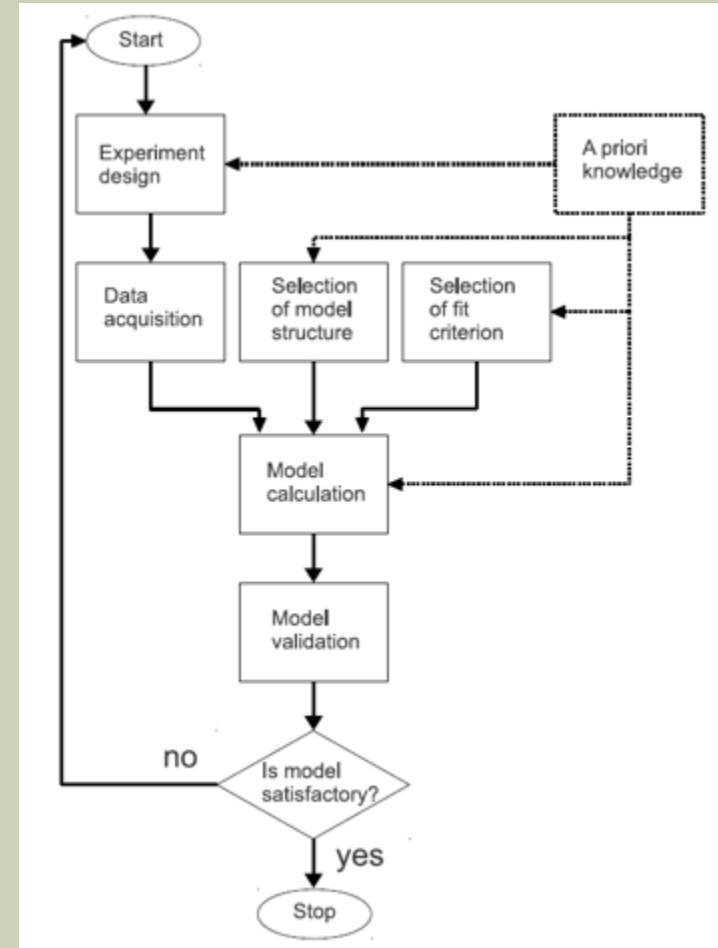
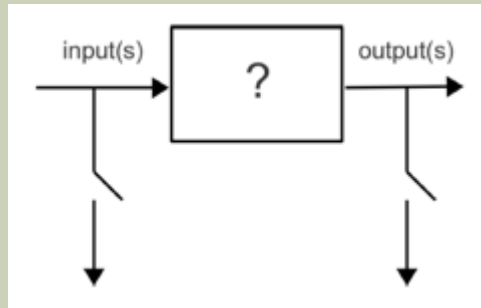
TBR – Truncated balanced residualisation

Arnoldi – Krylov subspace method

RIV – refined instrumental variable

DATA DRIVEN APPROACHES

- An experimental process, whereby making use of available input-output data and *a priori* knowledge, one aims to mathematically describe causalities that govern behaviour of system
- Different approaches to modelling based on *a priori* knowledge
 - White box
 - Black box
 - Grey box



CASE STUDY: BATTERY

1. Data acquisition

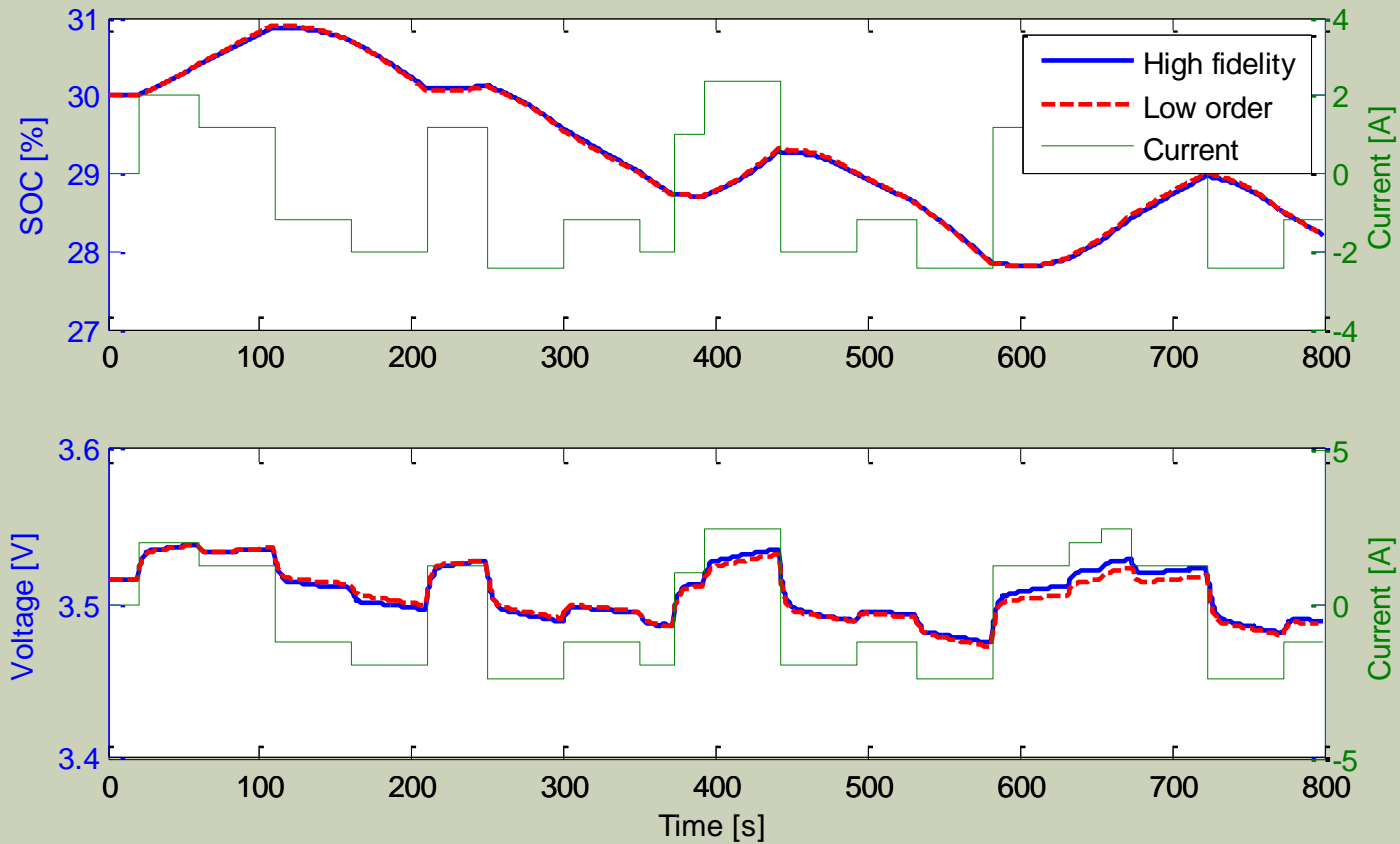
- **Voltage** and **SOC** (outputs) responses to **current** input
- 36 short (80 seconds) data sets starting at different SOC (positive current input – charge mode)
- Experiment repeated for negative current (discharging)

2. Obtained set of 144 LTI models using simplified refined instrumental variable (SRIVC) method

- Current to SOC models
- Current to voltage models

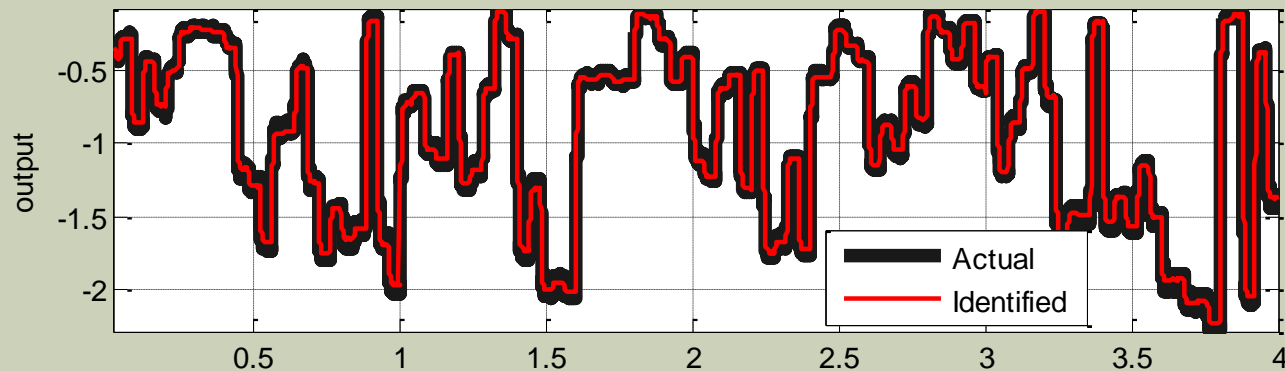
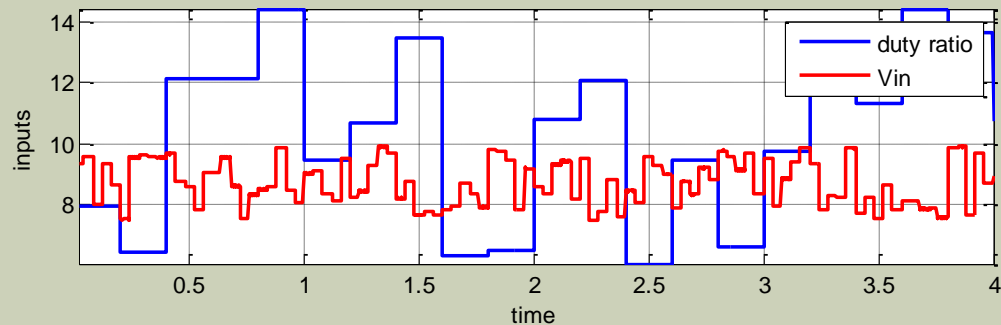
3. **Assumption:** low order model can have linear structure, where parameters depend on SOC and sign of current

CASE STUDY: BATTERY



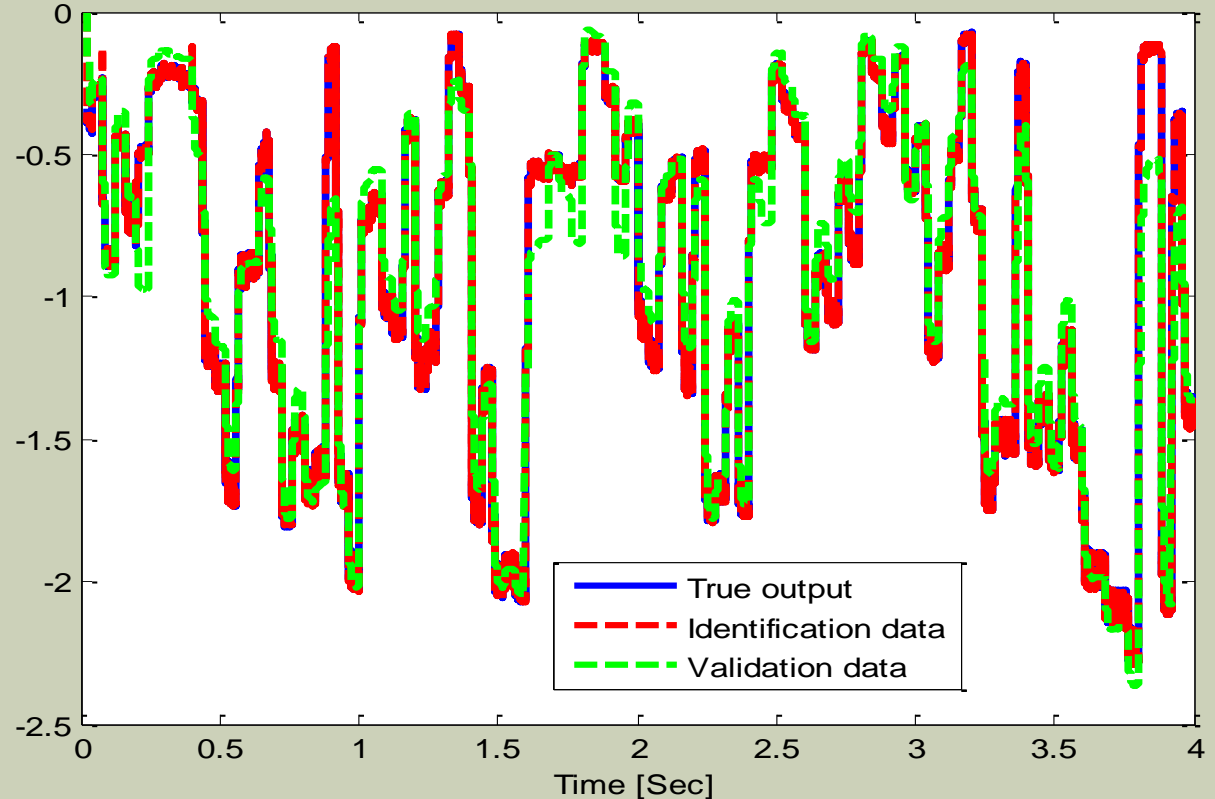
CASE STUDY: BUCK-BOOST CONVERTER

- **Model structure : 10 single input - single output (SISO)**
Hammerstein models with 4th order polynomial static nonlinearity
- **Inputs: Input voltage and duty cycle**
- **Output: Output current**
- **Model order: 1**
- **$R_T^2 = 99.83\%$**

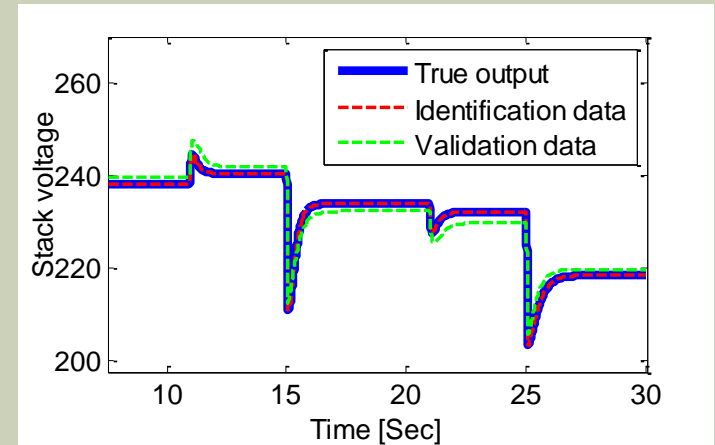
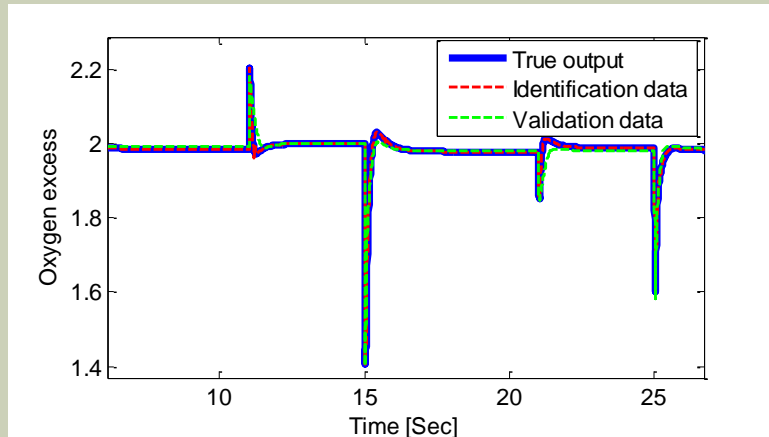


CASE STUDY: BUCK-BOOST CONVERTER

- **Model structure : Bilinear multiple inputs – single output (MISO)**
 - **Inputs: Input voltage and duty cycle**
 - **Output: Output current**
 - **Model order: 5**
 - $R_T^2 = 93.7\%$
 - $a - 5$
 - $b - 2$
 - **Bilinear term - 1**

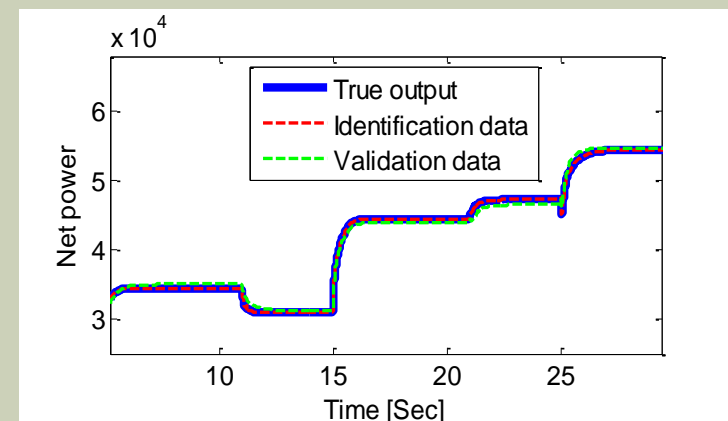


CASE STUDY: FUEL CELL STACK



■ Model structure : MIMO NARX

- Inputs: stack current, compressor voltage
- Outputs: stack voltage, oxygen excess, net power
- Logarithmic type nonlinearity on the input
- Model order: 15
- Oxygen excess $R_T^2 = 96.8\%$
- Stack voltage $R_T^2 = 94.2\%$
- Net power $R_T^2 = 99.3\%$



J. Pukrushpan, A. Stefanopoulou, and H. Peng, "Control of fuel cell breathing," IEEE Contr. Syst. Mag., vol. 24, no. 2, pp. 30-46, Apr. 2004.

SUMMARY OF FINDINGS

- MOR effectively retains fidelity of high order model whilst reducing the model order
- Data driven approaches are effective for reduced order modelling
- Purpose of model and a priori information determines the modelling method

- Outline of methodology for model order reduction
 - Control
 - Diagnosis
 - Prognosis
- Guidelines for MOR
- Methodology is robust for multiple case studies