FUTURE Vehicles WP3.1 –

Developments in Reduced Order Modelling

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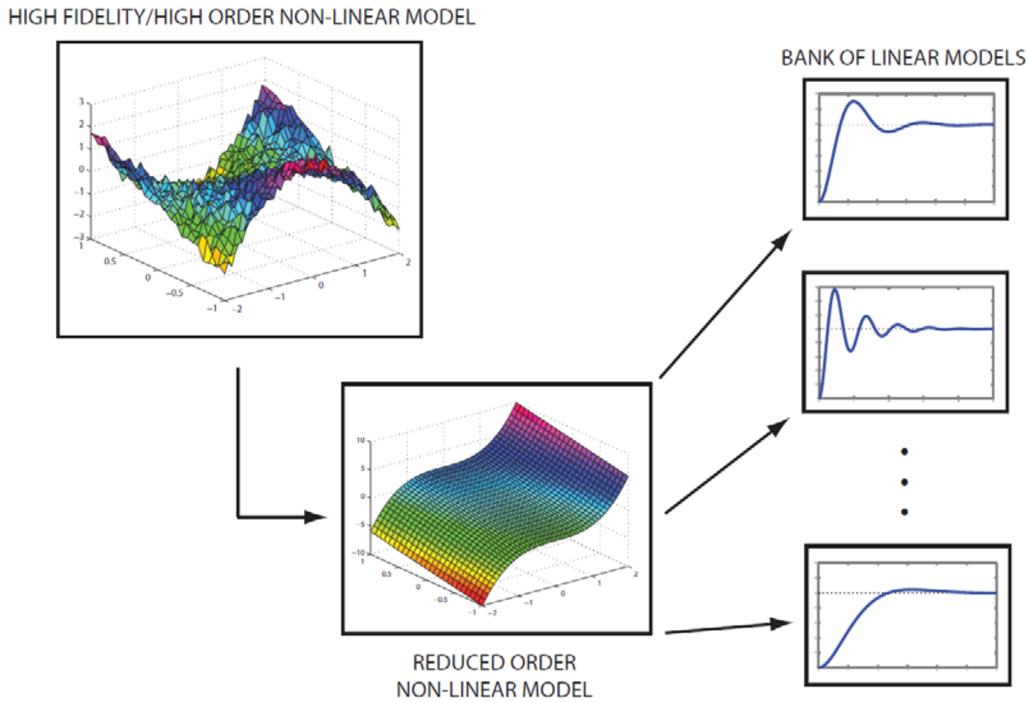
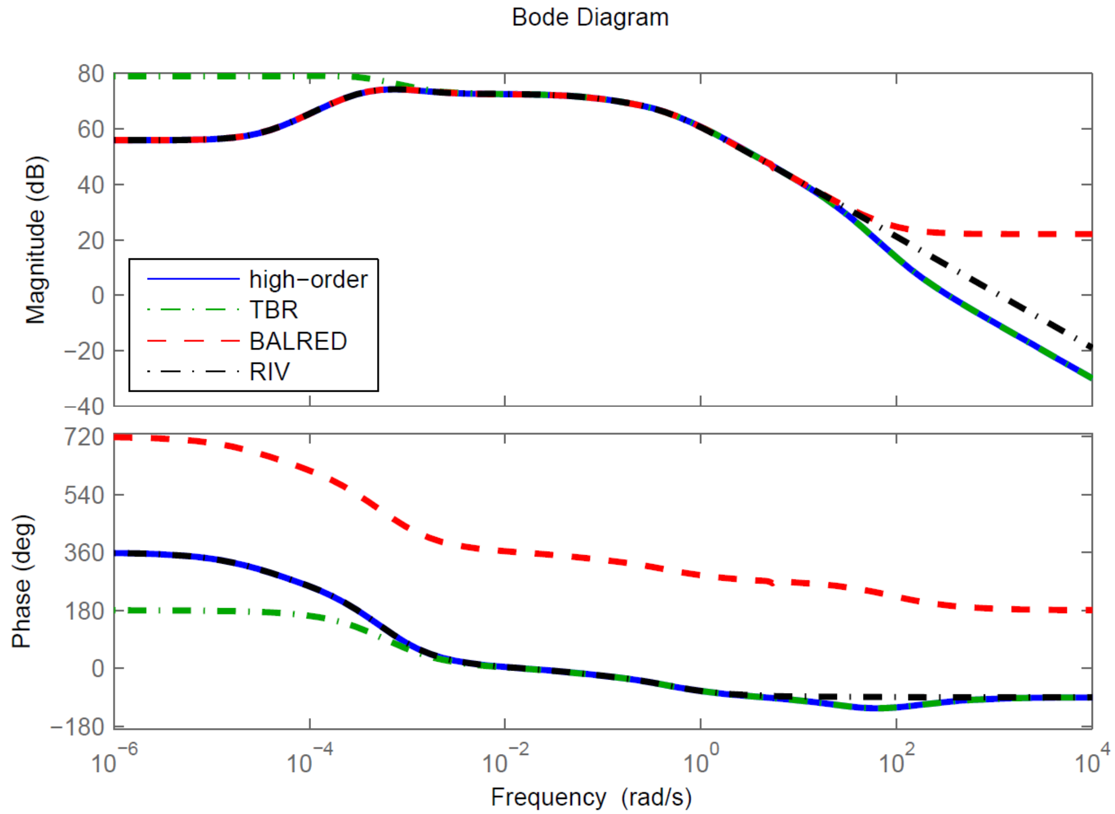
Introduction

- Aim – to reduce computational complexity of model yet retain sufficient accuracy for a specific purpose, i.e. control, diagnostics, prognostics



Overview of reduced order modelling

- Model features
  - > Dynamics
  - > Non-linearity
- Requirements
  - > Purpose of model
  - > Accessibility
  - > Accuracy
  - > Operational range
  - > Frequency range



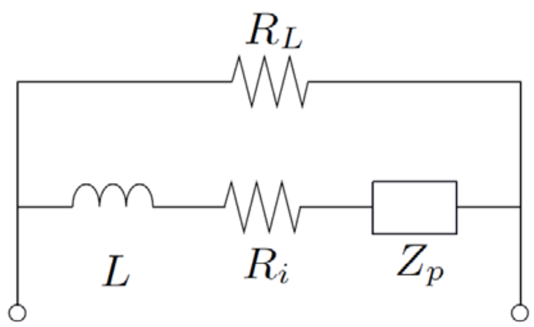
- Features and requirements inform and define the model reduction problem

Overview of techniques

- Classical approach
  - > Linear methods
    - Truncated balanced residualisation (TBR)
    - Singular perturbation analysis (SPA)
    - Krylov subspace methods
  - > Non-linear methods
    - Quadratic approximation
    - Trajectory piecewise linear approximation
- Data driven approach
  - > Parameter estimation and system identification methods
    - Estimation rules – least squares, recursive least squares, refined instrumental variables (RIV), Kalman filter for parameter estimation
    - Model structures – autoregressive, Box Jenkins, bilinear, Wiener, Hammerstein, state dependent parameter

Case study: Supercapacitor

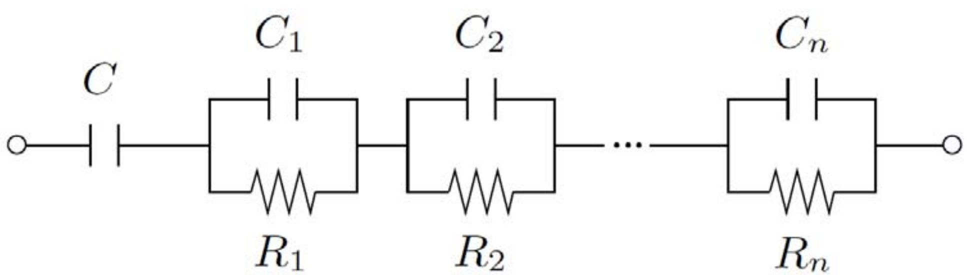
- Equivalent circuit of supercapacitor



$$Z_p(j\omega) = \frac{\tau \coth(\sqrt{j\omega\tau})}{C\sqrt{j\omega\tau}}$$

Z_p - complex pore impedance
C - capacitance
τ - time constant
L - inductance
R_L - leakage resistance

- Complex pore impedance approximated by n RC branches in series with capacitor



$$C_k = \frac{C}{2}$$

$$R_k = \frac{2\pi}{\pi^2 k^2 C} \qquad k = 1, 2, \cdots, n$$

model order = $n+2$

Case study: Supercapacitor

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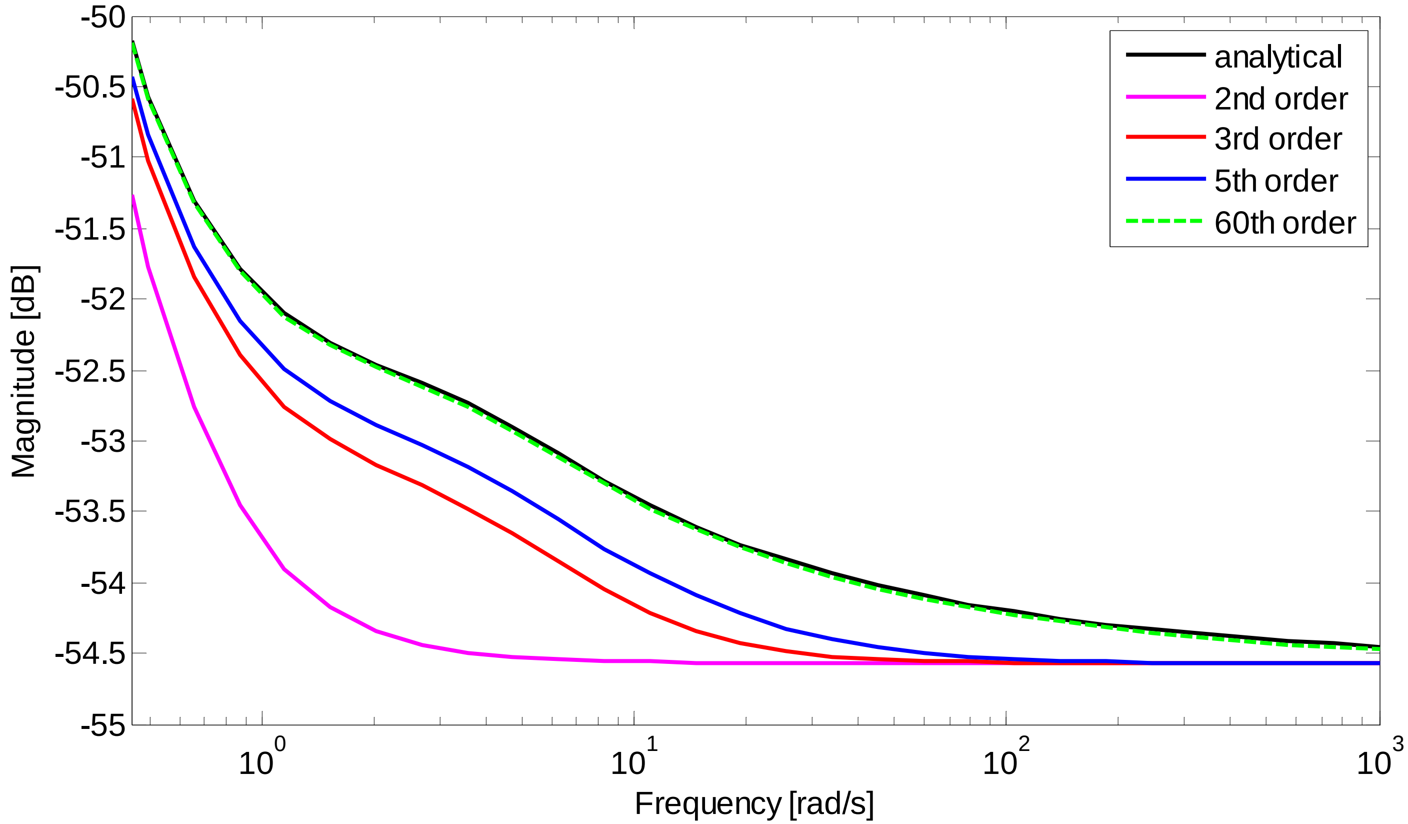
- Accuracy of n-branch model
- 58-branch (60 order) model used as baseline for model order reduction (MOR)



Case study: Supercapacitor

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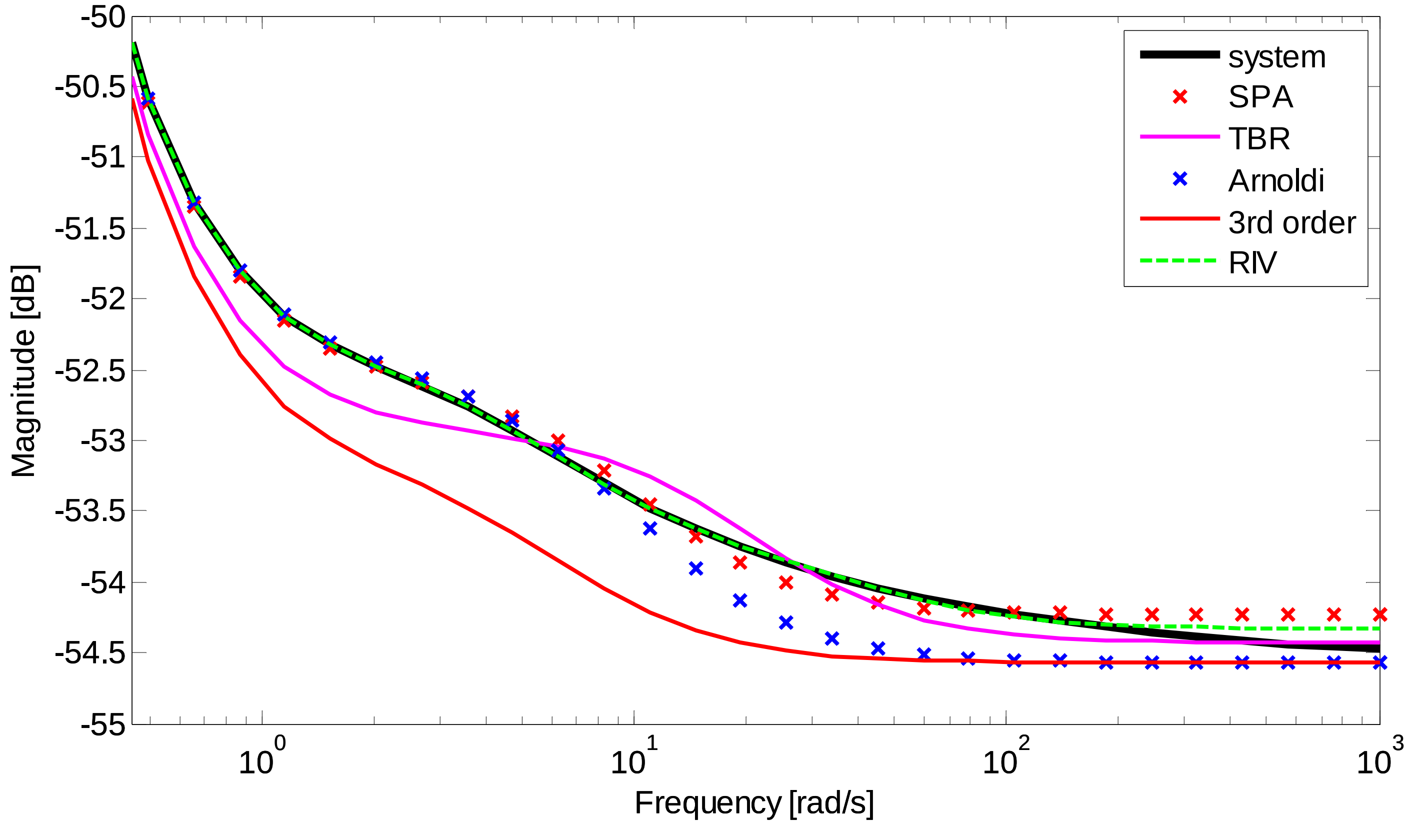
- Comparison of 3rd order model variants obtained via selected reduced order modelling techniques



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Case study: Transmission line model

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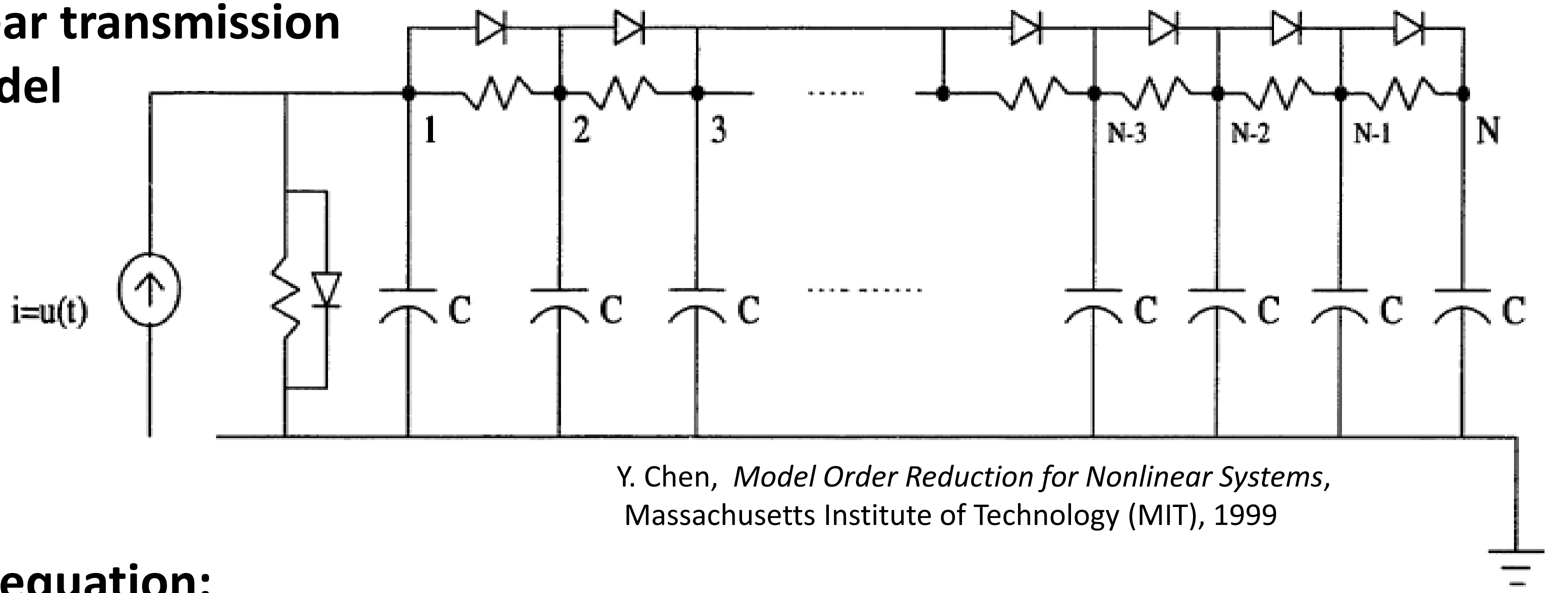
Nonlinear transmission line model



Y. Chen, *Model Order Reduction for Nonlinear Systems*, Massachusetts Institute of Technology (MIT), 1999

System equation:

$$C\frac{d\mathbf{v}}{dt} = \begin{bmatrix} -g(v_1) - g(v_1 - v_2) \\ g(v_1 - v_2) - g(v_2 - v_3) \\ \vdots \\ g(v_{k-1} - v_k) - g(v_k - v_{k+1}) \\ \vdots \\ g(v_{N-1} - v_N) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u(t)$$

Model parameters:

$g(v) = e^{40v} + v - 1$ (Nonlinear resistor profile)

$N = 100$ (Number of branches)

$C = 1$

Case study: Transmission line model

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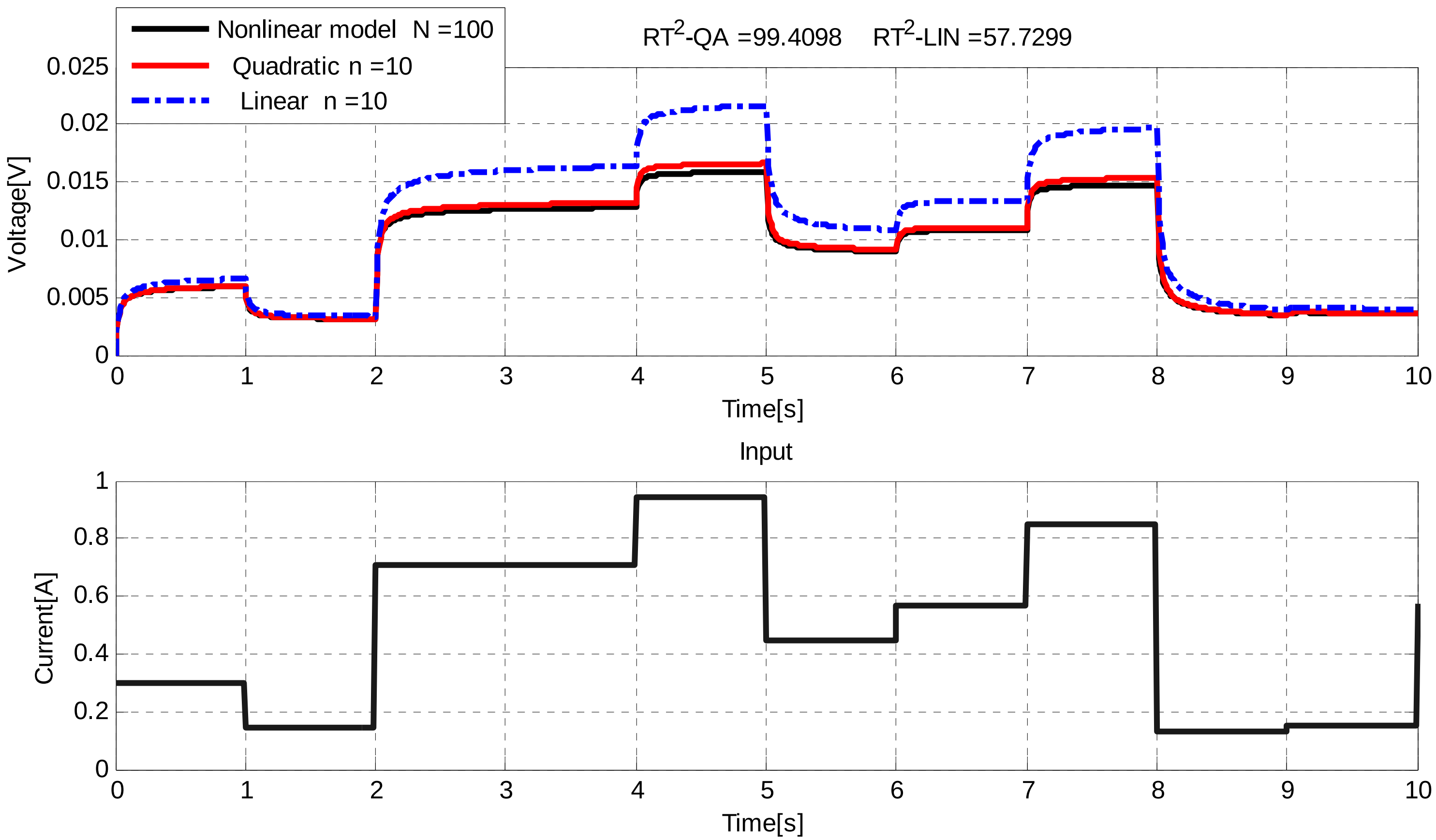
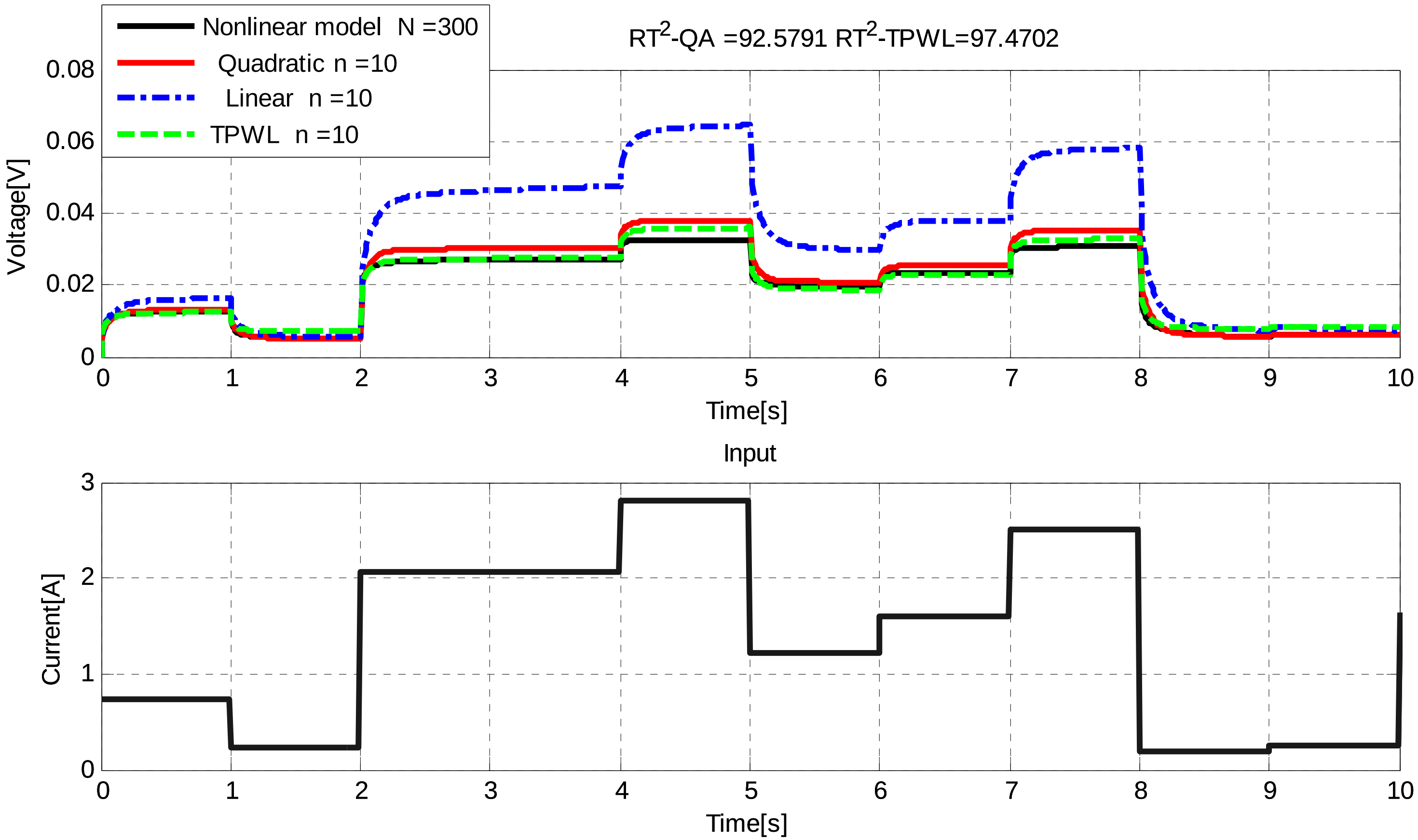


Case study: Transmission line model



Case study: Electric machine

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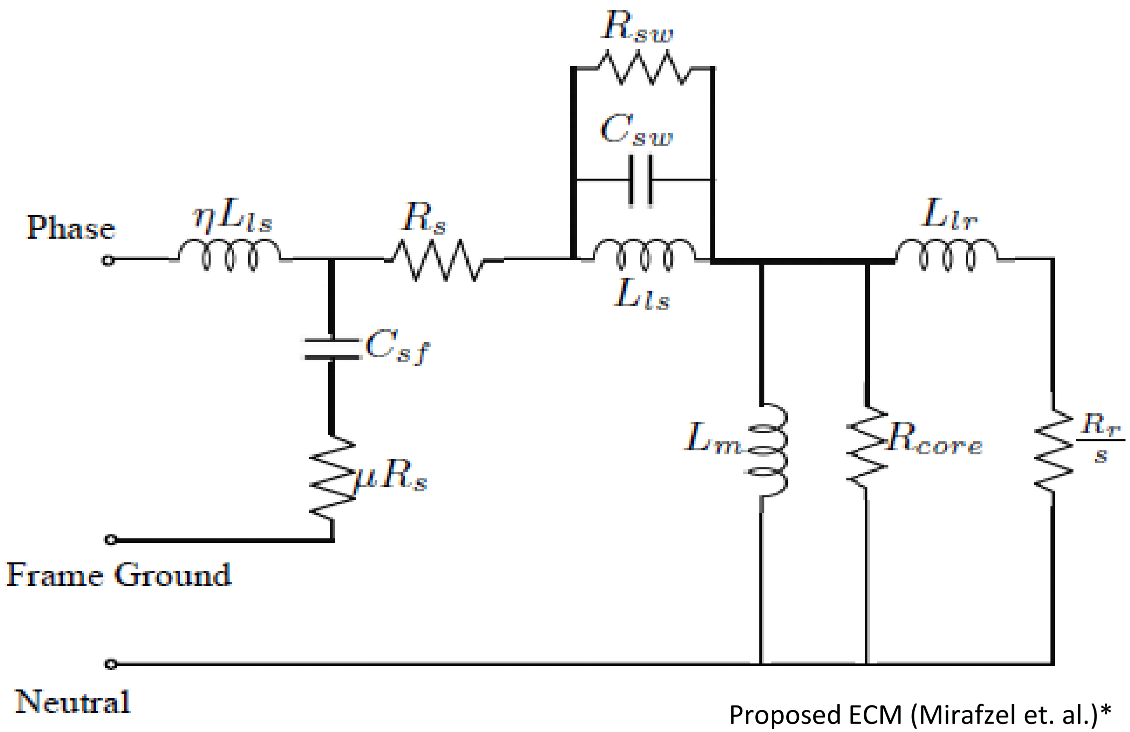

Proposed ECM (Mirafzel et. al.)*

[*] ***B. Mirafzal, G.L. Skibinski, R.M. Tallam, D.W. Schlegel, and R.A. Lukaszewski.*** "Universal induction motor model with low-to-high frequency-response characteristics". *IEEE Transactions on Industry Applications*, 43(5), pp. 1233–1246, (2007).

Case study: Electric machine

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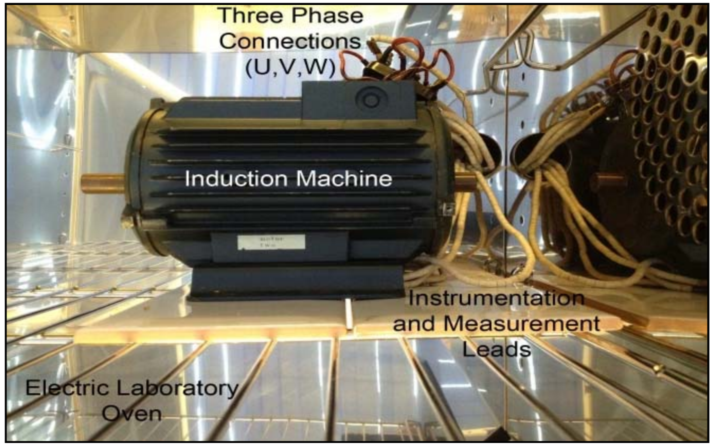
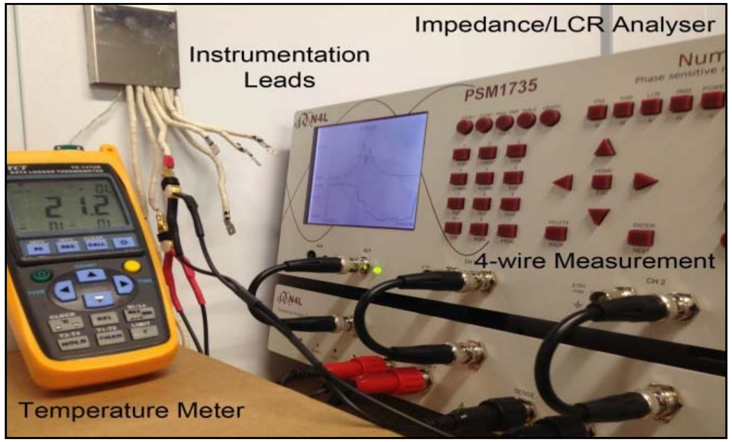
- AC induction motor
- Thermal chamber
- Temperature data acquisition (DAQ)
- Temperature range: 22.4 °C – 210 °C with 20 °C increments
- NL4 precision impedance LCR analyser
- Frequency range of interest 100 Hz – 10 MHz



Induction machine inside electric oven



Impedance LCR analyser and temperature data logger

Case study: Electric machine

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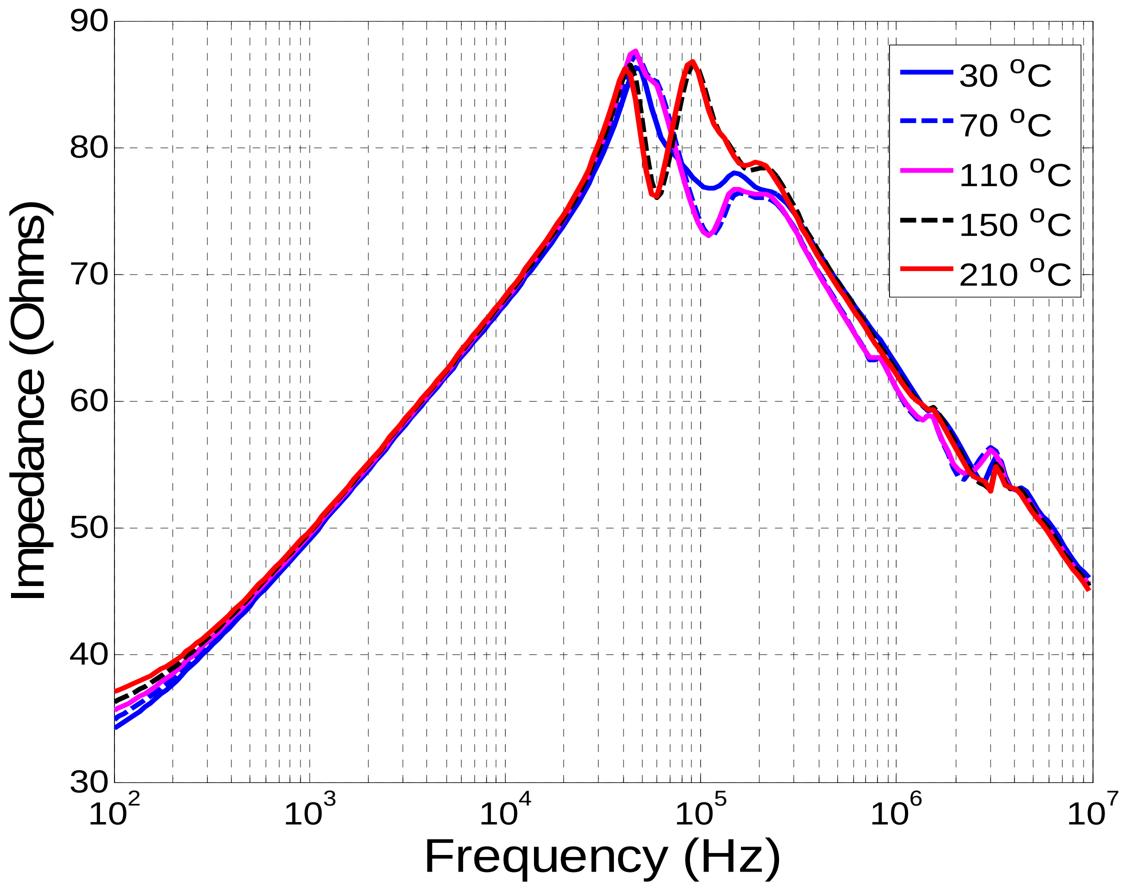
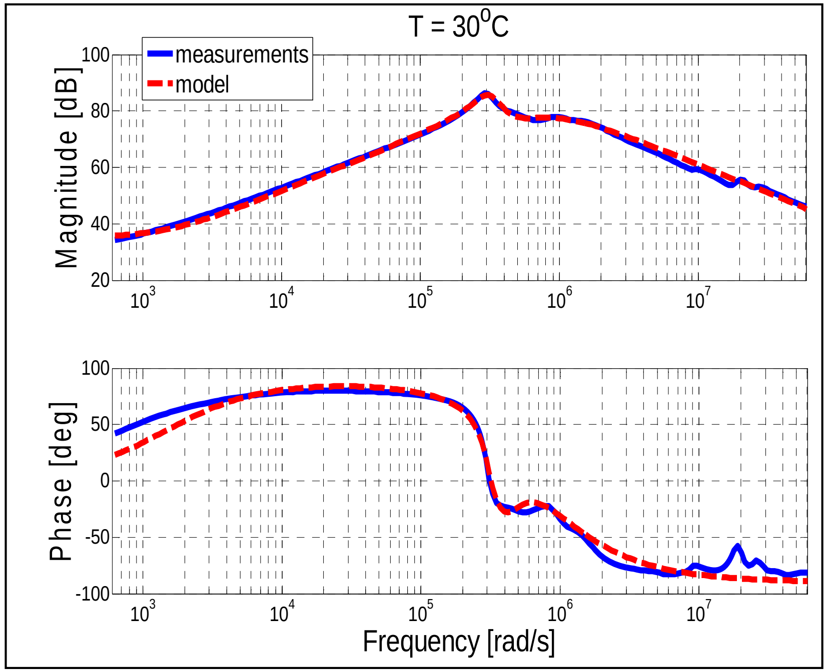
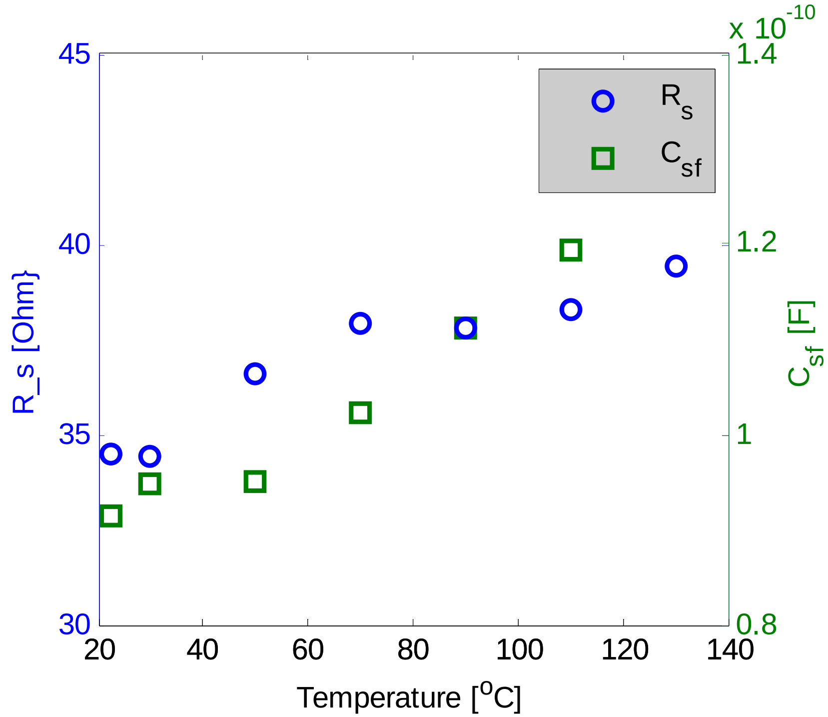
Impedance measurements at different temperatures



Case study: Electric machine





- ECM proven to be suitable for electric machine modelling up to 130 ⁰C
- Nonlinear optimisation used to find temperature dependent parameters

Case study: Battery

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1. Data acquisition
   - Voltage and SOC (outputs) responses to current input
   - 36 short (80 seconds) data sets starting at different SOC (positive current input – charge mode)
   - Experiment repeated for negative current (discharging)
2. Obtained set of 144 LTI models using simplified refined instrumental variable (SRIVC) method
   - Current to SOC models
   - Current to voltage models
3. Assumption: low order model can have linear structure, where parameters depend on SOC and sign of current

Case study: Battery

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- Observations:
  - current to SOC transfer function does not depend on SOC neither on sign of current
    - → transfer function described by linear 3rd order model

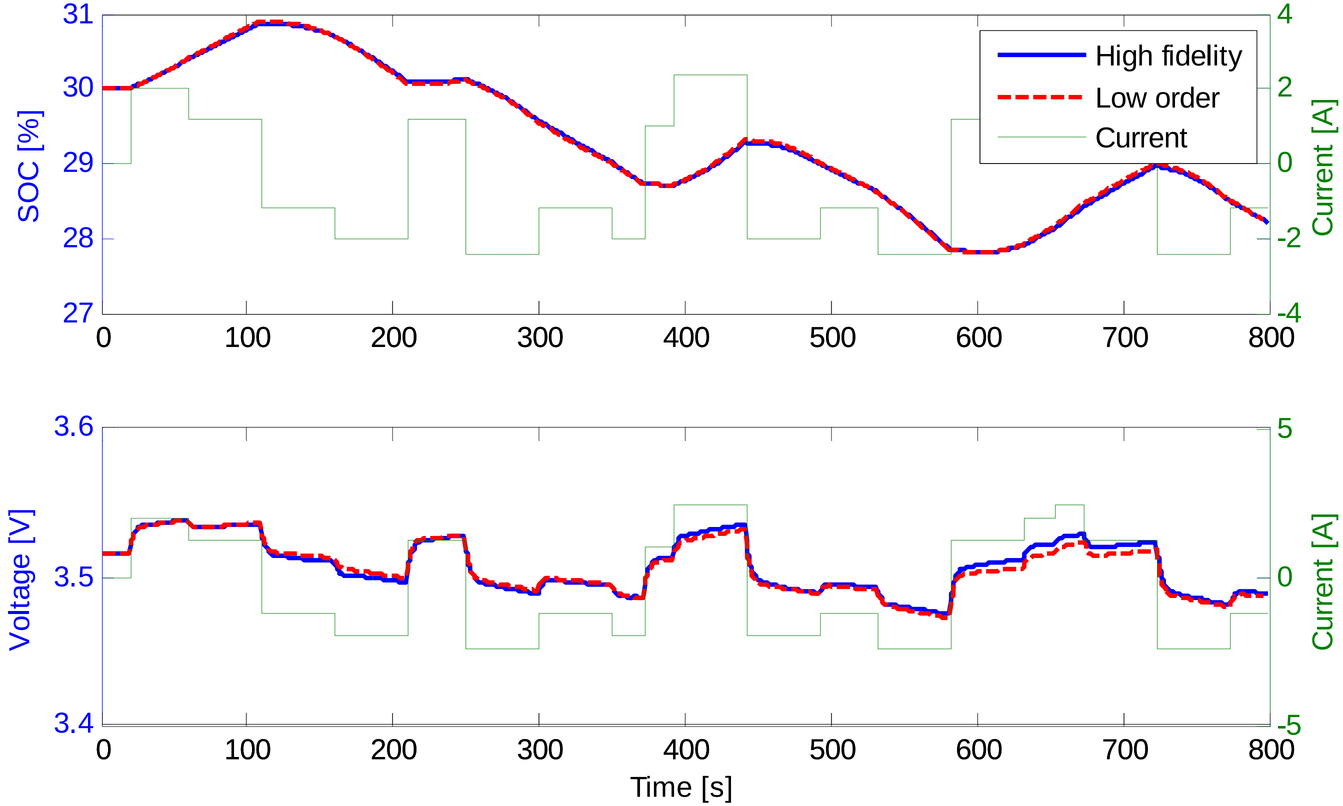
$$G_{soc}(s) = \frac{b_{11}s^2 + b_{21}s + b_{31}}{s^3 + a_{11}s^2 + a_{21}s + a_{31}}$$

  - current to voltage relationship depends on
    - → SOC
    - → sign of current

$$Z(s) = \frac{b_{02}(soc,m)s^2 + b_{12}(soc,m)s + b_{22}(soc,m)}{s^2 + a_{12}(soc,m)s + a_{22}(soc,m)} \qquad m = \mathrm{sign}(i(t))$$

- **Result: 5th order piecewise state dependent parameter model**
  - → because model is piecewise – it can model hysteresis

Case study: Battery



Summary

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- Knowledge of the system/ high order model (features) as well as the purpose of reduced model (requirements) both define and inform the **reduction problem**
- A variety of techniques available for addressing model order reduction provides flexible approach to obtain models for specific purposes

Work to date:

- Techniques so far investigated are targeted towards control purposes

Further work:

- Models for diagnostics and prognostics also to be considered