

Hexagonal Image Sampling: A Practical Proposition

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(Presented at: SPIE Conf: Expert Robots for Industrial Use, Cambridge, Mass,
USA, Nov 1988)

(Proc. SPIE, Vol.1008, pp.23-27, 1989)

ABSTRACT

A case is made for the use of regular hexagonal sampling systems in robot vision applications. With such a sampling technique, neighbouring pixels reside in equidistant shells surrounding the central pixel and this leads to the simpler design and faster processing of local operators and to a reduced image storage requirement. Connectivity within the image is easily defined and the aliasing associated with vertical lines in the hexagonal system is not a problem for robot vision.

With modern processors only a minimal time penalty is incurred in reporting results in a rectangular co-ordinate system, and a comparison between equivalent processing times for hexagonal and rectangular systems implemented on a popular processor has shown savings in excess of 40% for hexagonal edge detection operators. Little modification is required to TV frame grabber hardware to enable hexagonal digitisation.

1. INTRODUCTION

Image digitisation by equipment such as TV frame grabbers, conventionally involves the sampling of the 2D analog image at points located on a square grid system. This square sampling scheme seems to have been generally adopted because it is conceptually easy to develop 2D processing algorithms from the 1D case. An example of such an algorithm is the 2D FFT where 1D FFT's are applied firstly to the columns of square sampled data, and secondly to the rows¹.

Other sampling schemes have been considered, in particular the regular hexagonal system. Here, the sampling points covering the image are equidistant from each of their six nearest neighbours, as shown in Figure 1. Mersereau² demonstrated that the system requires 13.4% fewer sampling points to maintain the same high frequency information as an image sampled on a square grid system, and that the system is optimal in this respect. He then developed a 2D FFT and filtering algorithms for the system and showed that these require less computation time than their square system counterparts.

$$l_1 = 1.0, \quad l_2 = 2.0 / (3.0^{1/2}).$$

Figure 1. Regular hexagonal sampling points.

Figure 2. Three shell filter.

Golay³ considered hexagonally shaped pixel systems. He was concerned with connectivity within binary images, this is easy to define with hexagonal pixels as each neighbour is joined to the central pixel along an equal length edge, whereas in the square system connectivity can be along an edge or corner to corner.

Hexagonal sampling systems have been used by Staunton⁴ with various local operator implementations such as edge detectors and shown to compute more quickly than the square system counterparts. He develops algorithms for these operators and describes how images may be hexagonally sampled using a conventional TV frame grabber device.

This paper describes the practical implementation of a hexagonal data structure for use in robot vision systems. Various spatial domain local operators are developed, data storage is reviewed and a TV frame grabber system is described. Computation time and accuracy comparisons are made between similar edge detection operators in the hexagonal and square systems and the hexagonal is shown to be superior. Finally, for a robot vision system, processed results will probably be required to be reported in a rectangular co-ordinate system, this poses little difficulty for the hexagonal system.

2. SPATIAL PLANE LOCAL OPERATORS

2. 1. Filters.

The regular hexagonal structure leads to easy spatial plane local operator design. The local area can be defined to include the central pixel and any number of concentric 'shells' of pixels at increasing distances from the centre. All the members of a particular shell are equidistant from the centre and can be assigned equal weighting factors in many local operator designs. For example, consider a local Gaussian approximation filter of the type proposed by Davies⁵. For a three shell filter operating on a hexagonal grid, three weighting factors are initially calculated as shown in Figure 2 and the final algorithm will be of the form of Equation 1. In comparison a similar filter on a square grid (5x5) requires six different weighting factors and a correspondingly more complicated algorithm.

$$P = W_k \cdot k_1 + W_l \cdot l_a + W_m \cdot m_b \quad (1)$$

Where k_1 , l_a , and m_b are members of the three shells, a is an integer number from 1 to 12 and b an integer number from 1 to 18.

With median filters, operating in a square sampling system, the choice of optimum local area is difficult to define, a square area (eg 3x3) has been found to give better results than other shapes⁶. With hexagonal sampled systems, it is probable that the easily definable circular areas will enhance performance.

2. 2. Differential operators.

Differential operators are used to boost edge or line information and as part of many edge detection operators⁷. An example of such an operator is the Sobel operator⁷. A corresponding operator for use on a hexagonal system has been described by the author⁴. It was designed according to Davies design principle⁸. This principle predicts both this operator and the Sobel operator to be nearly optimal for use on their respective systems. Both operators are implemented by algorithms employing integer only arithmetic which leads to short computation times, but the hexagonal algorithm is very much simpler, requiring only one addition and to negation instructions. Both operators are shown in Figure 3.

For edge detection, the magnitude of the edge is found and then compared with a threshold value to determine if it is significant. In the square system two orthogonal operators are applied and the magnitude, m calculated as in Equation (2). Edge orientation angle, a is given by Equation (3)⁷. The corresponding equations for the hexagonal system are (5) & (6)⁴. Again only two operator responses are required, t_1 and t_2 . The vector responses of these operators are at a 60° angle to one another, which leads to more complicated algorithms for edge magnitude and orientation. However, for the complete calculation, starting with the much quicker operator application, processing time savings of 44% have been demonstrated with the hexagonal system⁴. Table 1 shows a processing time comparison per pixel for edge magnitude computations on the two systems. The computer used was an Olivetti M28 PC with an Intel 80286 processor.

$$m = [th^2 + tv^2]^{1/2} \quad (2)$$

$$a = \arctan [tv/th] \quad (3)$$

Where t_v and t_h are the responses of operators designed to respond maximally to vertical and horizontal edges.

$$m = [3 (t_v^2 + t_h^2 - t_v t_h)]^{1/2} \quad (4)$$

$$a = \arctan [3^{1/2} \frac{ (t_v - t_h) }{ (t_v + t_h) }] \quad (5)$$

Where t_v, t_h are operator responses.

The less accurate, but previously considered faster, template matching operator⁷ approach to edge detection was tried, but found to compute more slowly than the true magnitude calculation. This was as a result of the fast operation of the integer multiply instruction of the 80286, which operates in the same order of time period as the add instruction.

Table 1. Edge Magnitude Computation Time For Each Pixel.

Square 41.31μS.	Hexagonal 28.33μS.
Intel 80286 processor, clock rate 8.0 MHz.	

2.3. Edge accuracy.

Tests have been performed to determine the magnitude and angular accuracy of the hexagonal edge detection operator and to compare these results with those obtained from the application of the near optimum Sobel operator on equivalent image data in the square system. Several possible edge models exist⁹, the one chosen was of a 'sudden' step edge of height 10 units in a system with an intensity range of 256 units. In a real image, an edge is likely to have a slanted profile, the operator may even be calculating on planar data. For both operators, operating on planar data, theoretically there will be no errors except for those caused by the quantisation of the intensity values, and this was found to be true in practice. The 'sudden' step edge is considered to constitute the worst case situation⁸, and so is implemented here.

Error, %

Error, degrees

Figure 3. Edge magnitude error.

Figure 4. Edge orientation angle error.

The model was implemented by a computer program. The edge was limited so that it could pass only through the centre of the central data pixel. The area of this, and each surrounding, pixel situated on either side of the edge was then calculated, together with the resulting intensity value. The appropriate operator was then applied to this data and the

magnitude and angle results compared with the values used to generate the model data. The process was repeated for one degree angular steps from 0 to 90°. The results are shown in Figures 2 and 3, they do not include intensity quantisation noise, which can add an additional 2° of uncertainty to the angular results (256 quantisation levels). The hexagonal results are probably accurate enough for robot vision applications, but are less accurate than those for the square sampling system. This was due to the choice of integer '1' for the operator weighting factor being slightly less optimum than the choice of the integer weighting factors used in the Sobel operator case. If a fast floating point co-processor is available, accuracy could be improved for both operators with little processing time penalty by modifying these integer weighting factors.

3. APPLICATION TO ROBOT VISION

3. 1. General.

Robot vision systems should always be designed with commercial considerations in mind, systems must be cost effective and efficient. As stated previously, 13.4% less sampling points and hence 13.4% less image storage memory is required for a system using hexagonal sampling. Low level frequency plane operators and low level spatial plane operators are processed with significant time savings and high accuracy, as described above.

Commercial image processing systems based on cellular automaton operating on hexagonal arrays have been produced. Two, one called CLOPR and the other, its successor, DIFF3, have been developed from the original ideas of Golay³ and Preston. Preston¹⁰ describes these machines and gives examples of their use for biological cell analysis. Preston also indicates that the main reason for the unpopularity of hexagonal systems would appear to be that it is perceptually more satisfactory for humans to observe straight vertical features on a square grid system, and that in the real world, such features predominate. With a robot vision system, this will not be a concern as human involvement is limited to problem evaluation and system setup checking. The robot system will be able to exploit the more robust, easily defined, and quicker operating hexagonal algorithms, in particular those involving connectivity considerations.

The above machines employ mainly morphological operators to separate objects and background within the image. Now, frequency plane and local spatial plane operators are available giving a wide range of techniques with which to solve robot vision problems.

3. 2. System input and output.

Digitisation hardware for hexagonal image sampling is relatively easy to design. Raster scan TV digitisers can be produced by simple modifications to existing square digitiser designs⁴. The first sampling point on alternate scan lines needs to be delayed by a period of half the sampling interval, and, for a regular hexagonal geometry, the sampling interval must be increased by a factor of $2/(3^{1/2})$. No change to band limiting filters will be required².

Hexagonal sampling systems pose no additional problems with output from the vision system to the other robot systems. The sampling points are located within a rectangular co-ordinate frame, and, as stated above, only a minimal time penalty is incurred in computing vector quantities such as edge response.

4. CONCLUSIONS

This paper has investigated the use of a regular hexagonal sampling grid in computer vision and extended these investigations to discuss the advantages of such a grid in robot vision. The high symmetry of the grid led to the design of simple and accurate local operators which can be computed more efficiently than their rectangular grid counterparts.

Other techniques which can be optimally implemented on a regular hexagonal grid system include frequency plane techniques, such as global filters, and morphological techniques. Connectivity between pixels is more easily defined and less digital memory is required to store an image. Digitisation of raster scanned images on such a grid requires only minimal modification to frame grabber hardware.

Hopefully the case has been made that hexagonal sampling should be considered when robot vision systems are being designed.

5. REFERENCES

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6. FIGURES

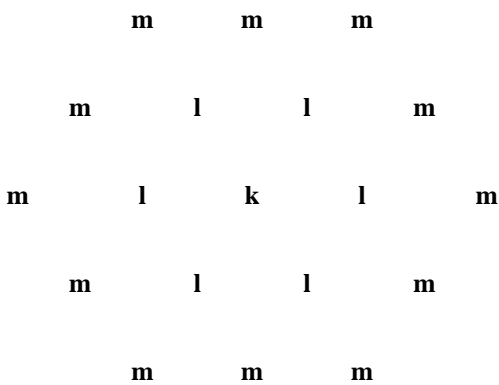


Figure2. Three shell filter.

1	2	1	1	0	-1
0	0	0	2	0	-2
-1	-2	-1	1	0	-1

magnitude $m = [th^2 + tv^2]^{1/2}$

angle $a = \arctan [tv/th]$

Figure 5a Sobel operators