Flow in Gutters & Downpipes

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April 2001

Abstract

This project deals with the development and empirical testing of a spatially varying flow model for analysis of rainwater flow in gutters. This area is of interest for rainwater harvesting, which is particularly relevant for providing cheap access to water fit for domestic use in developing countries. A theoretical model was developed, and a computational model created to allow testing of the model against experimental data. Experiments were conducted on a test rig designed to simulate the flow conditions in the gutter. Good agreement was found between the computational model and the experimental results, validating the model as sufficiently accurate for the applications being considered.

The principal outcome of the work is a computational design tool that can be used to improve gutter performance considering a variety of factors (gutter geometry, slope, rainfall intensity and gutter material).

1. Introduction

1.1. Why Rainwater Harvesting?

Access to clean (hygienic-low pathogen and toxic chemical content) water in reasonable quantities is essential to human health. In 1992 an estimated 1.2 billion people were without access to a clean, convenient water supply (Bastermeyer & Lee). In general, rainwater is of high quality.

Although there are economies of scale in water treatment, large plants tend to be highly capital intensive, and so are not feasible for certain situations in developing countries. Water harvesting in general is a feasible option for improving the living conditions of millions currently facing serious water supply problems, and reduces pressure on groundwater sources (Lee & Visscher). Under these conditions, rainwater harvesting becomes an attractive option for supplying water to a household. As the demand for water grows with increased global population and more "water-intensive" lifestyles, there may be an increase in world-wide use of rainwater harvesting. The increase in the number of potential catchment surfaces also increases the potential for its application (Gould & Nissen). There are also certain special cases in which rainwater harvesting may be particularly attractive, such as earthquake zones (e.g. Japan).

Rainwater harvesting, then, has as its source a cheap supply of good quality water, and there is little in the way of transport costs. Some of the components used are already present or simple, and the required technical skills may be low. This is not to say that the systems are always simple, or that there is not a significant amount of engineering involved.

It is thus an area worthy of investigation, particularly given the lack of theoretical work (see section 1.2).

Although rainwater harvesting may become more popular in developed countries in the future, the focus of this project is for its use in developing countries.

1.2. Theoretical modelling

There seems to be a paucity of information and work on the type of fluid mechanics required to analyse flow in gutters. The particular type of analysis used is termed spatially varying flow, and applies when the flow rate is not constant along a channel. In such situations, a range of factors such as depth, velocity, hydraulic radius etc, are constantly changing.

There was some development of such models, as can be found in Chow (1959), but as solutions tended to require numerical methods, little was made of them.
Increased computing power and more suitable tools, such as spreadsheets, mean that generating such solutions is no longer prohibitively tedious or time consuming.

If such solutions were available, they could be useful in sizing gutters efficiently, and hence reducing their cost. A second use of such models would be in determination of the effect of slope, and in giving guidance as to an optimal slope, if variations in slope are practicable.

2. Spatially Varying Flow Literature Review

Much of the theory available at present relating to channel flow assumes a constant flow, with no variation. Whilst some of the concepts used to analyse the flow are similar, and a sound understanding, as may be gained by reading Chanson[1], is useful, it does not give sufficient information. For flows such as this, the Manning equation can be applied.

When considering the flow in gutters, for which the value of the flow rate is constantly changing, some kind of spatially varying flow analysis is required. There has been less work conducted on this topic. Chow[2] explored the topic in a slightly different context in the 1950s, considering the flow over a weir into a spillway: here the scale of the whole operation is much larger than that being considered here, but the principles still apply. In his work he reaches a differential equation, which at the time of writing would not have been easy to solve by numerical techniques as is currently the case. There are also some comments of interest on the modification of predicted results to include some correction for entrainment of air in the flow. (This may also be of interest if siphonic guttering systems are considered.)

Garcia[3] conducted further work in the third year project of the previous academic year. This gave a spatially varying model for the flow, and some predictions as to the optimum slope which should be created for the gutter. However, in developing the model further, and re-examining the work in Chow, it was found that the hydrostatic pressure term, which had been neglected by Garcia, was not insignificant in describing the behaviour of the flow.

Considering work being conducted in the area of describing the flow in gutters, there still seems to be little work on spatially varying flow. Beecham and O'Loughlin[4-6] have conducted some work, using a rig that appears, from the abstract to [4], to be considerably superior to that at Warwick. However, the focus of their work centres on adaptation of the Manning formula for use with guttering systems, rather than developing a new spatially varying flow analysis. Whether this is a valid approach or not is beyond the scope of the project to consider. Beecham was contacted, and was extremely helpful in giving information and supplying papers. Unfortunately the experimental work conducted was for a private client, and as such is not available for free distribution.

Although it is of interest to consider the behaviour of the flow purely as a theoretical fluid dynamics question, it is important to be aware that the rainwater harvesting aspect means that the results obtained, to be of use, must be applicable to a practical situation. In light of this, the paper by Thomas[7] is useful, as it introduces several of the practical difficulties involved, including those of trying to use a non-uniform gutter profile. The theoretical model developed to describe the flow uses the Manning formula, and so is arguably not valid except as a very rough approximation. Heggen[8] includes some further information on guttering technologies in his paper. This does include some inaccuracies of quotations (the formula for the spatially varying flow from Chow is incorrect), and the overall content of the report seems to be a synopsis of work conducted by others on the subject, rather than contributing any new material. As such it may be of use to the reader wishing to find more about rainwater harvesting, but not for providing any new information.

1. Chanson, Hubert 1999. *The Hydraulics of Open Channel Flow –An Introduction.* Arnold, London.

2. Chow, Ven Te 1959. *Open-Channel Hydraulics*. McGraw-Hill Book Company, New York.

3. Garcia, Inigo 2000. *Guttering Design* (3rd Year Project) University of Warwick.

4. Beecham, S.C. and O'Loughlin, G.G. *The Hydraulics of Flows on Roofs and Gutters,* 6th International Conference of the Urban Stormwater Drainage, Niagara Falls, Canada. (1993)

5. Beecham S.C. and Khiadabi, M.H. *Frctional Characteristics of Spatially Varied Flow*. School of Civil Engineering, University of Technology, Sydney.

6. Beecham, S.C and O'Loughlin, G.G. *Hydraulics of Spatially Varied Flow in Box Gutters*. School of Civil Engineering, University of Technology, Sydney.

7. Thomas, T.H. 1999. *Working Paper No. 50 Guttering Design for Rainwater Harvesting.* Development Technology Unit.

8. Heggen, Richard J. *Hydraulics of Rainwater Catchment Gutters.* University of New Mexico

3. General Analysis of Gutter sections

In this section the main theory used to model and predict the flow in gutters will be introduced. Although the results will obviously be different for different cross-sectional shapes, the initial approach is the same, as can be found in the report by Inigo Garcia from last year. The first stage in modelling flow in the gutter system is to set up a control volume:

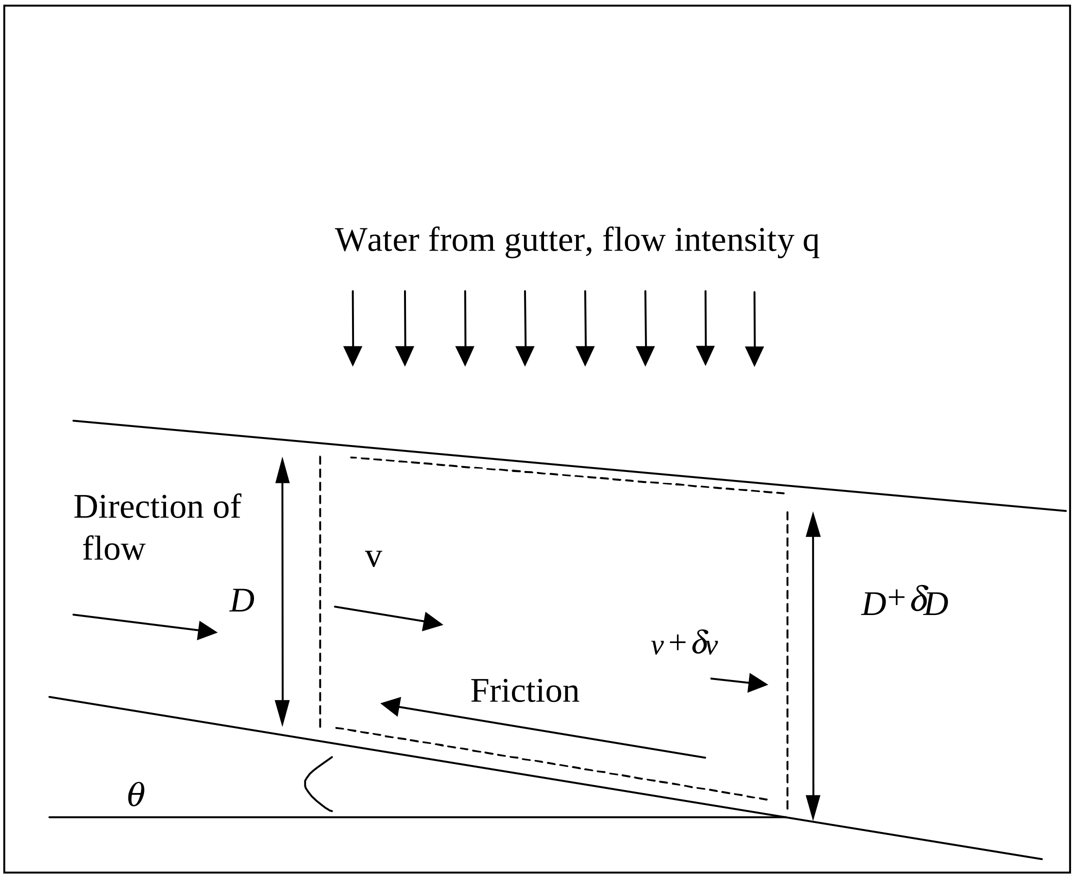


Figure 3.1 The Control Volume for Analysis

3.1. Symbols to be used in analysis

v	velocity
D	depth of flow
θ	slope of gutter bed
x	distance along gutter from non-outlet end
δx	horizontal length of control section
q	intensity of water falling into gutter from roof
p	wetted perimeter of flow
A	cross-sectional area of flow
R_H	hydraulic radius of flow
n	Manning surface roughness factor
τ_w	wall shear stress
ρ	density of fluid in flow
g	gravitational constant
h	drop from edge of roof to flow in gutter
v_f	velocity of water falling into gutter

3.2. Assumptions made in the analysis

- The head loss in a section is the same as for a uniform flow having the velocity and hydraulic radius of the section.
- The channel is prismatic.
- The velocity distribution in the channel is fixed.
- The slope of the channel bottom is small (less than 5% from Chaudhry).

- The slope of the gutter is constant. (This assumption is made for the initial analysis. Further work will use non-constant slopes, but in every case the slope in the control volume is constant. However, as the length of the control volume tends to zero, this means the model can be used for continually varying slopes.)

This analysis uses first order terms only, so does not claim to be entirely accurate, but is a simplified model which requires less complexity in its development, and can be used to test how useful modelling with the former assumptions can be as a design tool for rainwater harvesting.

3.3. Forces acting on the control volume

As can be seen, there are several forces acting on the control volume:

- Gravity, acting to accelerate the flow.
- Friction, opposing the flow.
- A hydrostatic pressure term, opposing the motion. [ψ]

However, there is not only the flow already present in the gutter, but also that being added by runoff from the roof.

The first expression to establish is one of mass continuity:

$$v(x)A(x) = qx \qquad \text{Equation 3.1}$$

[ψ] This was not included in the analysis conducted by Garcia

Where v is the velocity at distance x along the gutter from the non-discharge end, A is the cross-sectional area at that point, and q is the flow intensity running into the gutter.

3.4. Momentum Equation

The momentum equation states:

$$\sum F_f = \dot{M}_{out} - \dot{M}_{in}$$ Equation 3.2

The rate of momentum flowing in to the control volume can be expressed as:

$$\dot{M}_{in} = \rho Q v = \rho (Av) v = \rho A v^2$$ Equation 3.3

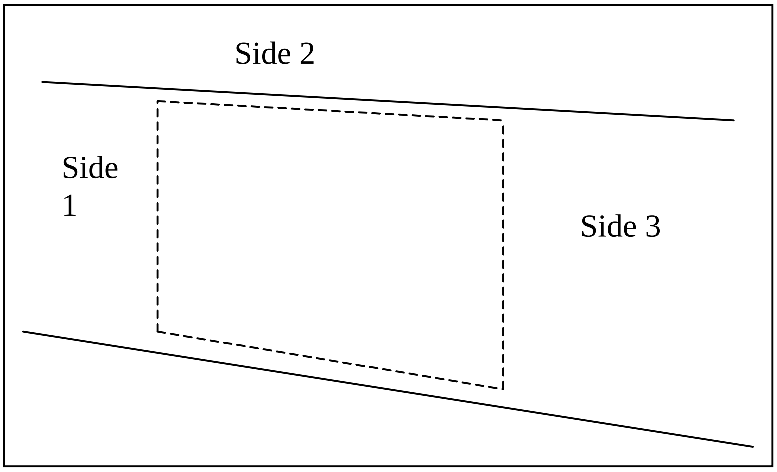


Figure 3.2 Labelling sides of the control volume

[NB in deriving this, the velocity of the additional water entering the flow is assumed to be zero in the direction of flow]

$$\dot{M}_{out} = \rho.(Q + \delta Q)(v + \delta v) = \rho (A + \delta A)(v + \delta v)^2$$ Equation 3.4

Neglecting higher order terms, equation 3.4 becomes:

$$\dot{M}_{out} = \rho(Av^2 + v^2\delta A + 2Av\delta v)$$ Equation 3.5

Thus the right hand side of equation 3.2 can be rewritten:

$$\dot{M}_{out} - \dot{M}_{in} = \rho(v^2\delta A + 2Av\delta v)$$ Equation 3.6

3.5. Forces acting on the fluid

3.5.1. Frictional Force

The frictional retarding force can be expressed by taking an average for the wetted perimeter (assuming a linear increase for the small length of the control volume):

$$Frictional\ \ force = -\tau_w \frac{(p + (p + \delta p))}{2}\delta x = -\tau_w (p + \frac{\delta p}{2})\delta x$$ Equation 3.7

Note the negative sign, as the force is opposing the direction of flow.

However, the wall shear stress can be expressed as:

$$\tau_w = \frac{\rho v^2}{8} f$$ Equation 3.8

With $f = \frac{7.4n^2}{R_H^{1/3}} \quad n = 0.0382\varepsilon^{\frac{1}{6}}$ Equation 3.9

Thus: $\tau_w = \frac{C\rho v^2}{R_H^{1/3}}\, with\, C = 9.7n^2$ Equation 3.10

So

$$Frictional\ \ force = -\frac{C\rho v^2}{R_H^{1/3}}(p + \frac{\delta p}{2})\delta x$$ Equation 3.11

Taking first order terms from this:

$$F_f = -\frac{C\rho v^2}{R_H^{1/3}}\, p\delta x$$ Equation 3.12

3.5.2. Gravitational Force

This can be expressed by resolving the gravitational force in the direction of the flow:

$$F_g = \rho g(Volume)\sin\theta$$ Equation 3.13

Assuming the increase in depth is linear for the small section being considered:

$$F_g = \rho g\, \frac{A + (A + \delta A)}{2}\delta x \sin\theta = \rho g(A + \frac{\delta A}{2})\delta x \sin\theta$$ Equation 3.14

Changes in flow surface angle with respect to horizontal (a departure from θ) become negligible in this first order analysis, giving:

$$F_g = \rho g A\delta x \sin\theta$$ Equation 3.15

However, for small angles, $\sin\theta \approx \theta$ and $\tan\theta \approx \theta$, therefore

$$\sin\theta \approx \tan\theta \qquad \text{Equation 3.16}$$

Substituting this into 3.15 gives

$$F_g = \rho g A \delta x \tan\theta \qquad \text{Equation 3.17}$$

As $Slope(S_o) = \tan\theta$, equation 3.17 becomes:

$$F_g = \rho g A \delta x S_o \qquad \text{Equation 3.18}$$

S_o is positive for a slope dropping from left to right, giving the gravitational force as opposite to the frictional force, as expected.

3.5.3. Falling Water Term

The water dropping in to the gutter will have a small component in the velocity of the flow. Assuming free fall, the velocity may be taken as a function of the fall distance, which will vary along the gutter, but may be taken as:

$$h_x = h_o + x\sin\theta \qquad \text{Equation 3.19}$$

For a body under constant acceleration

$$s = \frac{v_f{}^2 - u^2}{2a} \qquad \text{Equation 3.20}$$

Where *s* is the distance fallen, *v* the final velocity, *a* the acceleration and *u* the initial velocity. Making the assumption that the initial velocity of the water falling from the roof edge is zero, substituting equation 3.20 for *s* and *g* for *a* gives:

$$v_f = \sqrt{2g(h_o + x\sin\theta)} \qquad \text{Equation 3.21}$$

This can be resolved into a force in the direction of flow:

$$F_d = \rho q v_f \sin\theta\delta x$$ Equation 3.22

3.5.4. *Hydrostatic pressure term*

Having the situation shown below, there will be a net pressure force acting on the

control volume $= \frac{dF_p}{dx}\delta x$: Equation 3. 23

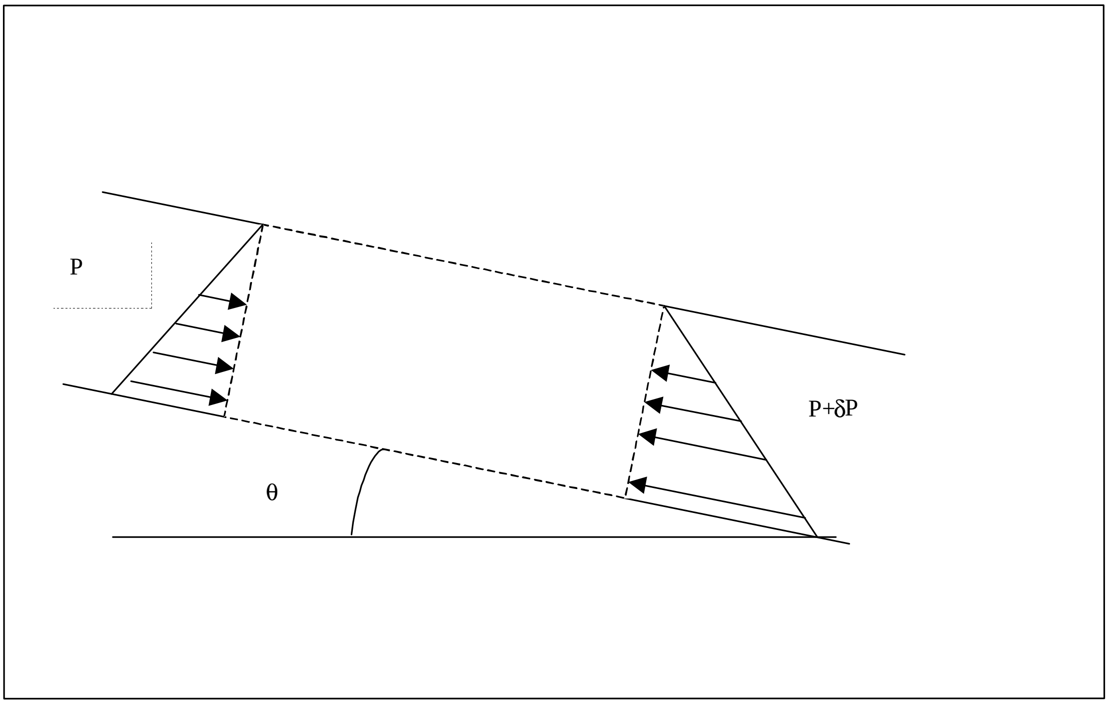


Figure 3.3 Illustrating the pressure forces

To find this pressure, consider the following example, using an arbitrary cross-sectional shape:

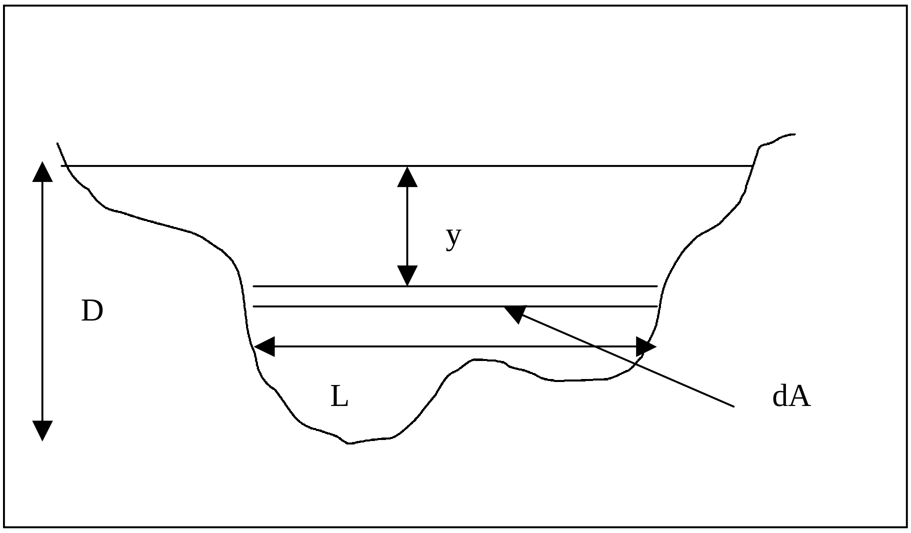


Figure 3.4 Labelled arbitrary cross-section for deriving pressure force

$F_p = \rho g y dA = \rho g L y dy$ Equation 3.24

Therefore:

$F_p = \rho . g \int_0^D L(y).ydy$ Equation 3.25

Once this has been found, we wish to find the differential, $\frac{dF_p}{dx}$, in terms of D, the depth in the gutter. To achieve this, we first express $\frac{dF_p}{dx}$ as a function of equation 3.25:

$\frac{dF_p}{dx} = \frac{d}{dx}\left(\rho g \int_0^D L(y)ydy \right) = \rho g \frac{d}{dx}\left(\int_0^D L(y)ydy \right)$ Equation 3.26

Then expressing 3.26 in terms of D:

$$\frac{dF_p}{dx} = \rho g \frac{d}{dD}\left(\int_0^D L(y)ydy\right)\frac{dD}{dx}$$

Equation 3.27

3.6. Re-constructing the equation

Substituting the terms so far generated into 3.2 (3.6, 12, 18, 23 & 27) gives:

$$-\frac{C\rho v^2}{R_H^{1/3}} p\delta x + \rho g A \delta x S_o + \rho q v_f S_o \delta x + \left(-\rho g \frac{d}{dD}\left(\int_0^D L(y)ydy\right)\frac{dD}{dx}\right)\delta x = \rho(v^2 \delta A + 2Av\delta v)$$

Equation 3.28

Cancelling ρ, dividing by δx and letting small terms become infinitesimal, gives

$$-\frac{Cv^2}{R_H^{1/3}} p + gAS_o + qv_f S_o + \left(-g \frac{d}{dD}\left(\int_0^D L(y)ydy\right)\frac{dD}{dx}\right) = (v^2 \frac{dA}{dx} + 2Av\frac{dv}{dx}) = \frac{d}{dx}(v^2 A)$$

Equation 3.29

Recalling $v = \frac{qx}{A}$ from 3.1, the right hand side of 3.29 can be rewritten:

$$\frac{d}{dx}(v^2 A) = \frac{d}{dx}\left(\frac{q^2 x^2}{A}\right) = q^2 \frac{d}{dx}\left(x^2 A^{-1}\right) = q^2\left[\left(x^2 \frac{dA^{-1}}{dx}\right) + \left(A^{-1}\frac{dx^2}{dx}\right)\right]$$
$$= q^2\left[\left(x^2 \frac{dA^{-1}}{dA}\frac{dA}{dD}\frac{dD}{dx}\right) + \left(\frac{2x}{A}\right)\right] = \left(-\frac{q^2x^2}{A^2}\frac{dA}{dD}\right)\frac{dD}{dx} + \frac{2xq^2}{A}$$

Equation 3.30

This can be substituted into equation 3.29, to give:

$$\left(\frac{q^2x^2}{A^2}\frac{dA}{dD} - g\frac{d}{dD}\left(\int_0^D L(y)ydy\right)\right)\frac{dD}{dx} = \frac{Cv^2}{R_H^{1/3}}p - gAS_o - qv_f S_o + \frac{2xq^2}{A}$$

Equation 3.31

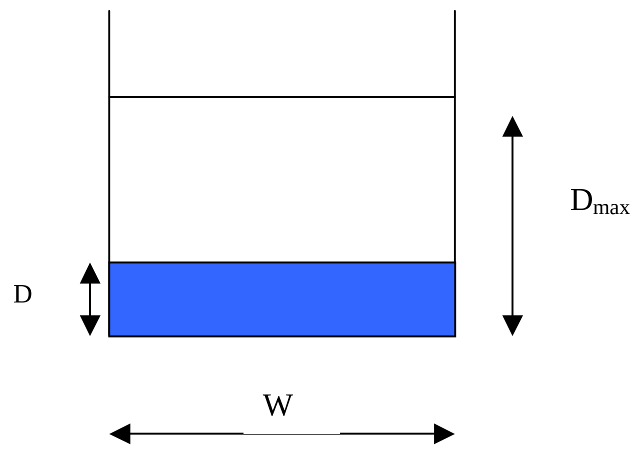
Multiplying by $\frac{A^2}{q^2x^2}$, noting that $\frac{A^2}{q^2x^2} = \frac{1}{v^2}$, and rearranging to give $\frac{dD}{dx}$:

$$\frac{dD}{dx} = \frac{\frac{Cp}{R_H^{1/3}} - \frac{gA^3S_o}{q^2x^2} - \frac{v_f A^2 S_o}{qx^2} + \frac{2A}{x}}{\left(\frac{dA}{dD} - \left(\frac{A^2 g}{\rho q^2 x^2}\frac{d}{dD}\left(\int_0^D L(y)ydy\right)\right)\right)}$$

Equation 3.32

3.7. *Specific Geometries*

At this point the unknown variables on the left hand side can be reduce by substituting specific equations for area, the pressure term, wetted perimeter, hydraulic radius, area etc. The two cross-sections considered so far are: rectangular and v-section. Below are the cross-sections, with characteristic lengths to allow formulation of equations to describe their area etc:



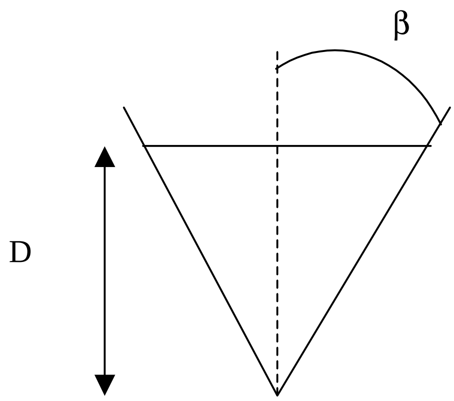


Figure 3.5 Specific cross-sections and characteristic dimensions

	Cross-section	
Property	**Rectangular**	**V-section**
Area (A)	WD	$D^2 \tan\beta$
$\frac{dA}{dD}$	W	$2D\tan\beta$
Wetted Perimeter (p)	$W+2D$	$\frac{2D}{\cos\beta}$
Hydraulic Radius (R_h)	$\frac{WD}{(W+2D)}$	$\frac{D}{2}\sin\beta$
$\frac{d}{dD}\left(\int_0^D L(y).ydy\right)$	WD	$D^2 \tan\beta$

This then allows the rate of change of depth to be found as a function of known variables, displacement along the gutter and depth at that point. Unfortunately the equation does not yield analytical results, but can solved using numerical methods to generate a profile, given a boundary condition, e.g. depth at the start of the gutter. There are difficulties with taking the initial depth as zero, as this leads to division by zero in equation 3.32, but a reasonable approximation may be obtained by setting the initial depth to a very low value.

Substituting each of these gives:

Rectangular cross-section

$$\frac{dD}{dx} = \frac{\frac{C(W+2D)^{4/3}}{(WD)^{1/3}} - \frac{g(WD)^3 S_o}{q^2 x^2} - \frac{v_f (WD)^2 S_o}{qx^2} + \frac{2WD}{x}}{W - \left(\frac{W^3 D^3 g}{q^2 x^2}\right)}$$

Equation 3.33

V-section

$$\frac{dD}{dx} = \frac{\frac{2CD}{\cos\beta\left(\frac{D\sin\beta}{2}\right)^{1/3}} - \frac{g\left(D^2\tan\beta\right)^3 S_0}{q^2 x^2} - \frac{v_f\left(D^2\tan\beta\right)^2 S_o}{qx^2} + \frac{2D^2\tan\beta}{x}}{2D\tan\beta - \frac{gD^6\tan^3\beta}{q^2 x^2}}$$

Equation 3.34

3.8. Weir Effect

There is also an effect to further complicate the analysis of the flow: at the discharge end of the gutter the flow upstream will “feel” the fact that there is less resistance downstream; information will be transmitted upstream in the flow. This will cause the flow to accelerate and so decrease the depth at this upstream point, departing from the predicted depth. The weir effect is discussed in Chaudhry, and it would be of interest to quantify where this effect starts to be significant.

As will be seen in the later experimental work, the drawdown effect does have an observable effect. If this effect is only to reduce the depth at a later point in the gutter, then it may be that a conservative design estimate is possible by neglecting the effect and assuming a maximum depth at the end of the gutter. Alternatively, it may be possible to assume the maximum depth at some point upstream from the outlet to give a more realistic estimate.

4. Practical Work

In the previous year's project on this topic, Garcia (2000) constructed a test rig to allow measurement of depth profiles in various gutters, along with the flow intensity of the water falling in to the gutter. This rig was modified and used for further experimental work, as detailed below.

The experimental work conducted to this point in the project is also included. This has focussed on calibrating the rig, measuring the flow profile for varying uniform slopes and cross sections of gutter, and examining the distribution of flow entering the gutter from the simulated roof.

The schematic diagram of this system is shown below:

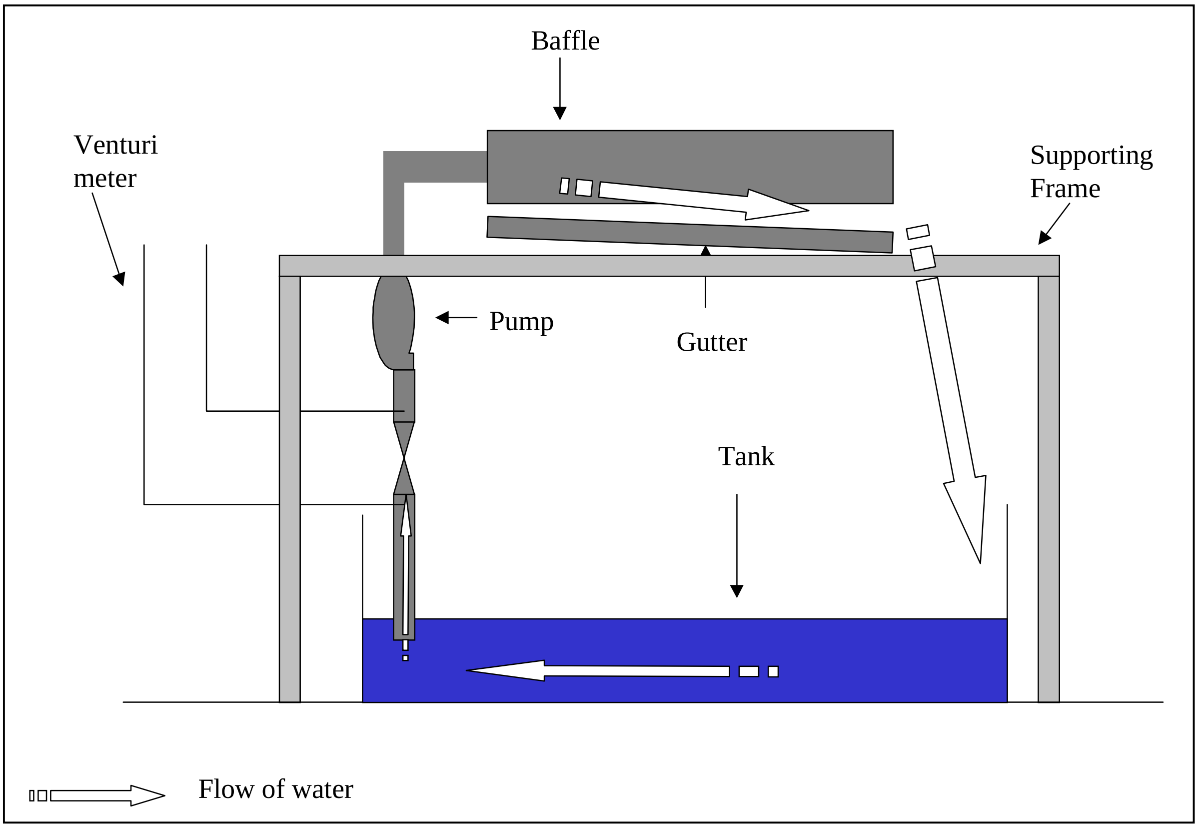


Figure 4.1 Schematic Diagram of test rig

4.1. Experiments

The experiments needed to test the theory require measuring the profile of the water in the gutter for varying:

1. Flow intensities.
2. Gutter slope.
3. Gutter cross-sections.

Before this could be carried out, the test rig had to be calibrated, to establish an effective means of measuring the flow intensity along the gutter. The experiments for this are detailed in Appendix B. The results enabled calculation of the flow intensity from readings taken from the venturi meter on the test rig.

4.2. Single Sloping section

A more detailed account of the experimental process is included in Appendix C. The main point is that several sets of experiments were conducted, for varying slopes and flow intensities. For each experiment the depth of the flow was measured at several points along the gutter.

The results analysed in this project were drawn from using a v-section gutter, and a small rectangular cross-section gutter.

Some further work was conducted on the distribution of the flow along the gutter, this is detailed in Appendix D.

4.3. Initial Observations

It quickly became apparent that the flow in the gutter did not give a simple smooth profile as predicted. The flow from the baffles did not supply the continuous sheet used in modelling the flow. At low flow rates (particularly at 3 valve turns, the lowest possible to obtain a flow), the jets were highly discrete, and could be seen to be at points equivalent to the holes in the blind pipe. As the flow increased, the flow became more uniform, but the jets remained discrete.

The effect of the jets would be to rapidly increase the depth in the section of gutter with the jet impacting, as the flow slowed given the added mass with no velocity in the direction of the flow. In the regions with no flow, the depth would decrease again, as the flow accelerated under gravity.

It could also be seen that the flow distribution along the gutter was not uniform: at the end of the blind pipe closest to the pump the flow was greater than that further down the gutter. This departure from the uniform distribution hoped for seemed to decrease as the flow rate increased.

There was also some temporal instability in the flow i.e. the depth at any point varied over time, seeming to oscillate around an average point.

5. Reconciling Theory and Experimental Data

5.1. Model Testing

An excel spreadsheet was constructed to enable the use of a numerical technique to test the validity of the theoretical model, by giving predicted flow profiles which could then be compared with that obtained from experimentation.

To give a greater understanding of the behaviour of the model, the individual variables, such as depth, area, hydraulic radius etc, were included individually, as were the numerator and denominator.

As it seemed that the model would collapse at the top end of the gutter (the denominator in several terms going to zero), it was decided to test the profiles generated from the experimental data by taking the data from a point where the flow was reasonably well developed, but not suffering from draw-down. Once this point had been established, the profile from the experimental data could be compared with that predicted by the model, given this one point, the slope, flow intensity and gutter geometry.

5.2. Initial Comments

Several points became clear from examination of the behaviour of the spreadsheet:

At certain points the model becomes unstable and starts exhibiting asymptotic behaviour (the precise nature of which will depend on the iteration step size). This can be explained by noting that as $\frac{A^2}{q^2x^2}\frac{d\left(F_p/\rho\right)}{dD} \to \frac{dA}{dD}$, the denominator tends to zero.

This indicates a breakdown in the model. When the Froude number was calculated for each step along the iteration, it was found that the instability occurred at Fr=1 i.e. the flow at this point becomes critical and a hydraulic jump occurs.

A more formal examination of the problem can be used to examine the asymptotic behaviour:

The asymptote will occur when $\frac{dA}{dD} - \frac{A^2}{q^2x^2\rho}\frac{dF_p}{dD} = 0$

Therefore

$$\frac{dA}{dD} = \frac{A^2}{q^2x^2\rho}\frac{dF_p}{dD}$$ Equation 5.1

Considering the rectangular cross-section,

$$\frac{dA}{dD} = W,\ A^2 = W^2D^2,\ \frac{dF_p}{dD} = \rho gWD$$ Equations 5.2-4

Substituting these into Equation 5.1 Gives:

$$W = \frac{W^2D^2\rho gWD}{q^2x^2\rho} = \frac{W^3D^3g}{q^2x^2}$$

$$1 = \frac{q^2x^2}{W^2D^3g}$$ Equation 5.5

Noting that $qx = Q, WD = A\, and$ v = Q/A

Equation 5.5 becomes:

$$1 = \frac{Q^2}{A^2Dg} = \frac{v^2}{Dg}$$

$$\therefore 1 = \frac{v}{\sqrt{Dg}} \text{ i.e. Fr} = 1$$

A similar analysis can be applied to the v-section gutter:

$$\frac{A^2}{q^2 x^2}\frac{d(F_p/\rho)}{dD} = \frac{A^2}{Q^2}\frac{d(F_p/\rho)}{dD} = \frac{1}{v^2}\frac{d(F_p/\rho)}{dD} = \frac{dA}{dD}$$ Equation 5.6

For the v-section, $A = D^2 \tan\beta \Rightarrow \frac{dA}{dD} = 2D\tan\beta$

$$F_p = \rho g \tan\beta \frac{D^3}{3} \Rightarrow \frac{d(F_p/\rho)}{dD} = g\tan\beta D^2$$

Substituting these two results:

$$\frac{1}{v^2} g\tan\beta D^2 = 2D\tan\beta$$ Equation 5.7

Rearranging gives:

$$1 = \frac{v}{\sqrt{g\frac{D}{2}}}$$ Equation 5.8

This is of similar form to the Froude number result previously obtained, but with a modified depth term. This is actually the hydraulic depth:

$$D_h = \frac{\text{Cross sectional area}}{\text{Surface Width}} = \frac{D^2 \tan\beta}{2D\tan\beta} = \frac{D}{2}$$ Equation 5.9

The significant terms in the numerator are the gravitational term and the momentum flux term. The flow profile is not very sensitive to fluctuations in the surface roughness of the gutter material.

5.3. Matching Practical Results to Theoretical Predictions

The spreadsheet data can readily be plotted to give a depth profile. Likewise, the known data points from the experimental data can be plotted and a trend-line fitted to them (along with error bars, as the method of measurement was not highly precise). This was done for all the data obtained from experimentation with the v-section and rectangular cross-section gutters. All the plots obtained are given in Appendices E and F. Two typical plots are shown below:

As can be seen, the triangular section suffers from some instability at the upper end of the gutter (as $x \to 0$). Although this shows the model cannot always accurately predict the behaviour in this region, this is not of significant practical importance for using the model as a design tool: the depth further down the gutter is significantly more important.

In the first two sets of results the theoretical model shows instability with the hydraulic jump, whereas the measured results show a drop in depth after this point. This may be explained by the draw-down effect, where the water in the gutter is able to “sense” the presence of the drop downstream, and so the hydraulic jump does not have the expected effect.

The occurrence of this point may also lead to some design recommendations as to the optimal arrangement of gutters if several sloping sections are possible. It may be that it is more efficient to put the steeper sloping sections further from the upstream end of the gutter, to inhibit the formation of the hydraulic jump.

6. Practical Significance of the Work

Having developed the theoretical model, and shown that the model gives a reasonable description of the flow, and can generate accurate profiles, it is useful to consider its practical use. Some consideration should be given as to what useful information can be extracted for those involved in the practical matter of working with rainwater harvesting systems.

6.1. Existing System or Design?

There are two main classifications of situations:

1. There is an existing system. Given information about the system, it should be possible to predict the performance of the system, giving the maximum gutter capacity, and the rainfall intensity at which this will occur. From this data some consideration could be given to the likelihood of such rainfall events occurring i.e. does the system have sufficient capacity. The change in capacity of moving the downpipe from the end of the building to a midway point can be predicted.
2. A system is being designed. There are certain parameters which may be regarded as fixed:
   - Roof size and geometry.
   - Maximum likely rainfall intensity. (A more sophisticated consideration is possible, with issues as to the importance of capturing the entire rainfall, how much rain is required, the effect of allowing the gutter to overflow on the building etc.)

   Other parameters may be regarded as variable: the designer may change their values to suit the situation:

- Gutter size, cross-section and material.
- The slope of the gutter.

Obviously, even the parameters that can be changed are constrained.

Several issues then arise:

1. Is there an optimal slope for the gutter, or to maximise the capacity, should it be set as high as practicable (there are issues of reduced efficiency of runoff capture with varying drop associated with this).
2. What will be the change in gutter capacity given moving the downpipe from a position at the end of the gutter to the midpoint. As this is likely to double the acceptable slope of the gutter, what will the overall effect on the system be.
3. How sensitive is the system to changes in slope, gutter material, cross-section etc.
4. If several sloping sections are possible, what is the optimum slope arrangement.

It would be useful if the outcomes from the work could be condensed to a series of simple guidelines, giving only such information as is necessary.

6.2. Two Cases of Flow Profile

In the previously developed model (Garcia, 2000), the depth profile gave an increase with depth along the gutter. For practical purposes the main consideration was the depth at the outlet end of the gutter, and how this would vary with flow intensity into the gutter, gutter slope and cross-section, and gutter length.

The later model developed gives a turning point for the depth profile, where the depth reaches a maximum, and then reduces, eventually becoming critical. This gives two situations to consider when using the model as a tool for rainwater harvesting system analysis and design.

1. The flow does not reach a turning point within the length of the gutter. In this case, the depth at the outlet is the critical depth to be considered, and will determine the maximum flow the gutter can accommodate without overflowing.
2. The flow does reach a turning point within the length of the gutter, and the depth then decreases. As the model does not accommodate behaviour after a hydraulic jump, this is not considered. [The hydraulic jump phenomena is to be avoided, as it gives a sudden increase in depth, reducing the capacity of the gutter without overflowing.]

From the perspective of a fieldworker, it would be useful to know whether one case would hold in all reasonable practical situations. To consider this, a few typical values will be examined. An extreme rainfall event will give not more than around 0.00005m/s of rain.

Given a maximum roof length of 10m, and a roof depth (length perpendicular to the gutter) of 5m, the maximum flow intensity will be 0.00025 $m^3/s/m$, with a maximum flow from the whole gutter of 0.0025 m^3/s (2.5 l/s).

There will also be a limit to the practicable drop from roof edge to gutter: a figure of 0.3m seems reasonable in this case. The gutter width is likely to fall within the range of 0.05 to 0.15m.

No data was available to give the initial data point to allow the numerical method to be used to predict a unique profile. However, the numerical method was used with a range of values for the initial data point (the depth at x=0.2m), and it was found that the profile features were not sensitive to initial depth (providing it was above a reasonable lower value of 1cm). The main feature to be drawn out of the work was that the profile did not exhibit a turning point within the length of the gutter. This is of interest, as it means there is still some interest in the siphonic outlets as a method for reducing the depth of flow in the gutter.

6.3. Comments on other Predictive Methods: Why use the Manning formula?

As has been shown quite clearly in the earlier practical work, the profiles generated by using the model developed (with one fixed empirical data point required by the numerical technique) and those obtained by experimental work are similar. In the case of the v-section gutter, there is some discrepancy, which is to be expected, as the flows involved were large and oscillating over a relatively large range. However, there is a close correlation between the two, and the results obtained with the rectangular cross-section also suggest some validity in using the model.

Given that the theoretical model developed is valid for the situations being considered, it is necessary to consider its practical application at this point. If the spreadsheet is to be used as a design tool in the field, then the initial data point required for the numerical technique may be difficult or unfeasible to obtain. It would be preferable if the entire profile could be predicted without requiring the use of any empirical data. One possibility to be considered in this case is that of using the Manning formula to generate an initial point.

6.4. Initial Generation of Data Point via Manning Formula

From the Manning equation:

$$Q = \frac{k}{n}\sqrt{S_o}R_H^{2/3}A \qquad \text{Equation 5.10}$$

It is possible to generate a solution to provide an initial point to use with the numerical methods detailed previously. For a rectangular cross-section, with SI units, the following substitutions may be made:

$k = 1$

$Q = qx$

$A = WD$

$R_H = \frac{WD}{W + 2D}$

Substituting these into equation 5.1 gives:

$$qx = \frac{1}{n}\sqrt{S_o}\left(\frac{WD}{W + 2D}\right)^{2/3}(WD) \qquad \text{Equation 5.11}$$

This can be rearranged to give:

$$D = \frac{1}{W}\left(\frac{qxn}{\sqrt{S_o}}\right)^{3/5}(W + 2D)^{2/5} \qquad \text{Equation 5.12}$$

Which can be used to give an iterative solution for D at any point along the gutter. It should be noted that this is only to be used as a tool to generate an initial point, not as a predictive model: the assumptions of the Manning equation are not valid for the situation in gutter flow. Equation 5.12 can be used as below:

$$D_{n+1} = \frac{1}{W}\left(\frac{qxn}{\sqrt{S_o}}\right)^{3/5}(W + 2D_n)^{2/5}$$

Which converges rapidly to give a solution.

Several points arise when considering using this approach to generate the initial point:

1. Using the Manning formula gives a flow profile considerably different from that obtained by experimental methods. At small flow levels, the flow depth is lower than that obtained, whilst as the flow develops, the depth predicted by the Manning formula rapidly becomes larger than that observed.
2. Under certain conditions, it is possible for the predicted depth using the Manning formula to be one of supercritical flow. For example, testing typical values of q = 0.00025, a slope of 2 degrees and a rectangular gutter of width 0.05m, the flow is supercritical at every point.
3. It is possible to gain what initially seems a more realistic estimate by assuming that the flow at the point being considered is critical. However, this is not a useful approach for use with the model developed, as the data point generated leads to a breakdown in the model.

One possible approach to the difficulty of the Manning solution giving critical flow conditions is to use conjugate depths. For a supercritical flow, there is a conjugate depth which the flow may "jump" to. The ratio of depths is given by[1]:

[1] Massey, B.S.1992. Mechanics of Fluids, 6th Edition. Chapman & Hall, London.

$$\frac{h_2}{h_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2Fr_1^2}$$ Equation 5.13

The use of conjugate depths may be justified by the argument that any supercritical flow is unstable, and disturbances to the flow will tend to cause a jump to subcritical flow conditions. In the situation being considered, the flow is being disturbed at every point, and therefore maintaining supercritical flow is unlikely to be possible.

6.5. Optimal Slope

If the assumption is made that the optimal flow condition in the gutter is one where the flow is critical at all points (i.e. just before the onset of a hydraulic jump), then it is possible to express the defining equation of the flow to give an expression for the slope at each point (considering a rectangular cross-section):

$$\frac{dD}{dx} = \frac{\frac{Cp}{R_H^{1/3}} - \frac{gA^3 S_o}{q^2 x^2} - \frac{v_f A^2 S_o}{qx^2} + \frac{2A}{x}}{\left(\frac{dA}{dD} - \left(\frac{A^2 g}{\rho q^2 x^2} \frac{d}{dD}\left(\int_0^D L(y) y dy\right)\right)\right)}$$ Equation 3.32

This can be rewritten as follows, and then simplified:

$$\frac{dD}{dx}\left(\frac{dA}{dD} - \left(\frac{A^2 g}{\rho q^2 x^2} \frac{d}{dD}\left(\int_0^D L(y) y dy\right)\right)\right) = \frac{Cp}{R_H^{1/3}} - \frac{gA^3 S_o}{q^2 x^2} - \frac{v_f A^2 S_o}{qx^2} + \frac{2A}{x}$$

As the flow is critical, the second term of the left hand side tends to zero, giving:

$$0 = \frac{Cp}{R_H^{1/3}} - \frac{A^3 g S_o}{q^2 x^2} - \frac{v_f A^2 S_o}{qx^2} + \frac{2A}{x}$$ Equation 5.14

As the flow is critical, the Froude number will be equal to 1, and hence $v = \sqrt{gD}$. As

$$v = \frac{Q}{A} = \frac{qx}{WD},$$

$$gD = v^2 = \left(\frac{qx}{WD}\right)^2$$

Which can be rearranged to give

$$D = \sqrt[3]{\frac{q^2 x^2}{W^2 g}} \qquad \text{Equation 5.15}$$

Expressing this in non-dimensional terms gives

$$D' = \sqrt[3]{\frac{q'^2\, x'^2}{W'^2}} \qquad \text{Equation 5.16}$$

Thus the depth at any point along the gutter can be found.

Equation 5.14 may be rearranged to give an expression for the slope as a function of *x*:

$$S = \frac{\dfrac{2A}{x} + \dfrac{Cp}{R_H^{1/3}}}{\dfrac{A^3 g}{q^2 x^2} + \dfrac{v_f A^2}{qx^2}} \qquad \text{Equation 5.17}$$

Noting that $\frac{1}{v} = \frac{A}{qx}$, equation 5.17 can be simplified to give

$$S = \frac{\dfrac{2A}{x} + \dfrac{Cp}{R_H^{1/3}}}{W + \dfrac{v_f}{qgD}} \qquad \text{Equation 5.18}$$

A difficulty arises when the expression for v_f is expanded, as this contains a term with S_o. However, on examining the spreadsheet for typical cases, it was found that the first term of the denominator is much larger than the second, and so the expression was simplified to the following, for initial consideration:

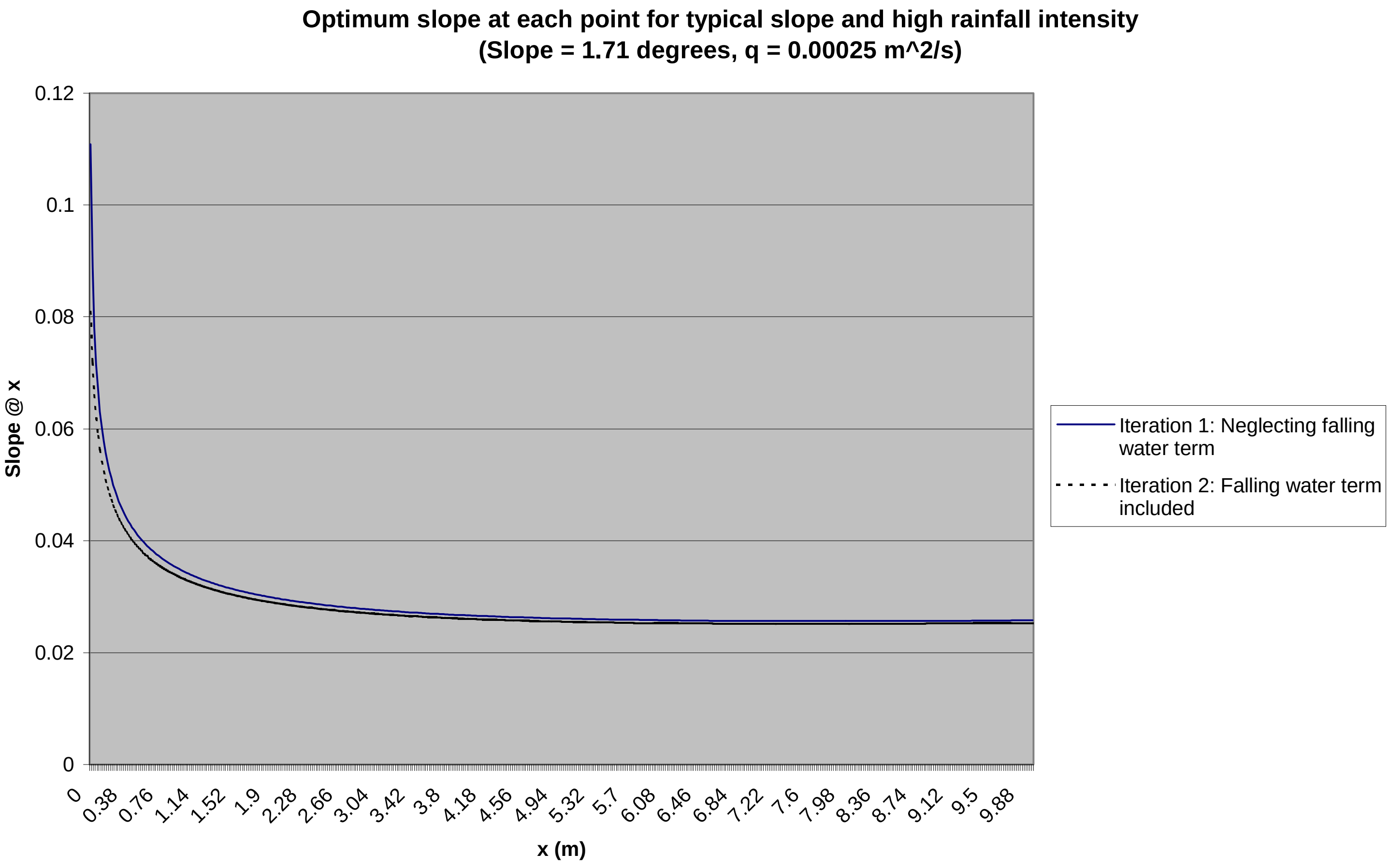
$$S = \frac{\frac{2A}{x} + \frac{Cp}{R_H^{1/3}}}{W} = \frac{2D}{x} + \frac{C(W+2D)^{4/3}}{W^{4/3} D^{1/3}}$$ Equation 5.19

Substituting equation 5.16 gives:

$$S(x) = \sqrt[3]{\frac{8q^2}{W^2 x g}} + C \frac{\left(W + \sqrt[3]{\frac{8q^2 x^2}{W^2 g}} \right)^{4/3}}{W^{4/3} \sqrt[9]{\frac{q^2 x^2}{W^2 g}}}$$ Equation 5.20

This then gives an explicit expression for *S* in terms of *x*. From this it is possible to recalculate the slope, using the initial slope generated and substituting into equation 5.18. It was found that this gives very similar results to those obtained neglecting the term for the velocity of the water in the direction of flow.

An example of the optimum slope predicted is shown below:



Optimum slope at each point for typical slope and high rainfall intensity
(Slope = 1.71 degrees, q = 0.00025 m^2/s)
Slope @ x
0.12
0.1
0.08
0.06
0.04
0.02
0
0 0.38 0.76 1.14 1.52 1.9 2.28 2.66 3.04 3.42 3.8 4.18 4.56 4.94 5.32 5.7 6.08 6.46 6.84 7.22 7.6 7.98 8.36 8.74 9.12 9.5 9.88
x (m)
Iteration 1: Neglecting falling water term
Iteration 2: Falling water term included

7. Further Theoretical Work: Non-Dimensional Solutions

7.1. Non-dimensional form:

Although the numerical method previously described validates the model, and will accurately predict the depth profile given an initial data point, the generation of this point is difficult, and the approaches attempted, using the Manning formula, were not successful. It might be possible to develop a further model to describe the flow at the inlet end of the gutter, but on examination this was found to be a non-trivial problem.

An alternative method to avoid the problem of generating the initial data point is to find a non-dimensional form of the solution to the flow, and then find an asymptotic solution to give an explicit expression for the depth.

Using the expression previously developed to describe the flow in the gutter:

$$\frac{dD}{dx} = \frac{\frac{Cp}{R_H^{1/3}} - \frac{A^3 g S_o}{q^2 x^2} - \frac{v_f A^2 S_o}{q x^2} + \frac{2A}{x}}{\frac{dA}{dD} - \frac{A^2}{\rho q^2 x^2}\frac{dF_p}{dD}} \qquad \text{Equation 3.32}$$

This can be non-dimensionalised with respect to two length scales: L, the gutter length, and αL, a characteristic length scale for the section dimension (e.g. $\sqrt{A'}$). First non-dimensionalise each of the respective terms:

$$x' = \frac{x}{L} \qquad \text{Equation 7.1}$$

$$D' = \frac{D}{\alpha L} \qquad \text{Equation 7.2}$$

$$A' = \frac{A}{(\alpha L)^2}$$ Equation 7.3

$$C' = \frac{C}{\alpha^{1/3} L^{1/3}}$$ Equation 7.4

$$R_H' = \frac{R_H}{\alpha L}$$ Equation 7.5

$$P' = \frac{P}{\alpha L}$$ Equation 7.6

$$v' = \frac{v}{\sqrt{g \alpha L}}$$ Equation 7.7

This becomes more complex (but not insoluble) when considering the flow intensity q:

$$v = \frac{qx}{A} \Rightarrow q = \frac{vA}{x} = \frac{\frac{v}{\sqrt{g\alpha L}}\sqrt{g\alpha L}\,\frac{A}{(\alpha L)^2}(\alpha L)^2}{\frac{x}{L}L}$$

Substituting for v', A' and x' gives:

$$\begin{aligned} q &= \frac{v' A'}{x'} \frac{\sqrt{(g\alpha L)}\alpha^2 L^2}{L} \\ &= q'\sqrt{g}\alpha^{5/2} L^{3/2} \end{aligned}$$ Equation 7.8

For v_f' use $\sqrt{gLS_o}$:

$$v'_f = \frac{v_f}{\sqrt{gLS_o}}$$ Equation 7.9

For $\frac{1}{\rho}F_p$: $\frac{1}{\rho}F_p \propto gD^3 = gD'^3 \alpha^3 L^3$

$$\frac{F_p'}{\rho} = (F_p / \rho)\frac{1}{g\alpha^3 L^3}$$ Equation 7.10

Substituting these terms on the left hand side of Equation 3.32:

$$\frac{dD}{dx} = \frac{\alpha L}{L}\frac{dD'}{dx'} = \alpha \frac{dD'}{dx'}$$

On the right hand side, considering the numerator terms individually:

Term 1: $\dfrac{Cp}{R_H^{1/3}} = \alpha L\left(\dfrac{C'\,p'}{R'^{1/3}_H}\right)$

Term 2: $\dfrac{A^3 g S_o}{q^2 x^2} = \alpha L\left(\dfrac{A'^3\, S_o}{q'^2\; Dx'^2}\right)$

Term 3: $\dfrac{v_f\, A^2 S_o}{qx^2} = \alpha L\left(\dfrac{v'_f\;\, A'^2\; S_o{}^{3/2}}{q'\,x'^2}\right)\sqrt{\alpha}$

Term 4: $\dfrac{2A}{x} = \alpha L\left(\dfrac{2A'\alpha}{x'}\right)$

Considering the right hand side denominators terms:

Term 1: $\dfrac{dA}{dD} = \alpha L\dfrac{dA'}{dD'}$

Term 2: $\dfrac{A^2}{q^2 x^2}\dfrac{d\left(F_p / \rho\right)}{dD} = \alpha L\dfrac{A'^2}{q'^2\; x'^2}\dfrac{d\left(F'_p / \rho\right)}{dD'}$

Substituting these terms and cancelling the common αL factor from the numerator and denominator of the right hand side:

$$\alpha \frac{dD'}{dx'} = \frac{\dfrac{C'p'}{R'^{1/3}_H} - \dfrac{A'^3 S_o}{q'^2 x'^2} - \dfrac{v'_f A'^2 S_o^{3/2}}{q' x'^2}\sqrt{\alpha} + \dfrac{2A'\alpha}{x'}}{\dfrac{dA'}{dD'} - \dfrac{A'^2}{q' x'^2}\dfrac{d(F'_p/\rho)}{dD'}}$$ Equation 7.11

Alternatively, using a Froude number type non-dimensional term for the velocity of the water falling into the gutter: $v_f^* = \frac{v_f}{\sqrt{gD}} \Rightarrow \frac{v'_f}{v_f^*} = \frac{\sqrt{\alpha D'}}{\sqrt{S_o}}$

Substituting for v'_f in equation 7.11, gives:

$$\alpha \frac{dD'}{dx'} = \frac{\dfrac{C'p'}{R'^{1/3}_H} - \dfrac{A'^3 S_o}{q'^2 D' x'^2} - \dfrac{v_f^* A'^2 S_o}{q' x'^2}\alpha + \dfrac{2A'\alpha}{x'}}{\dfrac{dA'}{dD'} - \dfrac{A'^2}{q' D'}\dfrac{d(F'_p/\rho)}{dD'}}$$ Equation 7.12

7.2. Non-Dimensional Asymptotic Solution

In order to consider the parameter space defined by reasonable values for gutter dimension etc, the non-dimensional form of the model may be used.

In this section the approach and techniques used will be outlined. The detailed algebra will not be given.

It is possible to find an explicit solution for the depth in terms of *x*, if the assumption is made that

$$D' = D_0' + \alpha D_1' + O(\alpha^2)$$ Equation 7.13

And that considering up to first order terms will be sufficiently accurate.

Equation 7.12 gives $\alpha \frac{dD'}{dx'}$ as a function of a range of factors, including D’.

Substituting equation 7.13 into 7.12, the left-hand side becomes

$$\alpha \frac{dD'}{dx'} = \alpha\left(\frac{dD_0'}{dx'} + \frac{d\left(\alpha D_1'\right)}{dx'}\right) = \alpha \frac{dD_0'}{dx'} + \mathrm{O}(\alpha^2)$$ Equation 7.14

The terms in the right hand side can be expanded likewise, with terms such as $\left(D_0' + D_1'\right)^{-1/3}$ converted to $D_0'^{-1/3}\left(1+\alpha \frac{D_1'}{D_0'}\right)^{-1/3}$, and binomial expansions used [$\left(1+x\right)^n \approx 1 + \left(\frac{n}{1}\right)x + \left(\frac{n(n-1)}{2!}\right)x^2$, providing $x \ll 1$], for example:

$$\left(D_0' + D_1'\right)^{-1/3} = D_o'^{-1/3}\left(1+\alpha \frac{D_1'}{D_0'}\right) \approx D_0'^{-1/3}\left(1-\frac{1}{3}\alpha \frac{D_1'}{D_0'}\right)$$

Once all the terms in 7.12 have been expanded in this manner, orders of α can be equated. Taking $\mathrm{O}\left(\alpha^0\right)$ i.e. $\mathrm{O}(1)$, the left-hand side goes to zero. This allows D_0' to be found as a function of x'. It was found that the zero-order solution given is equivalent to using the Manning solution at a given point x and assuming that an equilibrium has developed at that point.[2]

Once this has been done, the next order terms may be equated ($\mathrm{O}(\alpha)$), giving $\frac{dD_0'}{dx'}$ as a function of D_0' and D_1'. The previous solution for D_0' in terms of x' may be substituted in (and differentiated for the term on the left-hand side), to give an expression for D_1' as a function of x'.

Given these terms, equation 7.13 can be reconstructed, to give D' as an explicit function of x', which can then be solved.

[2] It should be noted that this does not say that the Manning equation predicts the depth profile as a function of *x*: the derivation of the Manning formula relies on there being no change of depth with *x*, this assumption is contradicted by the derivation of a function giving depth as a function of *x*.

The correction factor given by the first order acts to increase the depth of the flow in the gutter. It is possible to expand the solution to higher orders, but this would require a considerable amount of working, and has not been conducted for this project.

There is some breakdown in the terms at the upper end of the gutter, showing a breakdown in the model. This is not particularly troubling in terms of applying the model, as the depth at the upper end of the gutter is not the most important point to be considered: one assumes that the depth downstream will be greater, and so determine the performance of the gutter.

Although the corrective term seems relatively small, this does not mean it is insignificant.

8. Siphonic Guttering Systems

The use of siphonic guttering systems, where the flow in the gutter is drawn into the downpipe, rather than falling under gravity, are of interest for rainwater harvesting, but their examination falls beyond the scope of this project. Some initial information on the theory involved in siphonic guttering is included in Appendix A.

9. Conclusions & Further Work

The experiments conducted, allied with the computational model developed, have validated the theory developed in this report.

The work has shown that the generation of the initial data point required by the numerical method employed is non-trivial. This is a significant problem for developing the model into a useful design tool. The only remaining ways of generating the data point are estimation from experience or further experimentation, leading to reference data to give upstream depths. Neither of these are ideal: the estimation brings into question the reliability of any data generated by the model, and the experimental work to characterise the upstream conditions would be time consuming, and could never be exhaustive.

Approaching the problem by formulating a non-dimensional version of the governing equation and solving this asymptotically seems to be a fruitful line to pursue, as it not only gives an explicit solution, but also allows the parameter space to be covered more efficiently.

Given the development of the asymptotic solution, further work should include systematic parametric investigation. This should cover the sensitivity of gutter capacity to the characteristic variables: gutter geometry, aspect ratio, material and surface roughness, variations in rainfall intensity. It would be useful to cover the possibility of varying the slope: quantifying the benefit, if any, of dividing the gutter into several straight sections of different slopes.

Further experimentation would be useful, not only with the rig at present, but with a rig more capable of simulating actual rainfall conditions. Improvements to the rig would include:

- Greater length over which the rainfall could be generated.
- Lower rainfall intensities possible (with uniform distribution of flow into gutter).
- Modification of baffles to give control over flow distribution: the choice should be available as to whether the rainfall simulated would be in a continuous sheet, or in discrete jets. (This would allow the simulation of corrugated roofs.)
- More efficient and accurate measuring devices: some form of measuring device which automatically time-averaged the depth at the point to be measured would be preferable to the current situation of the experimenter having to estimate the mean depth.

Further investigation and experimentation to quantify the drawdown effect of even having the discharge under gravity is also of interest.

Some work could be done to consider whether higher order asymptotic solutions to the non-dimensional equation give significant changes in depth profile.

Once sufficient accuracy has been achieved with the asymptotic solution, improving its presentation into a more user-friendly package would be beneficial. This would generalise the input data so that one package could cope with a variety of different cross-sections, and including automatic searches to reduce manual iteration.

The area of siphonic guttering is still of interest. However, the writer has some reservations as to the potential for applying such systems in developing countries, both from system complexity, increased material costs, and effectiveness of the system in reducing water depth in the gutter for a given flow. In a developing country context, the cost of unskilled labour is comparatively lower than in a developed country, so the increased amount of materials required to obtain an effective siphonic system may outweigh the other benefits obtained.

In summary, the project has developed the tools that may be effectively used to assist with understanding the behaviour of gutter flow, and in the design of rainwater harvesting systems. There is considerable scope for further investigative work, but the fundamental techniques seem unlikely to change.

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Appendix A: Siphonic Guttering Systems

A.1 Siphonic Systems Literature Review

Siphonic Systems have been in use since the mid 20th Century, particularly in Europe. The fundamental principle of their operation is that by developing full-bore flow in the downpipe, a pressure difference may be developed, which acts to drive the flow through the downpipe more rapidly than falling under gravity alone. [This is explained in more detail later in this Appendix]. At first consideration, this would suggest that the depth of water in the gutter be reduced, at least in the vicinity of the downpipe. Bramhall and Saul[1] claim the siphonic system has the advantage of quickly removing large quantities of rainwater cost effectively. A clear introduction to the workings of a siphonic system, including the priming procedure, can be found in some of the work by Arthur and Swaffield[2], including some numerical modelling of the system beyond the writer's comprehension.
They make the point that the majority of rainfall events a siphonic system would have to drain would be well below the design condition. Establishing the required full-bore flow depends on matching the drainage network to the expected storm hyetograph, but this is not the only factor to consider: some form of extra hydraulic resistance is required. The most common device used seems to be a horizontal section, as mentioned by Arthur and Swaffield[2]. They also detail a further difficulty, with vortex formation at the inlet injecting air into the system: this would cause the flow throughout the downpipe to be at atmospheric pressure, and thus no driving pressure difference would be developed. The system outlets are designed to inhibit vortex formation.

Bramhall and Saul[1] found that BS 0367 gives accurate depths for flows, and so may be used as a design tool for siphonic systems. A further point made by Bramhall and Saul[3] concerns the position of the outlets within the gutter, and suggests positioning them as close as possible to central within the gutter, as the flow into them is already limited by the presence of a finite gutter sole. From experimentation Arthur and Swaffield[2] found that a significant amount of air entered the system due to entrainment within the system inflow, via turbulent flow within the gutter. They also found that with increasing rainfall intensity, partial unsteady de-pressurisation of the system will occur, resulting in substantial amounts of air being drawn into the system. The cyclic nature of the unsteady flow regime leads to some noise generation and structural vibration, which could lead to physical failure of the system. Even when the inflow to the system is approaching the design condition, ambient flow conditions are far from steady.

Bramhall and Saul[4] have done some experimental work with a test rig, and mention some further work to be pursued, which could be of relevance to both siphonic and possibly general flow in gutters. Martyn Bramhall was contacted, and proved extremely helpful with queries. Unfortunately, the data had from the experimental work had not been processed by the time of writing.

There are several issues to consider in relation to siphonic systems and their suitability for developing countries.

- The issue of noise in the system, as detailed in [2], could lead to durability problems, particularly in situations where there are less stringent quality control systems than in industrialised countries.
- If, as stated in [1], the existing British Standard for predicting gutter performance is reasonably accurate for use with siphonic systems, then this suggests that there is not a significant reduction in depth obtained with the siphonic system. If this is correct, then there would appear to be little attraction in adopting such a system in a developing country context. This conclusion does seem counterintuitive: if it is possible to obtain

full-bore flow with the same bore of downpipe, then presumably the influence of the pressure differential added to that of gravity would give an increased velocity of the flow in the downpipe, and hence in the gutter. As the flow rate in the gutter remains constant, the cross-sectional area of the flow must decrease.

- The complexity of the system, as described, with baffles being required, may make it too complex for a developing country situation.

1. M.A.Bramhall & A.J.Saul. *Performance of Syphonic rainwater Outlets*. 1999 Proceedings of the Eighth International Conference on Urban Storm Drainage.

2. Arthur, S. and Swaffield, J.A. *Understanding Siphonic Rainwater Drainage Systems.* Drainage Research Group

3. M.A.Bramhall & A.J.Saul. September 21-23 1999. *The Hydraulic Performance of Syphonic Rainwater Outlets Relative to their location within a gutter*. CIBW62 Water Supply & Drainage for Buildings.

4. M.A.Bramhall & A.J.Saul. *Examination of the performance of syphonic rainwater outlets*. 1998 International Symposium on Water Supply and Drainage for Buildings.

A.2 Siphonic Systems: A Brief Introduction

In a conventional guttering system, the flow is water is produced by the action of gravity alone on the water. In a siphonic system, a pressure difference is generated and used to increase the rate of flow. As can be seen in the figure below, if full bore flow is developed in a gutter, the pressure at the top of the gutter must be less than that at the bottom, by $\rho.g.h$. The surface of the water in the gutter is still exposed to atmospheric pressure though, so there is a pressure differential driving the fluid in the same direction as the flow in the gutter.

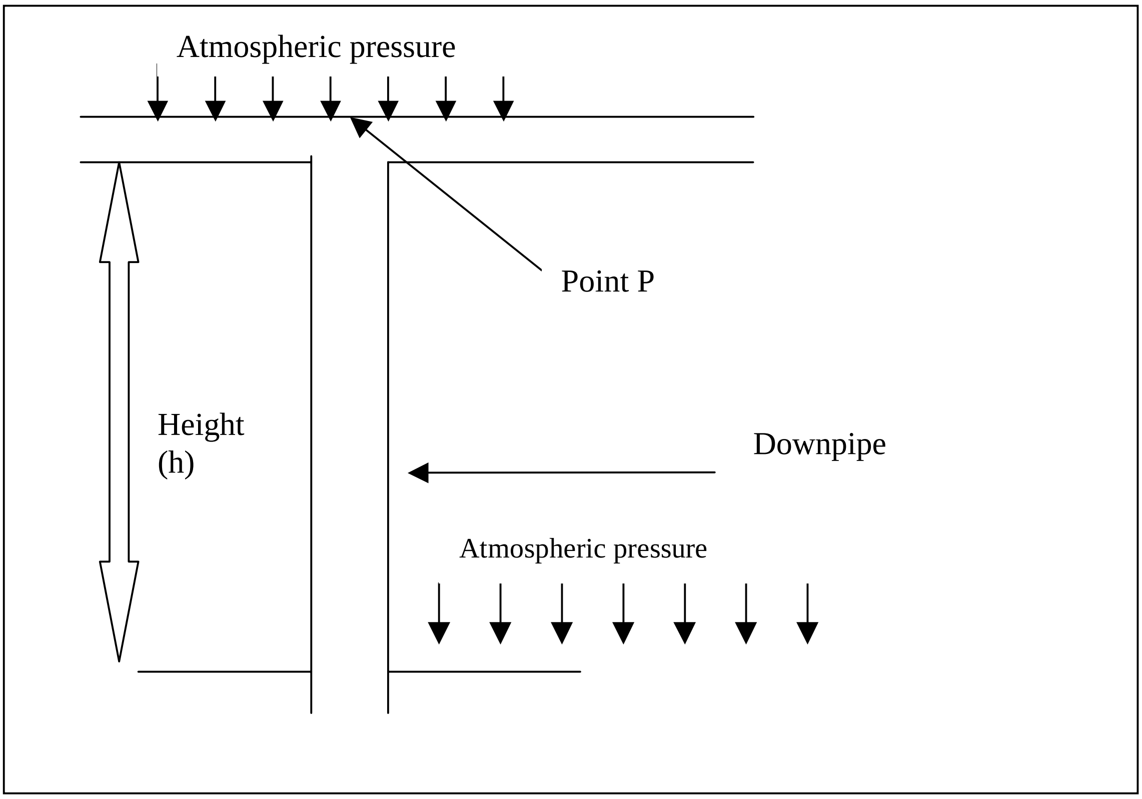


Figure A.1 Simplified view of downpipe to illustrate pressure differential.

There are problems associated with siphonic systems, which will be examined in the report. The main problem is that of creating a system that will be self-priming i.e. will automatically achieve full bore flow from an initial system in which there is no water in the system. A simple arrangement with a vertical downpipe does not have sufficient hydraulic resistance to allow this to occur, so some extra feature is required in the system. In most cases this feature is simply a length of horizontal pipe, so the guttering from the channel then becomes:

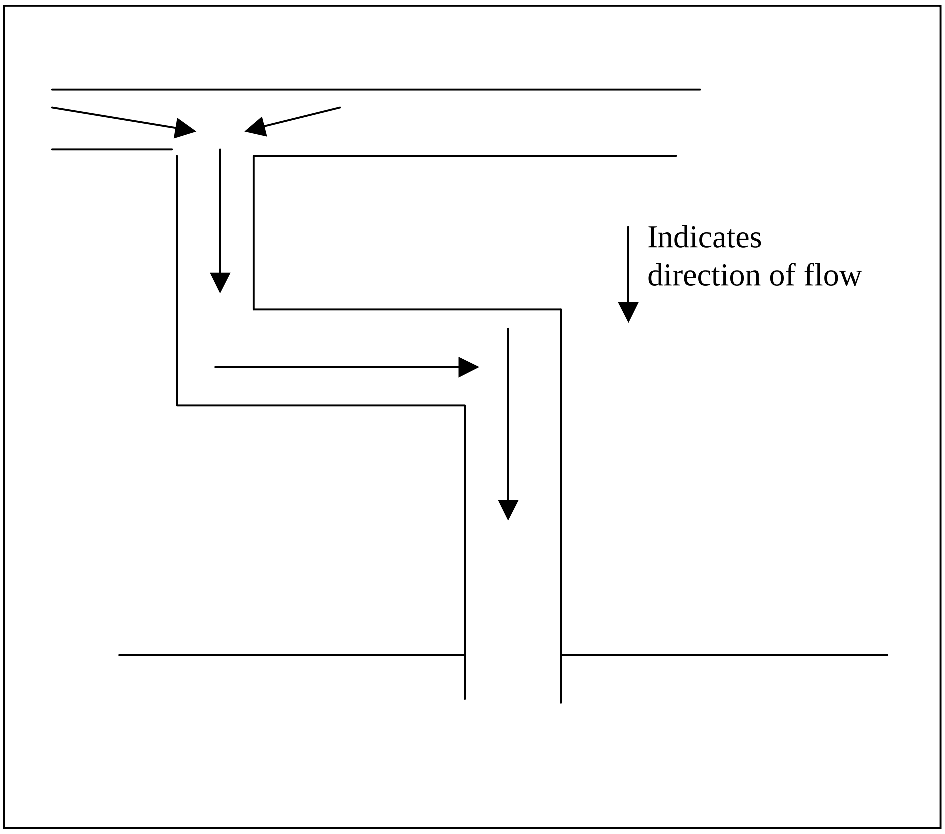


Figure A.2 Simplified pipe arrangement for siphonic system

Appendix B: Calibration Experiment

Test Rig Calibration

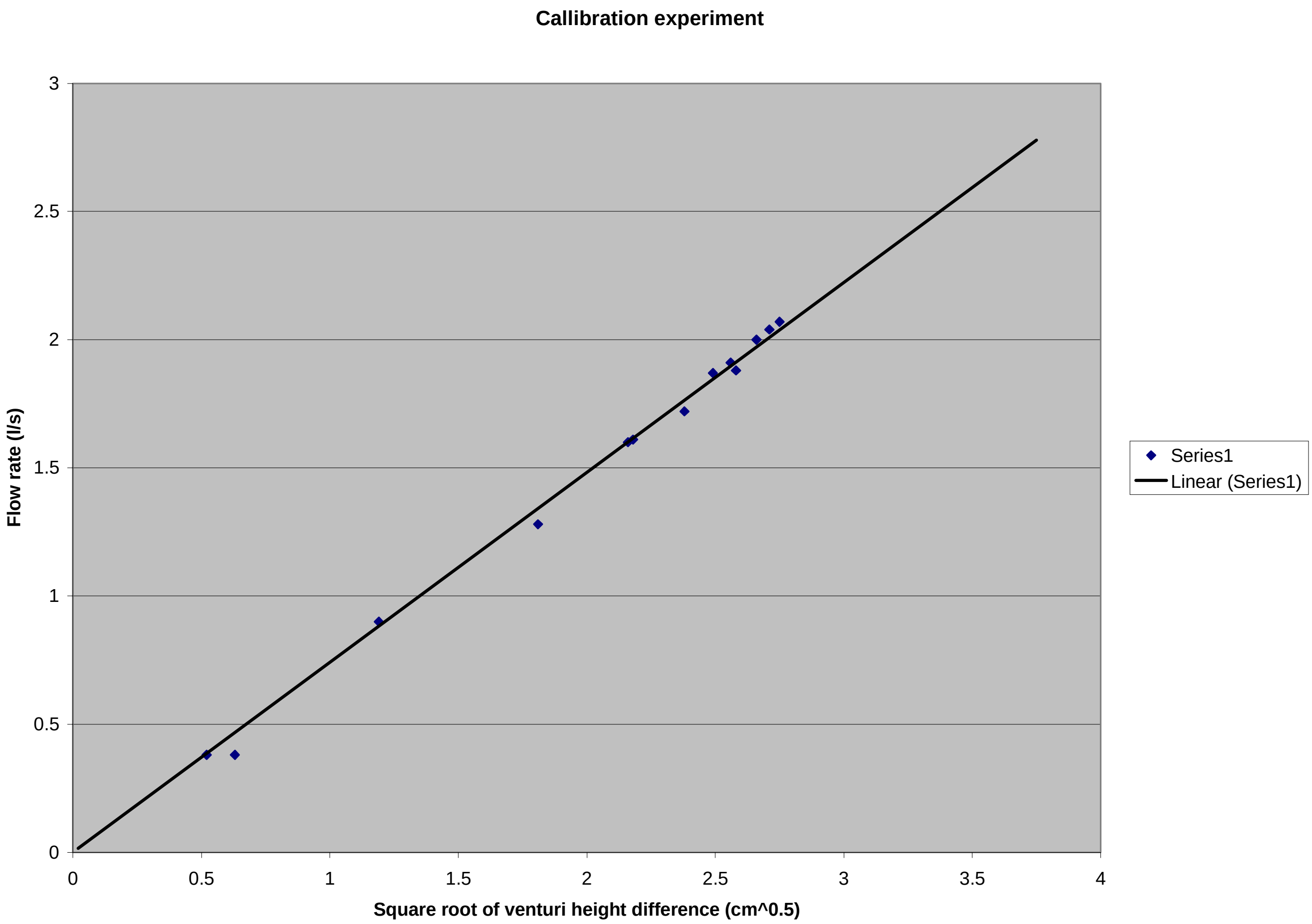
The test rig is fitted with a venturi meter on the inlet to the pump, and the number of valve turns can be measured. The calibration experiment was intended to allow a correlation to be established between flow intensity from the gutter, and either or both of the venturi reading, and the number of turns of the valve.

To allow this, the flow from the gutter was collected in a graduated container, and the time taken for it to fill the container to a certain level measured, for various recorded numbers of valve turns and venturi readings. The data from this experiment can be found at the end of this section.

As can be seen in the work by Garcia, Bernoulli's equation predicts that the relationship between height difference measure using the venturi meter, and flow through the meter, should be related by

$$Q = C.\sqrt{\Delta h} \qquad \text{Equation B.1}$$

Where Q is the flow rate, Δh the height difference measured across the venturi meter, and C the constant of proportionality. From the results below:



It can be seen that there is a linear relationship, and from the data below, that the constant is reasonably steady at 0.75, when Q is measured in l/s, and Δh in cm. This is likely to be more reliable than the number of valve turns, as problems were encountered with backlash in the valve. However, both readings were taken in the remaining experimentation.

Number of valve turns	Venturi mean height difference (cm)	Mean flow rate (l/s)	Squre root of height difference ($cm^{1/2}$)	Constant of proportionality
3	0.3	0.38	0.52	0.72
4	1.4	0.90	1.19	0.75
5	3.3	1.28	1.81	0.71
6	4.7	1.60	2.16	0.74
7	5.7	1.72	2.38	0.73
8	6.2	1.87	2.49	0.75
9	6.7	1.88	2.58	0.73
10	7.1	2.00	2.66	0.75
11*	7.4	2.04	2.71	0.75
12	7.6	2.07	2.75	0.75
9	6.6	1.91	2.56	0.74
6	4.8	1.61	2.18	0.74
3	0.4	0.38	0.63	0.59

Table B.1

Appendix C: Experimental Method

Initial approach

Given the large number of variables to be considered, it is not feasible to take a detailed profile along the gutter. The more practicable approach is to take readings at a few selected points, to establish whether there is any correlation between data recorded from the rig and that predicted by the model.

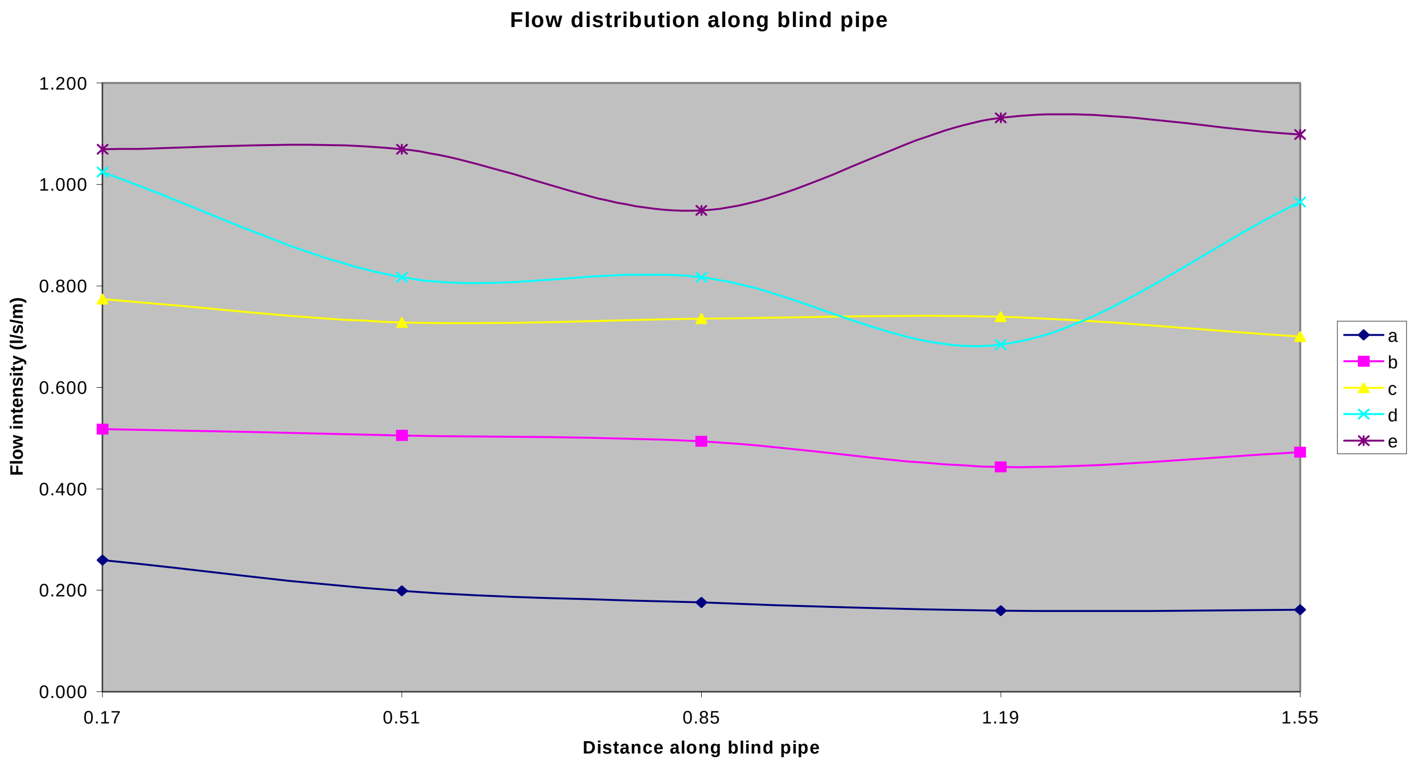
To make the readings of the depth possible, a simple device was manufactured, using a threaded rod and some Perspex. It was hoped that this would allow measurement of the depth without interfering with the flow in the gutter, as doing so would alter the variable to be measured.

Two sets of experiments were carried out, one using v-section gutter, manufactured from aluminium, and the other using plastic semicircular guttering. In both cases, the geometry of the rig limited the range of slopes available, as there is a limited height drop possible beneath the baffle supplying the water. Each slope was used for four experiments of varying flows, in the case of the v-section for 3 to 12 valve turns, and in the case of the semi-circular gutter, for an equally spaced number of valve turns from 3 to the number at which the gutter was observed to start overflowing.

Appendix D: Flow Distribution Experiments

Flow Distribution along the gutter

As noted earlier, the distribution along the gutter appeared to depart from the ideal used in the modelling. To quantify this departure, a further series of experiments were run, with varying flows, measuring the flow over five sections of the gutter.. This gave the flow distribution as shown below:



Series	Flow intensity (l/m/s)
A	0.21
B	1.18
C	2.00
D	2.79
E	5.64

As can be seen, there is some departure from the idealised situation used in the initial theoretical work.

The experimentation to establish this distribution was not very sophisticated, and there was insufficient time for repeat readings to be taken.

Initially the decision was taken to ignore the deviation from the ideal distribution as being negligible: if the theory and data agree fairly closely, then there is little to be gained from further experimentation.

Appendix E: Results- Triangular Cross-section

Appendix F: Results- Rectangular Cross-section

As can be seen, for two of the rectangular cross-section data sets there is a very good correlation between the predicted and measured data (with some deviation as the flow becomes critical and the model breaks down). The behaviour of the model after this point is not relevant: it is dependent on the magnitude of the iterative step size used in the numerical method, rather than behaviour of the model.

For the third set (slope of 1 degree, q of 0.0004) there is a considerable difference between the prediction and the measured results. This may be explained in terms of inaccuracy of measuring the slope: if it is changed by 0.5^o, then the other plot shown is obtained, showing a close correlation. Given the experimental rig, and the method for measuring the slope, an error of this magnitude is not impossible.