Form-finding design

Prof. Wanda J. Lewis
School of Engineering, University of Warwick, UK

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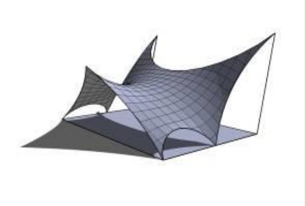
Overview

- **Introduction to form-finding**
- **Form-finding designs of fabric and shell structures**
- **Analytical form-finding: moment-less arch structures**
  - results and observations
- **Case study: Gateway Arch in St. Louis**
- **Conclusions**

What is form-finding ?

It encompasses methods of shaping structures by means of forces applied to them

- Many methods are possible, using both physical and computational modelling
- Form-found structures exhibit **maximum strength/mass ratio**, as observed in natural objects (plants, shells, trees).



Background: Tension fabric structures

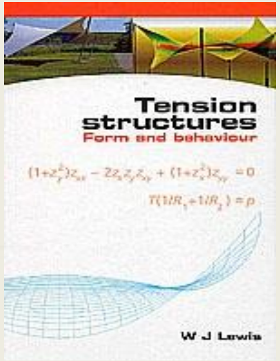
The shape of a fabric membrane under tension cannot be imposed; it has to be found

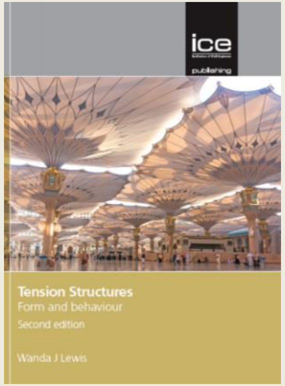
⇩

Form-finding

***‘Form follows force’* principle:**

Set of forces → Structural form → Structural response



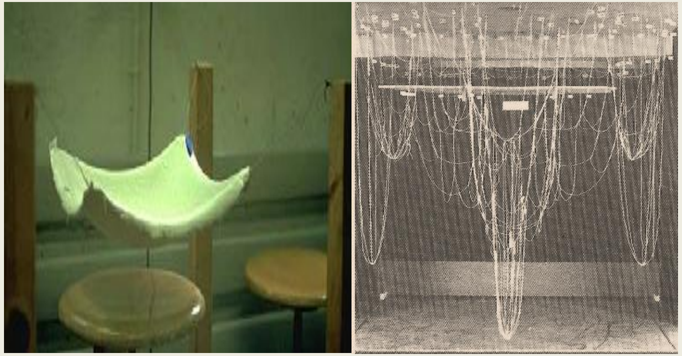


Experimental form-finding methods

Soap film models

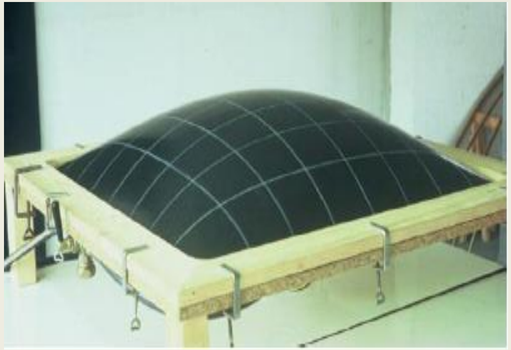
Fabric and 'free form' roof structures

Inverted hanging models
(made of fabric or chains)



Shell structures

Inflatable membrane models



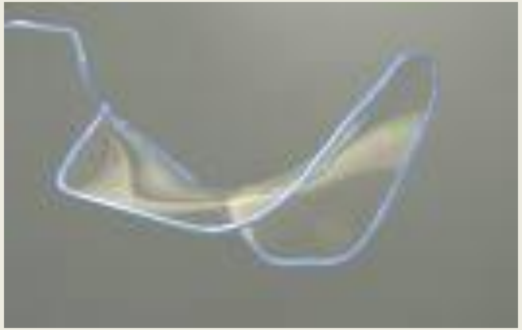
Pneumatic membrane roofs and domes

Features of soap film (minimal) surfaces

A soap film surface is a ***minimum energy form*** **or** ***'minimal surface'*****.** Such surfaces are ideal tension structures characterised by:

- **constant (uniform) surface tension**
- **minimum surface area**
- **zero mean curvature (saddle shape)**

It is advantageous to exploit the concept of a 'minimal surface' in form-finding design of tension fabric structures.



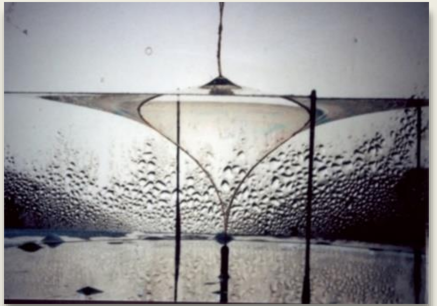
250 Convertible Umbrellas in Madinah
Saudi Arabia
Design: SL-Rasch, 2011

PROJECT DATA:

Umbrella span: 26m x 26 m

Total area: 150,000 m^2

Cost: $1.3 billion



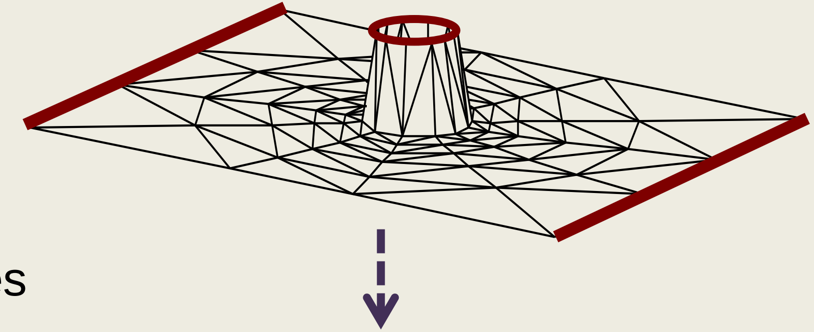
Soap film model
- used interactively with computational form-finding

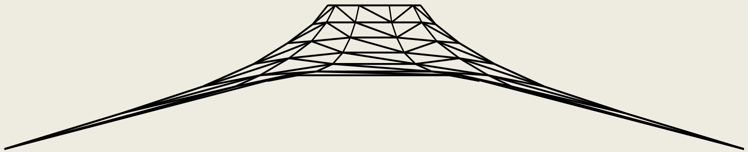
Minimal surfaces in computational form-finding of tension structures

(i) Initial 'guessed' surface is spanned between chosen boundaries and constant surface pre-stress is prescribed

(ii) The initial geometry and prescribed pre-stress give rise to the out-of-balance forces in the surface and lack of static equilibrium. This requires incremental adjustments of surface geometry to restore equilibrium

(iii) Iterative adjustments/calculations continue until static equilibrium is reached.





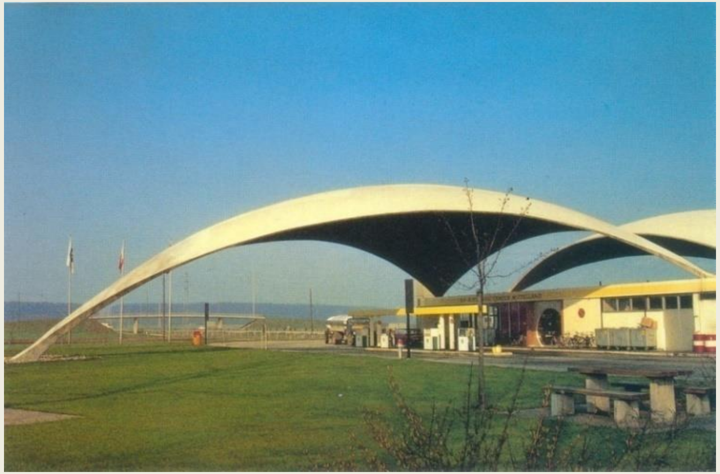
Form-found 'minimal' surface

Inverted hanging model in the design of shell structures

Heinz Isler (Switzerland)

Shell roof for **Bürgi Garden Centre**
Camorino, Switzerland,1973

Shell thickness: 90 mm
Span: 27m



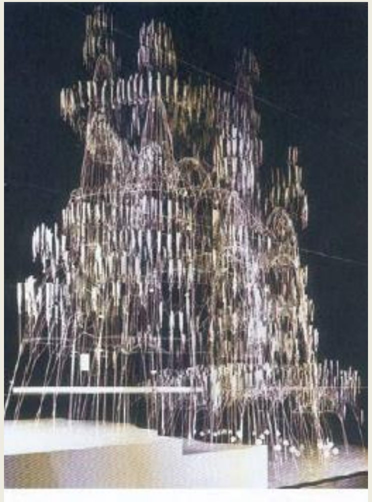
Petrol station in **Deitingen**
Switzerland,1968

Shell thickness: 80 mm
Span: 30 m)

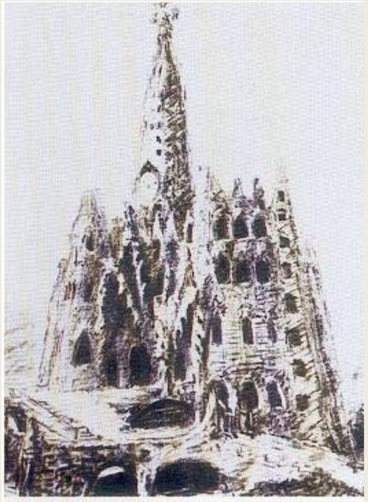
Inverted hanging model in the design of shell structures

Antonio Gaudi (Barcelona)

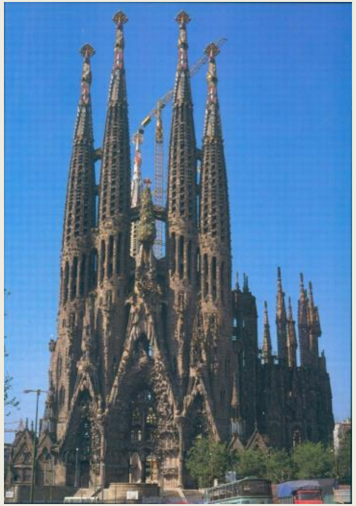
A series of chains loaded with small bags of sand gives a set of spires, or 'pointed' catenaries



The model (left) is photographed and inverted to get the final form

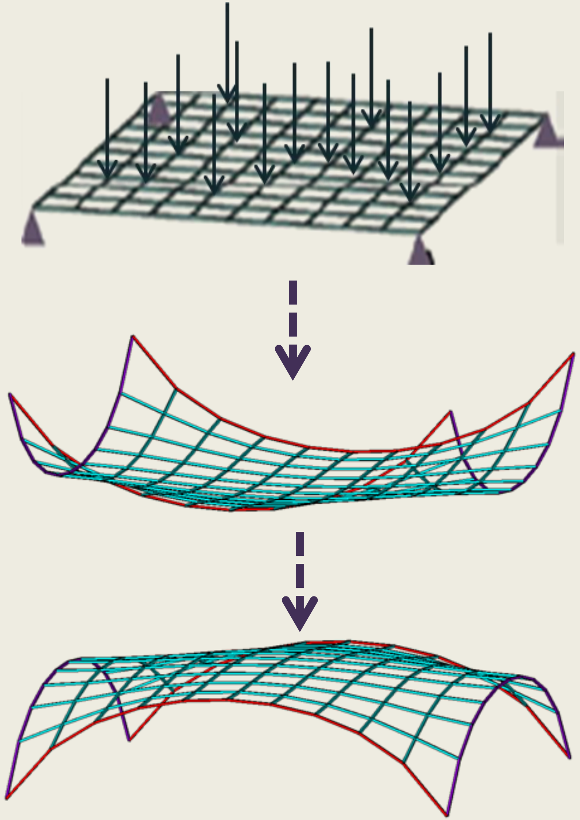


Güel Colony Church *- designed by this method, but project not completed*



Sagrada Familia, *Ongoing project since 1883*

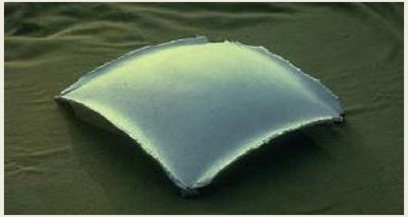
Inverted hanging model in computational form-finding of shell structures



The model is represented by line or surface elements. Gravitational loads are applied to each node of the surface

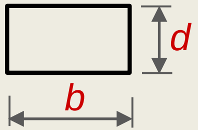
The surface is put into pure tension; its geometry is then adjusted iteratively during computations, until full equilibrium is reached

In the final, inverted form, tension is converted to pure compression - a desired action for a shell(or an arch structure).

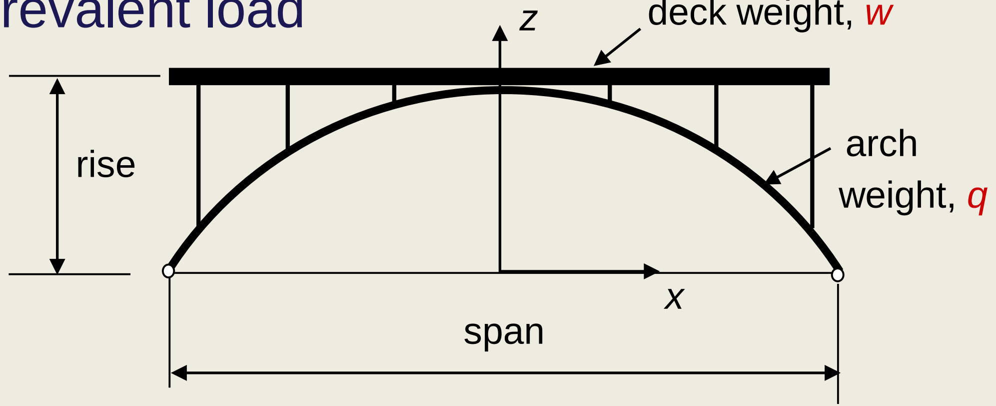


New approach: analytical form-finding

of moment-less arch forms



Moment-less arches develop pure compression, no bending moments under statistically prevalent load



Conventional design:
Shape of the arch is known (circular, parabolic, or catenary form)

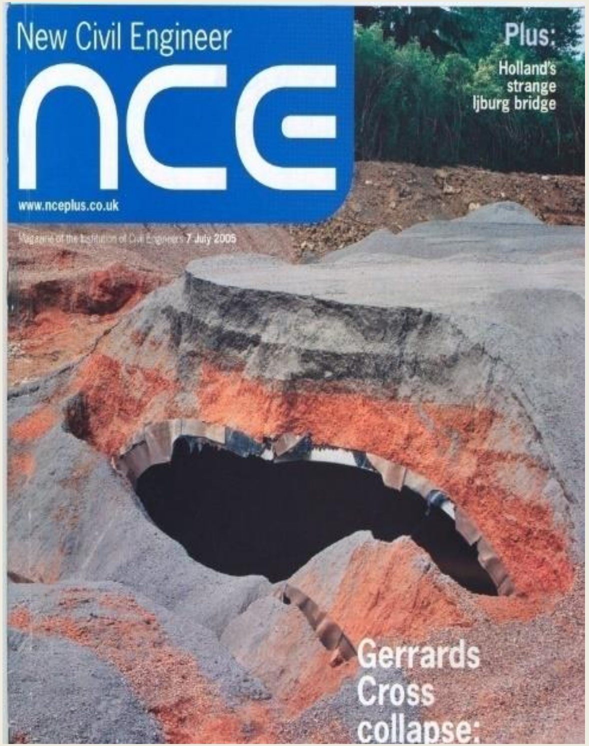
Form-finding design:
Shape of the arch <u>is not known</u>; x, z co-ordinates are derived as functions of span to rise ratio, and values of q and w.

Form-finding data:

Span = 50 m
w = 50 kN/m
q = 25 kN/m

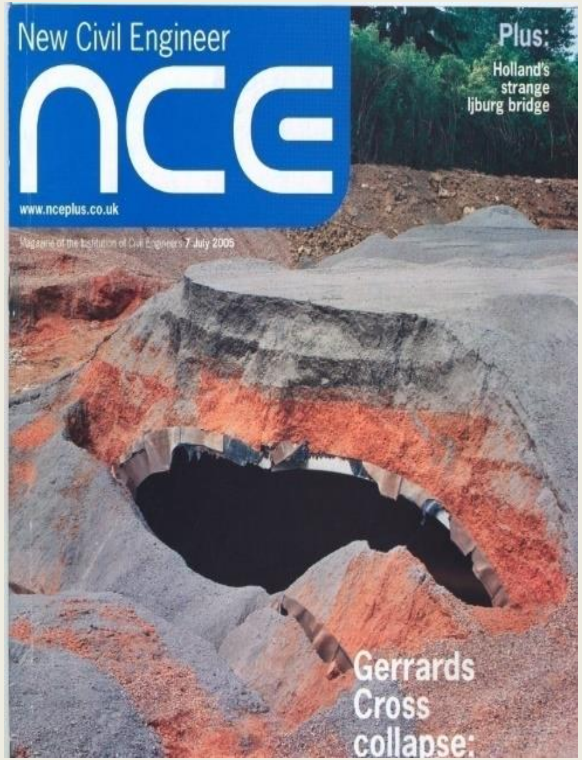
Cross-section:
b = 1.47 m
d = 0.68 m

Motivation: failure of Gerrards Cross tunnel, UK, 2005



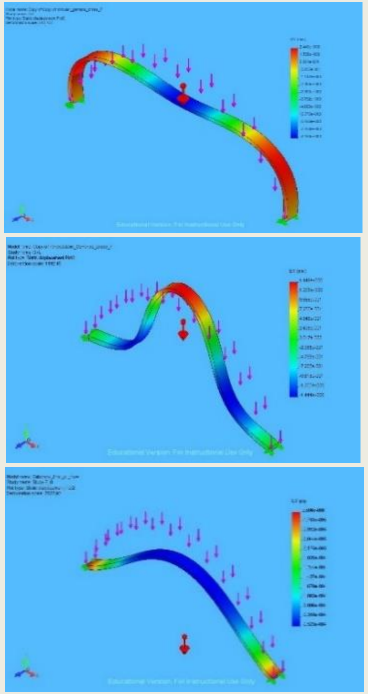
THE FACTS:

- 30 m section of a 320 m tunnel collapsed
- Structure analysed as an arch of 10 m span and 300 mm thickness
- Failure after heavy rain
- Backfilling deviated from the ‘assumed’ construction sequence.



Observed mode of failure corresponded to that of a circular arch

However, the solution did not involve re-shaping of the arch, but rebuilding it using a deeper cross-section.



Circular

Parabolic

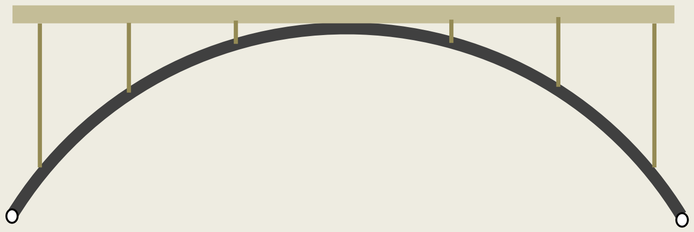
Catenary

Displacement modes under permanent loads

Moment-less arches of constant cross-section

Catenary (inverted chain curve)

- moment-less when the only, or dominant, load acting on the structure is the arch weight. Gateway Arch in St. Louis approaches a catenary form

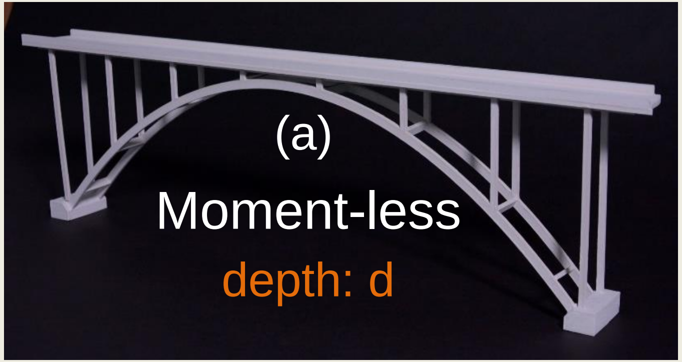


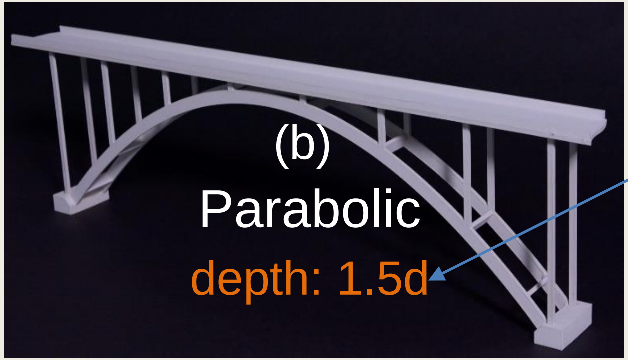
Parabolic- moment-less when the only, or dominant, load acting on the structure is the deck weight; this is the most common arch form in use

General case of a moment-less arch under both arch and deck weight has been recently solved using an analytical form-finding model[1].

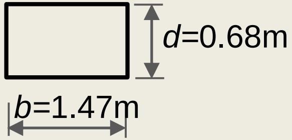
[1]*Lewis, W. J. 2016. Mathematical Model of a Moment-less Arch. Proc.Roy. Soc. A*

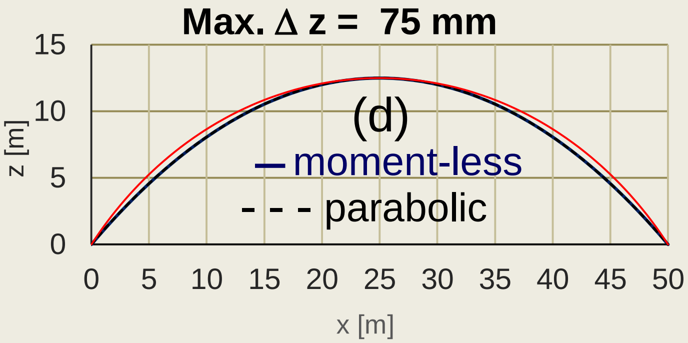
Span to rise 4:1

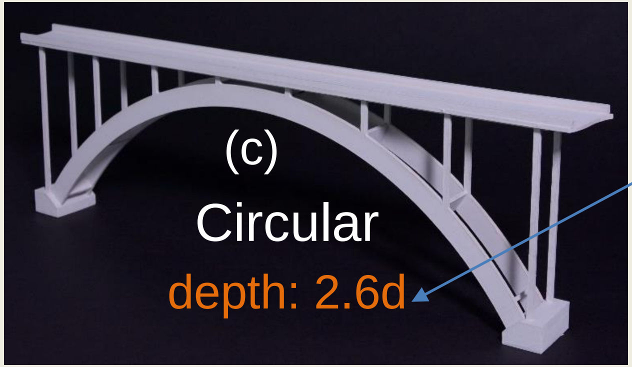




Req'd to match max. stresses in the moment-less arch



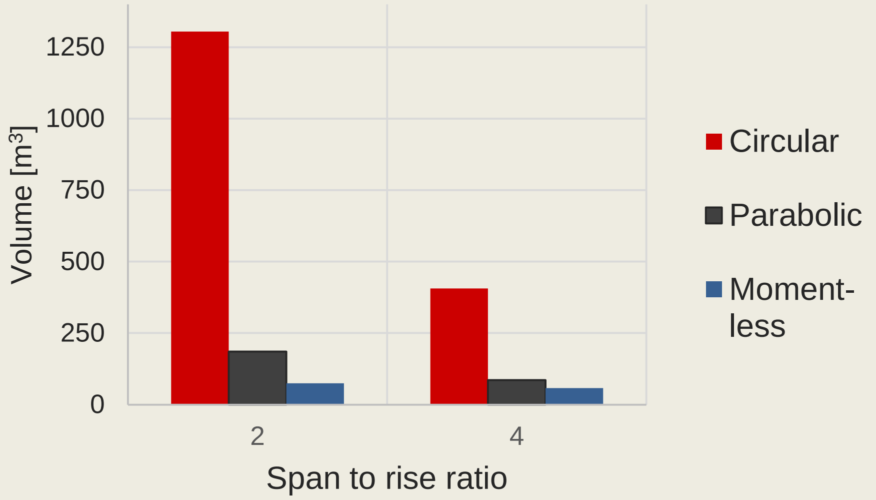


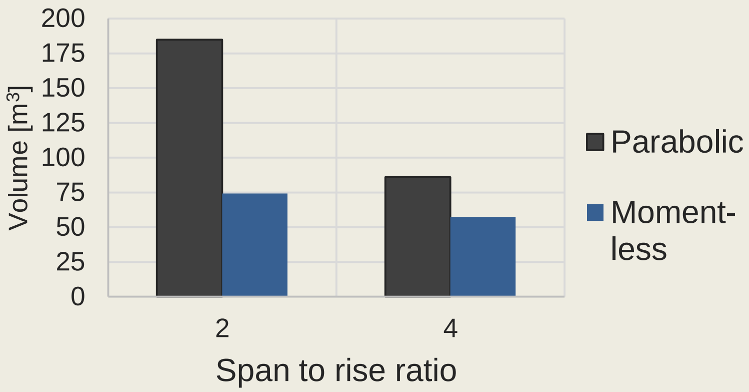


Req'd to match max. stresses in the parabolic arch

Results: (a), (b), (c) - arches carrying the same deck load
(d) - centre line profiles

Material volumes required to match max. compressive stress in the moment-less arch





	Circular	Parabolic	Moment-less
Span to rise	m^3	m^3	m^3
2 : 1	1,304	185	74
4 : 1	406	86	57

Results and observations

Amongst the dictated forms, a **circular arch** (although aesthetically pleasing), is the most inefficient structure

At span to rise of 4:1, there are very small differences in the overall geometry of parabolic and moment-less arches. Yet, the difference of just 75 mm requires 50% more material in the parabolic arch to match the max. compressive stress of the moment-less arch

Moment-less arch is the most efficient structure, over a range of span to rise ratios used in practice.

.Deformed masonry arch. Customs House, Exeter, 1681

Stone arch bridge at Dentdale

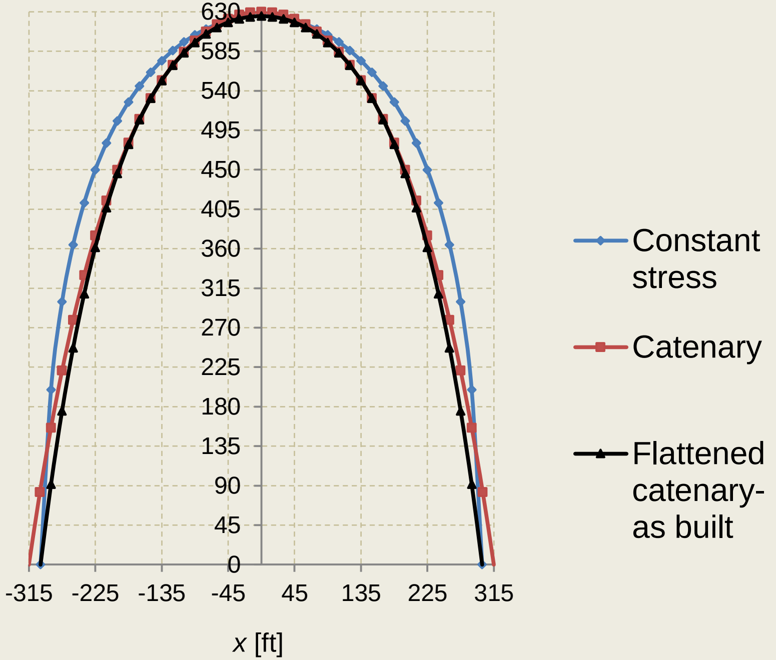
Gateway Arch in St. Louis

- a moment-less arch form?

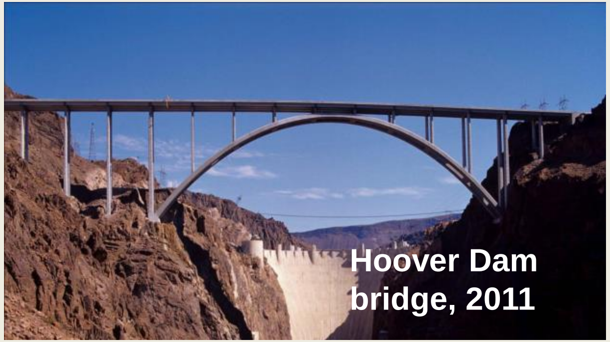
Originally designed as a catenary form, the arch is not moment-less under its own weight, because

- The cross- section area of the arch is not constant
- Having experimented with form-finding, the architect adjusted the intended arch shape ‘by eye’. Surprisingly, this gave it a shape mathematically defined [2] as ‘flattened catenary’.

[2] *R. Osserman, (2010), How the Arch got its Shape. Nexus Network Journal 12, 167-189*



‘Flattened catenary’ shape required foundations to be moved closer together. Constant stress moment-less arch (of varying cross section), shaped through analytical form-finding, lies above the two catenary forms (research in progress).


Hoover Dam bridge, 2011


Munich Olympic Roof, 1972


O_2 Arena, 2000

Thank you

Questions?


Zhaozhou bridge China, 605

O_2 Arena, 2000

I wish to thank the following contributors for the provision of their images: IL/ILEK, Heinz Isler, NCE, Bill Harvey, ICE Publishing, David Goodyear, and Warren Knoles.

The O_2 Arena, 2000

Architecture: Richard Rogers

Design: Buro Happold

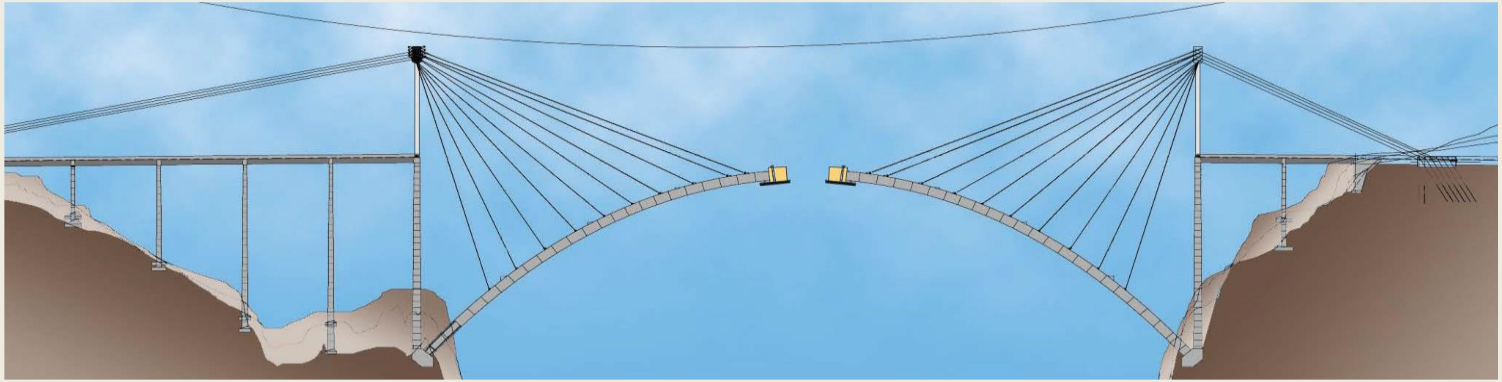
Structure: teflon-coated glass-fibre membrane, forced into a synclastic (dome) shape using pre-stressed and stay cables supported by masts. A total of 700 kms of cable used.

Hoover Dam bridge: a moment-less arch form

Design: TY Lin International, 2011

DESIGN DATA:

- **R.C segmental rib** arch, 268 m above the Colorado river
- **Span**: 323 m (span to depth ratio: 75)
- **Rise:** 84 m (initial span to rise ratio: 3.8; achieved ratio: 3.7)
- **Deck weight**: 295 kN/m
- **Arch weight**: 410 kN/m
- **Concrete strength**: 70 MPa
- **Material volume:** 6,300 m^3



Cable-stayed construction of 53 reinforced concrete segments. The technique allowed to control bending moments during the erection process.

Arch shape adjusted during construction to meet the requirement of zero bending moments under permanent load; involved increasing camber in the final stage

Final arch shape: a moment-less arch . Arch configuration deviating from a parabolic shape by less than 0.6 m (not noticeable by eye).

Munich Olympic Stadium Roof, 1972

Design: Frei Otto and Gunther Behnisch

A minimal ‘saddle-shape’ roof based on soap films and fabric models

Structure: acrylic cladding resting on pre-stressed cable supported by masts. A total of 200 kms of cable was used

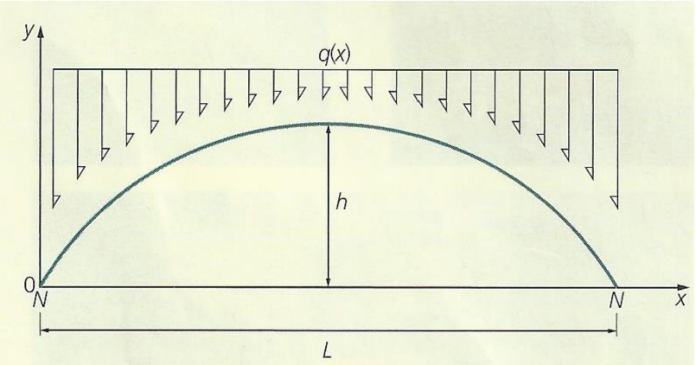
Due to physical modelling inaccuracies, the project marked a start of computational form-finding.

Zhaozhou bridge, China**,** 605

BASIC DATA[2]**:**

- **Span** (central arch): 37 m
- **Rise** (central arch): 7.23 m; span to rise ratio 5:1
- **Foundations** only 1.5 m deep

[2] *Man Zhou et.al. (2018). Spanning over 1400 years: China's remarkable ZhaoZhou bridge. ICE Proceedings, Civil Engineering 170, CE3, 113-119*



Latest 2D analysis indicates that the central arch shape was based on a thrust line to ensure the arch works in pure compression under permanent load

The arch curve was shaped by the varied load applied from above

This is perhaps the the oldest example of a form-found bridge.