Experiments in Fluids 14, 42–48 (1993)

Experiments in Fluids

© Springer-Verlag 1993

On the motion of particles in a fluid under the influence of a large velocity gradient

P. J. Thomas*, **K.-A. Bütefisch** and **K. H. Sauerland**

Deutsche Forschungsanstalt für Luft- und Raumfahrt, Institut für Experimentelle Strömungsmechanik, Bunsenstr. 10, W-3400 Göttingen, Germany

Abstract. The motion of seeding particles as used in laser Doppler anemometry is investigated in the presence of a large velocity gradient across aerodynamic shocks under different flow conditions. Experimentally obtained results are presented and compared with theoretical predictions based upon the size distribution of the seeding particles used. It is found that the agreement of experimental and theoretical results depends on the flow conditions as well as on the particle material.

1 Introduction

Any fluid flow velocity measurement using tracer techniques such as laser Doppler anemometry (LDA) relies on the information obtained from seeding particles suspended in and transported with the fluid. Hence one basic assumption of these techniques must be that the particles follow any changes of the flow velocity immediately, i.e. with only a negligible velocity lag. While this assumption can usually be regarded as being satisfied for small to moderate velocity changes, this is not necessarily the case for a large velocity gradient as it occurs for instance across aerodynamic shocks.

There are numerous publications on LDA dealing either with the theoretical investigation of the motion of single particles in some aspect [e.g. Maxwell and Seasholtz (1974), Walsh (1975), Nichols (1985), Tedeschi et al. (1990)], or the experimental measurement of the flow velocity across shocks [e.g. Krishnan et al. (1987), d'Humieres et al. (1990)]. Nevertheless none of the theoretical publications available takes into account that the seeding particles used always consist of a more or less broad size distribution and, furthermore, the work that has been done on direct comparison of experimental results with numerical predictions is very scarce. Only very recently have the first known comparisons of experiment and theoretical prediction been published by Bloomberg et al. (1989) and Meyers (1991). Bloomberg et al. present some results obtained for the motion of particles of various types and sizes across a shock in a flow with a Mach number of $M = 2.6$ upstream of the shock. They found that the motion of the larger of their latex particles was reasonably well described by theory, while the motion of smaller particles was less accurately predicted. Using theoretical results they tried to estimate the size of some fluid seeding particles. Meyer's results are for aluminium oxide particles in a $M = 6.0$ flow. He found a disagreement between experiment and theory which in his case was due to agglomeration.

The purpose of this paper is to present new experimental LDA data and a comparison with numerical predictions obtained for the motion of particles of a known size distribution in the presence of large velocity gradients across shocks for different flow conditions. With the interpretation of experimental LDA data in mind, this is done with the aim of determining how accurately the particle motion can be predicted for the type of flow considered here.

2 Experimental setup

The experiments described were carried out in the small supersonic wind tunnel (KÜG) of the DLR Göttingen. The measuring section of this wind tunnel has a cross-section of 10×10 cm^2. Using different nozzles it can be run continuously at various supersonic Mach numbers between $M = 1.25$ and $M = 3.0$. The air taken in from the atmosphere is dehumidified and the temperature in the settling chamber can be controlled by a heater. The flow can be observed through plexiglass windows which are the side walls of the tunnel in the region of the test section.

To generate oblique shocks of defined strength and position, wedges with wedge angles δ varying between $\delta = 6.3°$ and $\delta = 14°$ were used. The wedges were mounted in the middle between the top and the bottom wall of the wind tunnel test section. The flow velocity across each shock was measured from location E to location F at a height $h = 10$ mm above the model as indicated in Fig. 1. The height h was chosen to be large enough not to produce scattered

* Present address: Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge, CB3 9EW, UK

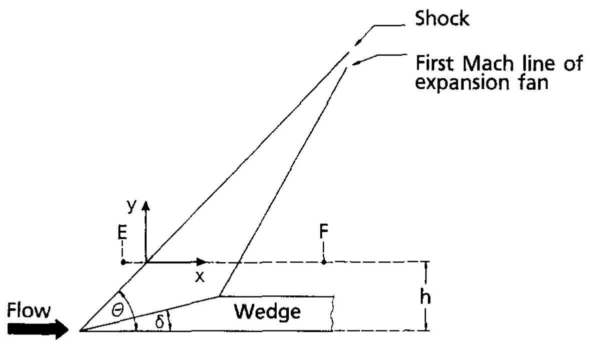

Fig. 1. Wedge with shock and measuring traverse $\overline{EF}$

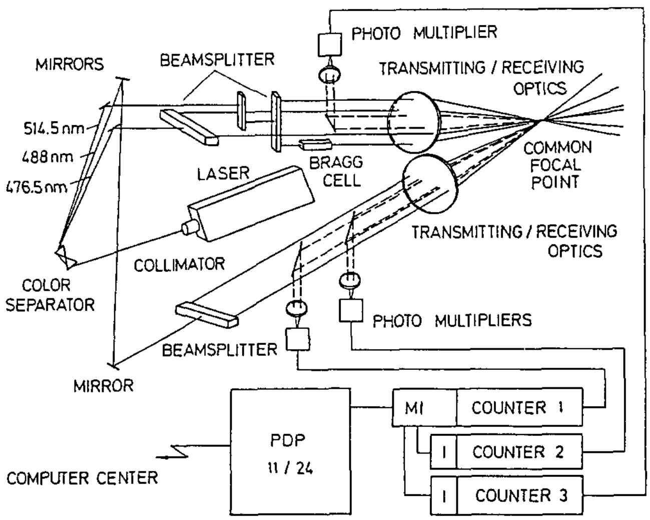

Fig. 2. Sketch of LDA and data acquisition system

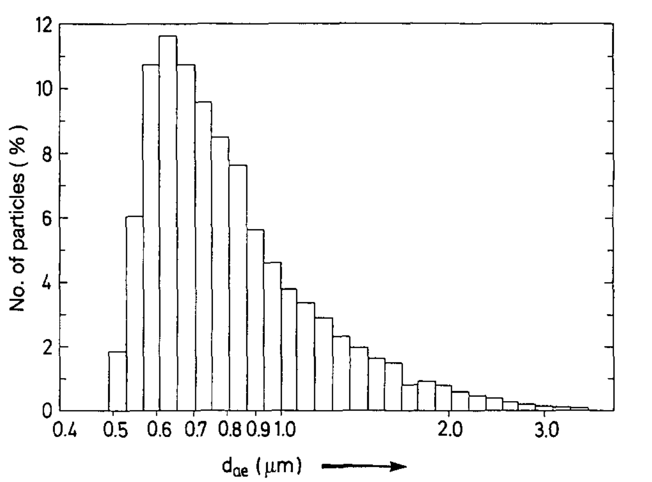

Fig. 3. Measured size distribution of the olive oil seeding particles

laser light from beams impinging on the wedge surface. On the other hand, it was small enough to yield a large distance along $\overline{EF}$ between the shock and the expansion fan to allow for the best possible relaxation of the particles to the flow velocity downstream of the shock. Due to blocking of the wind tunnel, the wedge thickness and thus its overall size was limited as no supersonic flow could be obtained for wedges thicker than about 6–7 mm.

The LDA used is a 3-component LDA working in backscatter off-axis mode. Its light source is a 4-Watt Argon-ion laser (Spectra Physics Mod. 165). The green ($\lambda = 514.5$ nm), blue ($\lambda = 488$ nm) and the violet ($\lambda = 476.5$ nm) lines of the emitted laser light are used for the three components of the LDA. The distance between the measuring volume and the receiving optics was 750 mm. If required, Bragg cells can be integrated into the optics to enable the LDA to distinguish between reversing flow directions.

The flow configuration considered is two-dimensional and thus only two components of the LDA are sufficient to determine the velocity components of the flow, namely the velocity component in the main flow direction and the component perpendicular to that, due to the deflection of the flow across the shock. Nevertheless, only the experimental data of Fig. 7 were collected with the LDA's position relative to the main flow being such that only two LDA components were used, i.e. the green and the blue LDA component measuring the two velocity components directly. Different positioning of the LDA relative to the flow was chosen for collecting the experimental data of Figs. 4–6. In this case all three LDA components were involved in collecting the experimental data to reduce problems connected with the diffraction of laser beams when they have to cross the shock front in order to reach the measuring volume (compare section 4.).

As only scattered light of the centre portion of the generated measuring volume is received by the receiving optics, the effective measuring volume of the LDA has a diameter of approximately 0.1 mm and is about 0.3 mm long. The data acquisition system consists of TSI signal processors (Model 1990B) and a PDP 11/24 computer system allowing for a quick scan of the results. Automatic positioning of the LDA system is also controlled by the PDP 11/24 system. A sketch of the LDA and the data acquisition system is shown in Fig. 2. A more detailed description is given in Bütefisch and Sauerland (1985) and Bütefisch (1989).

If not stated otherwise, olive oil seeding particles generated by a Laskin nozzle type particle generator were used for the experiments. The seeding particles were introduced into the flow in the settling chamber of the wind tunnel. The particle output of the generator was analyzed using a TSI APS 33B Aerodyamic Particle Sizer. Figure 3 shows the measured size distribution of the olive oil seeding particles, with a mean diameter of around 0.9 μm.

3 Particle motion

Consider a small (order of size: μm), non-deformable, spherical particle not disturbing the flow. Neglecting wall effects, the motion of this particle in a fluid can be described by the Basset-Bousinesq-Oseen (BBO) equation which is given in the notation of Soo (1967) by

$$\frac{4\pi}{3} r_p^3 \varrho_p \frac{dU_p}{dt_p} = \frac{4\pi}{3} r_p^3 \varrho_p G(U_F - U_p) - \frac{4\pi}{3} r_p^3 \frac{\partial P}{\partial r} + \frac{1}{2}\frac{4\pi}{3} r_p^3 \varrho_F \frac{d}{dt_p}(U_F - U_p) + 6 r_p^2 \sqrt{\pi \varrho_F \mu_F} \int_{t_{p0}}^{t_p} \frac{\frac{d}{d\tau}(U_F - U_p)}{\sqrt{t_p - \tau}} d\tau + F_a \quad (3.1)$$

with

$$G = \frac{3}{8} C_D \frac{\varrho_F}{\varrho_p} \frac{1}{r_p} |U_F - U_p|, \quad [\mathrm{s}^{-1}], \quad (3.2)$$

where U_p and U_F denote particle and fluid velocity, respectively, and ϱ_p, ϱ_F are the density of the particle material and the fluid. The radius of the particle is r_p, the viscosity of the fluid is denoted by μ_F and the empirical drag coefficient of the particle is given by C_D. The physical significance of the five terms on the right-hand side of Eq. (3.1) are as follows. The first term represents the force acting on the particle due to a stationary viscous flow for a particle motion with a relative velocity $|U_F - U_p|$. By defining a Reynolds number Re as

$$Re = \frac{2 r_p |U_p - U_F| \varrho_F}{\mu_F}$$

and using Stokes' expression for the drag coefficient $C_D = 24/Re$, this term can be seen to be the well-known Stokes law for the force acting on the particle. The second term on the right hand side of Eq. (3.1) is due to the pressure gradient in the surrounding fluid. The third term is referred to as added or apparent mass and represents the force required to accelerate the mass of the fluid surrounding the particle and moving with it, the increment being one half the mass of the fluid displaced ($1/2\, m_F$). The fourth term on the right hand side is the so-called Basset integral, or Basset history integral. This term accounts for the deviation of the flow from the steady state. The last term on the right hand side of Eq. (3.1) denotes external forces such as gravity.

The pressure term disappears in the region of constant flow velocity downstream of the shock. As discussed by Thomas (1991) the influence of the added mass term on the particle motion may be neglected for the experiments carried out. Hughes and Gilliand (1952) have shown that the force described by the Basset integral in the case of a flow accelerated at high rates can be many times larger than the viscous drag in the stationary case. The force described by the Basset integral and its influence on the particle motion across the shocks investigated is studied in detail by Thomas (1991). It is shown there that this force may be neglected in the cases examined here. Moreover the influence of external forces will be excluded.

The simplified equation for the particle motion across the shock, upon which the theoretical results of section 4. are based is thus:

$$\frac{dU_p}{dt_p} = G(U_F - U_p), \quad (3.3)$$

with G given by Eq. (3.2). In the general case, with the drag coefficient C_D appearing in Eq. (3.2) written as $C_D(Re, \ldots)$, the component form of Eq. (3.3) consists of two coupled ordinary differential equations for the two components of the flow velocity and has to be solved numerically. If Stokes' expression $C_D = 24/Re$ for the drag coefficient is used, the two equations for the velocity components uncouple and can be solved in closed form. The equation for the drag coefficient C_D used here for the calculations presented is due to Henderson (1976). This equation satisfies a large number of experimental and theoretical results, accounting for compressibility as well as rarefaction effects and appears to be among the most accurate of the C_D equations to date. As this equation constitutes a rather lengthy expression, it is not expressly stated here. Equation (3.3) is integrated numerically by the use of the Bulirsch-Stoer method using Turbo Pascal routines suggested by Press et al. (1986).

Our interest is focussed on the particle motion between the shock and the expansion fan. Nevertheless, for the numerical simulation of the particle motion the flow velocity inside the expansion fan can be modelled by a parabola, as is suggested by the experimental data, to obtain a realistic approximation of the flow in this reacceleration region.

Any particle detected upon crossing the measuring volume at a measuring location along the traverse $\overline{EF}$ (Fig. 1) has crossed the shock front at a position lying underneath the intersection of $\overline{EF}$ and the shock front. Thus the coordinate system of the particle motion and the measuring system of the LDA (shown in Fig. 1) are not identical. A transformation of the particle motion into the measuring system can be obtained by simple geometrical considerations [see Thomas (1991)].

The calculations presented below are based on the particle size distribution of Fig. 3. The particle motion was determined for i different size classes and transformed into the measuring system of the LDA. The results for both velocity components of a size class at any given location along the measuring traverse $\overline{EF}$ were weighted in accordance with the number of particles in that class, to obtain an average velocity curve and the respective RMS values for each of the two velocity components.

The RMS values are given in percent with respect to the free stream velocity upstream of the shock. As it is assumed that all particles travel at this speed upstream of the shock the RMS value vanishes there. Only its increase due to the size distribution is considered upon entering the region downstream of the shock. Thus the theoretical curves were matched to coincide with the experimental curves at the location of the shock at $x = 0.0$ mm.

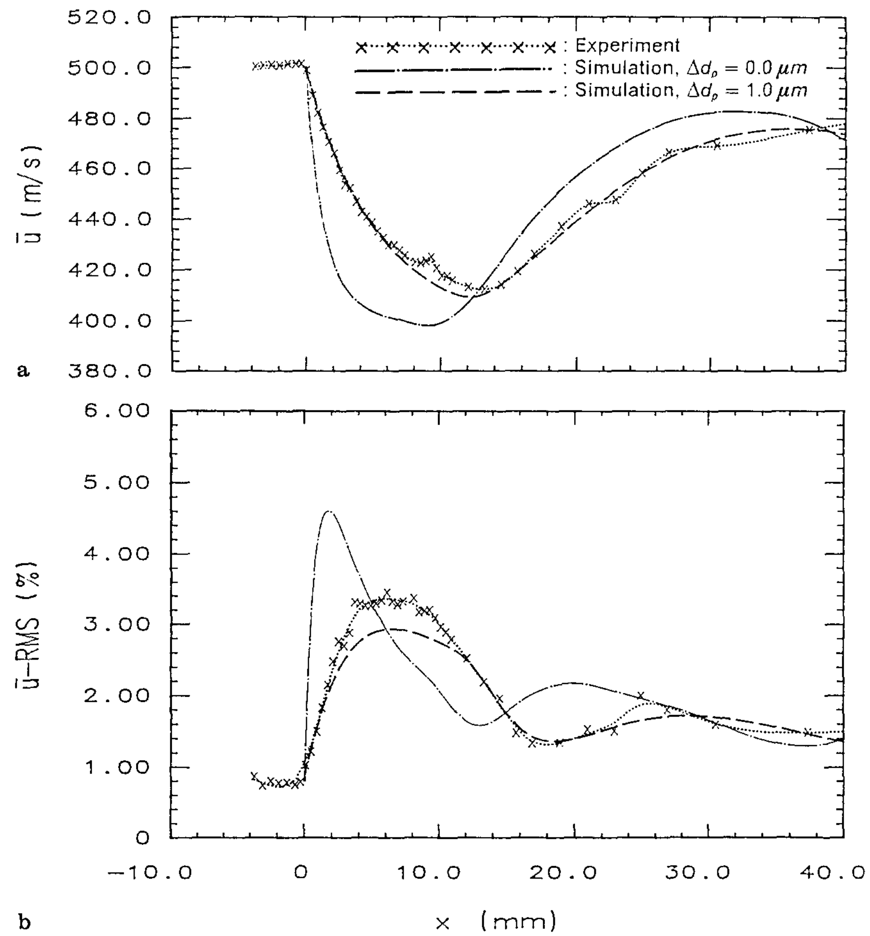

Fig. 4 a and b. Velocity and RMS values measured and theoretical predictions for olive oil seeding particles across a shock for an upstream Mach number of $M=1.93$ and a $\delta=14.0°$ wedge. ($\Pi=0.77$, $P_1=13{,}850$ Pa, $P_2=28{,}900$ Pa, total pressure/temperature in settling chamber: 97,400 Pa/291.4 K)

4 Experimental results and comparison with theoretical predictions

The results presented here refer to the component of the flow velocity in the main flow direction. The results for the second velocity component, perpendicular to the main flow direction, are documented by Thomas (1991) and reveal the same qualitative behaviour as will be found below.

Figure 4 shows the experimental data obtained for olive oil seeding particles across a shock generated at a $\delta=14°$ wedge with the Mach number upstream of the shock being $M=1.93$ compared with two numerical simulations for $\Delta d_p=0.0$ μm and $\Delta d_p=1.0$ μm. The value $\Delta d_p=0.0$ μm corresponds to the measured size distribution of the olive oil seeding particles shown in Fig. 3, and $\Delta d_p=1.0$ μm corresponds to the size distribution obtained if the diameter d_{p_i} of each size class i of Fig. 3 is replaced by $d_{p_i}+\Delta d_p$, i.e. if the whole size distribution is shifted towards larger sizes. The shock strength Π takes on the value

$$\Pi=\frac{P_2-P_1}{\varkappa\cdot P_1}=0.77,$$

where P_1 and P_2 are the static pressures upstream and downstream of the shock respectively, and $\varkappa$ is the ratio of the specific heats of air. It can be seen from Fig. 4a, b that the numerical simulation with $\Delta d_p=0.0$ μm does not describe the measured data well. The results seem to indicate that the seeding particles are actually larger than those measured by the size analyzer. Assuming the effective particle size distribution to be similar in shape to the one of Fig. 3 but centered around a larger mean value, the parameter Δd_p was varied in a first approximation, until a good agreement between experiment and theory was obtained. As can be seen from Fig. 4 this is the case for $\Delta d_p=1.0$ μm. To ensure that the behaviour of the particles appearing larger than expected is not due to the specific experimental setup used in Göttingen, experiments with closely reproduced flow conditions were carried out in a wind tunnel of the ONERA in Meudon, France. The LDA used in Meudon is a 3-component LDA working in forward-scatter mode. The particle generator and the olive oil from Göttingen were used for these experiments. The experimental data obtained in Meudon are almost identical to the results obtained before and show the same qualitative behaviour as found in Göttingen [see Thomas (1991)].

In order to understand the results found, further experiments were carried out in Göttingen for various flow conditions. Figure 5 shows the results obtained for the case of the upstream conditions ($M=1.89$) kept almost constant but with the wedge angle changed to $\delta=6.3°$, this giving a weaker shock strength of $\Pi=0.3$. It can be seen from Fig. 5 that

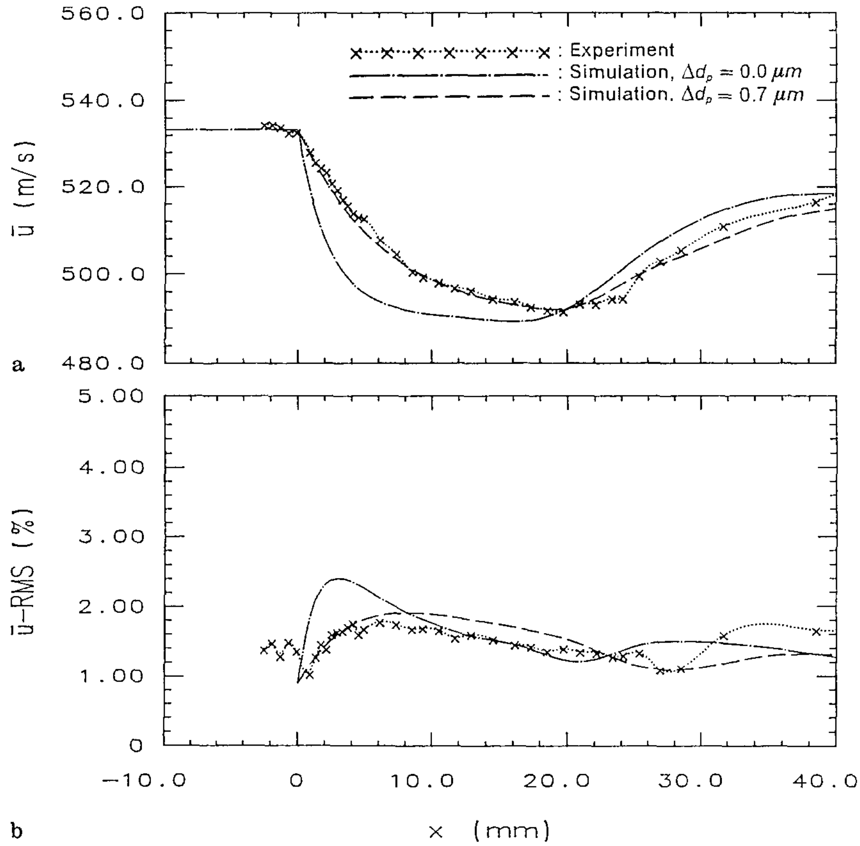

Fig. 5 a and b. Velocity and RMS values measured and theoretical predictions for olive oil seeding particles across a shock for an upstream Mach number of $M = 1.89$ and a $\delta = 6.3°$ wedge. ($\Pi = 0.3$, $P_1 = 14{,}700$ Pa, $P_2 = 20{,}750$ Pa, total pressure/temperature in settling chamber: 97,000 Pa/339.2 K)

the numerical simulation for $\Delta d_p = 0.0$ µm again does not describe the experimental data well. A good agreement is obtained in this case for a shift of $\Delta d_p = 0.7$ µm. With Δd_p now being smaller here than in Fig. 4, theory describes the experimental data better in the case of a weaker shock.

Figure 6 shows the result obtained for a $\delta = 14°$ wedge but with the upstream Mach number now being $M = 2.86$, giving a shock strength of $\Pi = 0.95$. As can be seen from Fig. 6 a, very good agreement of experiment and numerical simulation is obtained here for the original size distribution of Fig. 3 with ($\Delta d_p = 0$ µm) for the velocity curve across the shock. The measured data for the corresponding RMS values on the other hand do not follow the predicted curve (Fig. 6 b). The data obtained experimentally for the RMS values across the shock show an irregular behaviour in the region 2 mm $\lesssim x \lesssim$ 6 mm with their values first decreasing and then increasing again. This behaviour also coincides with a slight kink in the measured velocity curve of Fig. 6 a. However, no explanation can be given yet as to the reason for this irregularity.

Figure 7 finally shows the results obtained using commercially available solid "Blanc Fixe, Micro" particles for seeding. The average diameter of these particles as given by the manufacturer is $\bar{d}_p = 0.7$ µm and their density is $\varrho_p = 4.4 \cdot 10^3$ kg/m^3. The experimental velocity curve was measured using the $\delta = 14°$ wedge with the upstream Mach number being $M = 1.95$. The numerical simulation also shown in Fig. 7 was obtained for a single particle using the values for $\bar{d}_p$ and ϱ_p as given by the manufacturer. The influence of the expansion fan for $x \gtrsim 12$ mm was neglected for this calculation. Very good agreement of experiment and theory was found for these solid particles under flow conditions where fluid olive oil seeding particles only showed poor agreement (compare with Fig. 4). The strong velocity fluctuation in the region of the location of the shock seen for this particular experiment of Fig. 7 is spurious and is due to the positioning of the LDA with respect to the axis of wind tunnel, i.e. the main flow direction. It results out of a change of the angle between two laser beams due to defraction if one of them has to cross the shock front in order to get to the measuring volume for some measuring locations on the traverse $\overline{EF}$ (see Fig. 1) in the vicinity of the position of the shock.

5 Discussion

The results presented in the last section show a dependence of the quality of agreement between the experimental and the theoretical data on the flow conditions, as well as on the material of the seeding particles used. For the olive oil seeding particles an increasingly better agreement was found for decreasing shock strength. An increase in the agreement was also found for increasing shock strength under an increasing upstream Mach number and a simultaneously decreasing

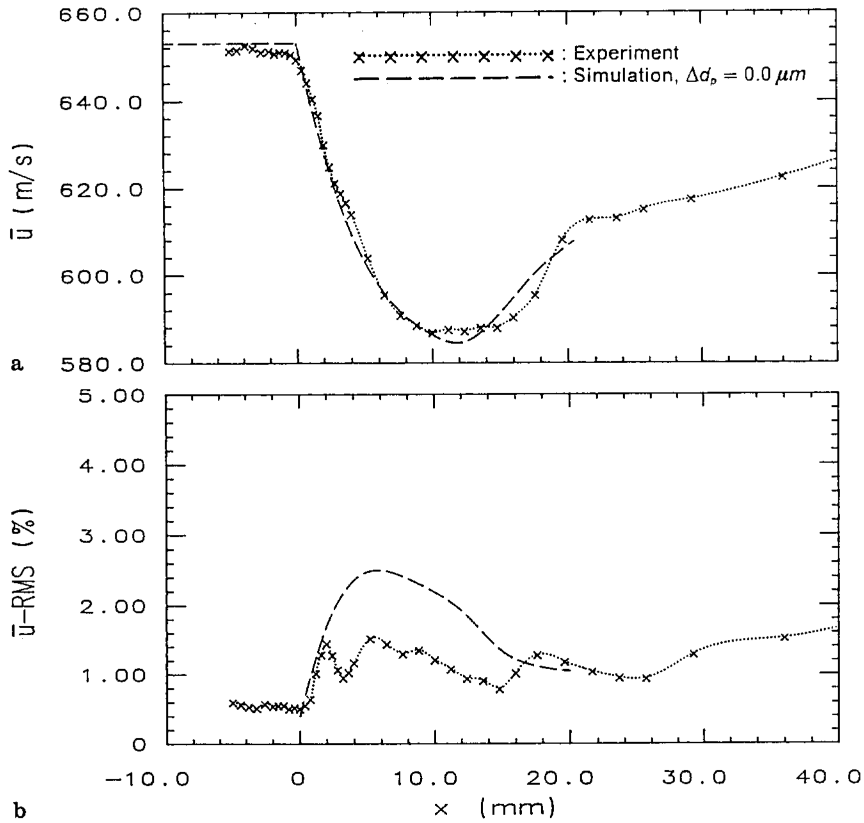

Fig. 6 a and b. Velocity and RMS values measured and theoretical predictions for olive oil seeding particles across a shock for an upstream Mach number of $M = 2.86$ and a $\delta = 14.0°$ wedge. ($\Pi = 0.95$, $P_1 = 3{,}400$ Pa. $P_2 = 7{,}950$ Pa, total pressure/temperature in settling chamber: 100,800 Pa/342.1K)

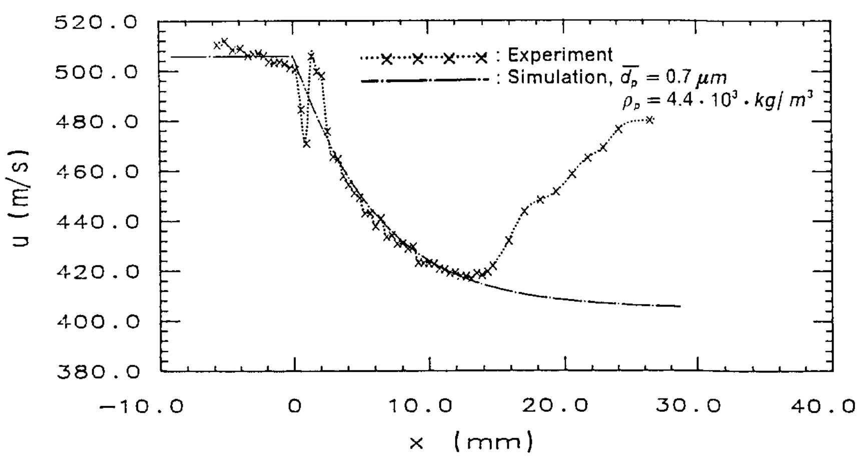

Fig. 7. Velocity measured and theoretical prediction for "Blanc Fixe, Micro" particles across a shock for an upstream Mach number of $M = 1.95$ and a $\delta = 14.0°$ wedge. ($\Pi = 0.76$, $P_1 = 13{,}500$ Pa, $P_2 = 27{,}900$ Pa, total pressure/temperature in settling chamber: 97,700 Pa/295.2 K)

downstream static density of the air. Furthermore, the experimental results suggest that if the olive oil particles can be assumed to be liquid under the upstream flow conditions, then the agreement between experiment and theory is better for solid particles than for liquid particles.

One should discuss what causes this varying agreement of the experimental results and the numerical simulation.

The theoretical investigation of the particle dynamics shows [Thomas (1991)] that the rather large disagreement found for the experiments shown in Fig. 4 and Fig. 5 can neither be attributed to the neglected terms of the equation of motion nor can it, for the flow conditions and particles considered, be explained solely by the specific C_D equation used.

During the experiments condensation was observed to occur in the test section of the wind tunnel. This condensation could not be suppressed completely by dehumidifying the air before it entered the wind tunnel. The condensed water droplets became visible in the test section by the scattered laser light when they crossed the laser beams inside the test section. Due to the results obtained, we had to suspect that the olive oil particles act as condensation nucleii and so grow on their way from the settling chamber to the test section. It has to be stressed here that all the LDA signals validated are solely due to the seeding of the flow. When seeding was stopped, with condensation still visible, no signal was obtained. Supposing that it is possible for olive oil particles to act as condensation nucleii, and supposing that they grow like water droplets, the particle growth can be estimated [Gyarmathy (1963)]. Although this estimate predicts a growth able to explain the behaviour found, it is most likely not the reason since no condensation was observed during the experiments in Meudon, the results being the same nonetheless. That the results are the same shows, moreover, that the deviations cannot be attributed to the specific configurations of the LDA used at the DLR and at the ONERA, i.e. on the LDA's ability of possibly "seeing" only particles of a certain size range. Taking the light scattering properties of the particles into account, as well as the fact that one of the LDA used works in the forward-scatter mode while the other works in the back-scatter mode, the results strongly indicate that almost all the seeding particles present in the flow field were "seen" by both of the LDA. Thus, for these particular experiments and seeding particles, none of the two LDA seems to have favoured only particles of a certain size range in the particle size distribution present.

When the oil particles enter the shock region they are decelerated with respect to the main flow direction. Since small particles are decelerated more rapidly than the large ones, this could cause the large particles to grow by "picking up" small particles through collisions with them. The observation of detected burst signals using oscilloscopes has shown that only very rarely does more than one particle cross the LDA's measuring volume at a time. This indicates a sufficiently low number density of the particles for "picking up" not to occur. Furthermore, good agreement between experiment and theory was obtained for the $M = 2.86$ case. Therefore particle growth due to this mechanism may be excluded.

Recalling the results summarized in the first paragraph of this section suggests that the deviations found for the experiments of Figs. 4 and 5 might be attributed to deformation of the liquid oil droplets. For a constant upstream Mach number and decreasing shock strength the droplet deformation and hence its deviation from the assumed spherical shape should decrease. Decreasing droplet deformation can be expected as well for increasing shock strength if the downstream density of the air decreases simultaneously, leading to smaller forces acting on the particle. In both cases better agreement between experiment and theory can be expected and is observed. The assumption that the deviations found result from particle deformation is also supported by the good agreement obtained for the solid "Blanc Fixe, Micro" particles in a case where experiments with olive oil have shown poor agreement.

Further experiments for comparison with theoretical predictions are needed to gain a deeper understanding of the particle dynamics under the influence of large velocity gradients. Such experiments should include experiments using a variety of different liquid and solid seeding particles of known size distributions, as well as attempts to measure particle velocity and particle size simultaneously. Simultaneous measurement of size and velocity could possibly be achieved with the inclusion of the phase Doppler technique, nevertheless this might pose major problems in the velocity range considered here.

Acknowledgements

The authors would like to thank Dr. O. Leuchter and his co-workers at the ONERA for performing the measurements at Meudon and for the good collaboration. The authors also wish to thank Dr. H. G. Horn at TSI, Aachen, for the particle size analysis.

References

Bloomberg, J. E.; Dutton, L. C.; Addy, A. L. 1989: An investigation of particle dynamics effects related to LDV measurements in compressible flows. Technical Report UILU ENG 89-4009 of Dept. of Mech. and Ind. Eng., Univ. of Illinois at Urbana-Champaign, USA

Bütefisch, K.-A. 1989: Three component laser Doppler anemometry in large wind tunnels. Prog. Aerospace Science 26, 79 113

Bütefisch, K.-A.; Sauerland, K. H. 1985: A three component dual beam laser Doppler anemometer to be operated in large wind tunnels. Proc. of the ICIASF '85, Stanford University, August 26 28

Gyarmathy, G. 1963: Zur Wachstumsgeschwindigkeit kleiner Flüssigkeitstropfen in einer übersättigten Atmosphäre. ZAMP 14, 281–293

Henderson, C.B. 1976: Drag coefficients of spheres in continuum and rarefied flows. AIAA J. 14, 707–708

Hughes, R. R.; Gilliand, E. R. 1952: The mechanics of drops. Chem. Eng. Prog., 48, 497–504

d'Humieres, C.; Micheli, F.; Papirnyk, O. 1990: Etude tu Comportement des Aerosols pour la Mesure en Velocimetrie Laser. Actes du 2ème Congres Francophone de Velocimetrie Laser, Meudon, Frankreich, 25.–27. 09. 1990

Krishnan, V.; Bütefisch, K.-A.; Sauerland, K. H. 1987: Velocity and turbulence measurements in the shock region using the two component laser Doppler anemometer. In: Stevenson, W. H. (ed.) Proc. ICALEO 1987, Vol. 63, Optical Methods in Flow & Particle Diagnostics

Maxwell, B. R.; Seasholtz, R. G. 1974: Velocity lag of solid particles in oscillating gases and in gases passing through normal shock waves, NASA TN D-7490

Meyers, J. F. 1991: Generation of particles and seeding. In: Karman Institute for Fluid Dynamics Lecture Series 1991-05, Laser Velocimetry, June 10–14, Rhode-Saint-Genese, Belgium

Nichols, R. H. 1985: Calculation of particle dynamics effects on laser velocimeter data. In: Wind Tunnel Seeding Systems for Laser Velocimeters. NASA Conf. Publ. 2393.

Press, H. W.; Flannery, B. P.; Teukolsky, S. A.; Vetterling, W. T. 1986: Numerical recipes, the art of scientific computing. Cambridge: Cambridge University Press

Soo, S. L. 1967: Fluid dynamics of multiphase systems. Waltham, Mass., USA: Blaisdell Publ. Co.

Tedeschi, G.; Gouin, H.; Elena, M. 1990: Etude theoretique du suivi de particules dans les ecoulements a forts gradients de vitesse. In: Actes du 2ème Congres Francophone de Velocimetrie Laser, Meudon, Frankreich, 25.–27. 09

Thomas, P. 1991: Experimentelle und theoretische Untersuchungen zum Folgeverhalten von Teilchen unter dem Einfluß großer Geschwindigkeitsgradienten in kompressibler Strömung. Doctoral Thesis Univ. Göttingen, published by Deutsche Forschungsanstalt für Luft- und Raumfahrt as Research Report DLR-FB 91-25

Walsh, M. J. 1975: Influence of drag coefficient equations on particle motion calculations. Proc. Symp. on Laser Anem., University of Minnesota, Bloomington, Minn., USA, 22.–24. 10

Received March 20, 1992