

Step-by-Step to Approximate the Second Derivative by 2nd-Order CD on Non-uniform grid

1. The domain size is $[-\pi, \pi]$
2. Number of grid point is 32
3. Apply the formula

$$ufscd(i) = \frac{((uf(i+1) - (2*uf(i)) + uf(i-1)) / (dx**2)) * (1 / (du(i)**2))) - ((ddu(i) / (du(i)**2)) * fscd(i))$$

Where $ufscd$ is the approximated second derivative
 uf is the function value from the Non-uniform grid
 dx is the uniform grid spacing
 du is the first derivative of the grid transformation function
 ddu is the second derivative of the grid transformation function
 $fscd$ is the first derivative of the function on the uniform grid

To find $fscd$, the following equation is applied,

$$fscd(i) = ((uf(i+1) - uf(i-1)) / (2*dx))$$

The first and the second derivative of the transformation function where the control parameter (γ) is 2 are following respectively,

$$du(i) = \pi * (2 - 2*\tanh(2*x(i))**2) / \tanh(2*\pi)$$

$$ddu(i) = -4*\pi*\tanh(2*x(i))*((2 - 2*\tanh(2*x(i))**2) / \tanh(2*\pi))$$

Where $x(i)$ is the grid point on the uniform distribution