Numerical Simulations of Unsteady Turbulent Flows: Temporal Acceleration

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Acknowledgement: T Talha, E Hurst, M Jones, Dr S Y Jung
1 Introduction
   - Governing equations
   - Acceleration parameter
   - Numerical methods

2 Results
   - Wall shear stress
   - Mean velocity profile
   - Fluctuations
   - Flow structures

3 Conclusions
The Navier-Stokes equations

\[
\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} u_i u_j = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j} + f \delta_{i1},
\]

(1)

\[
\frac{\partial u_i}{\partial x_i} = 0.
\]

(2)

where \( Re \) is the Reynolds number, \( Re = \frac{U_m h}{v} \).

Non-dimensionalisation

- \( h \): the half channel height,
- \( U_m \): the initial bulk mean velocity.
Temporal acceleration

Non-dimensional acceleration parameter, $f$

- $f$ is the non-dimensional acceleration parameter, $f = \frac{dU_m^*}{dt^*}$.

$$f = \frac{h}{U_m^2} \frac{dU_m}{dt},$$

$$\gamma = \frac{D}{u_{\tau 0}U_m} \frac{dU_m}{dt} = 2 \frac{Re_m}{Re_{\tau 0}} f.$$

The effect of $f$

<table>
<thead>
<tr>
<th>$f$</th>
<th>0.2</th>
<th>0.35</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNS/LES</td>
<td>LES</td>
<td>LES</td>
<td></td>
</tr>
</tbody>
</table>
Numerical methods

Finite volume method

- Implicit, fractional step method (FSM) with Crank-Nicolson method, Kim et al. (2001)
- Second-order accurate, finite volume scheme,
- Periodic boundary conditions in the streamwise and spanwise directions,
- Ensemble average with 15 realisations.

Validation: Steady DNS/LES comparison

<table>
<thead>
<tr>
<th>( Re )</th>
<th>2800</th>
<th>3500</th>
<th>7000</th>
<th>11000</th>
<th>12000</th>
<th>15000</th>
<th>17000</th>
<th>20000</th>
<th>22600</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Re_\tau )</td>
<td>180</td>
<td>210</td>
<td>395</td>
<td>590</td>
<td>640</td>
<td>780</td>
<td>860</td>
<td>1000</td>
<td>1110</td>
</tr>
<tr>
<td>DNS</td>
<td>DNS</td>
<td>DNS</td>
<td>DNS</td>
<td>DNS</td>
<td>LES</td>
<td>DNS</td>
<td>LES</td>
<td>LES</td>
<td>LES</td>
</tr>
</tbody>
</table>
Simulation parameters for LES

<table>
<thead>
<tr>
<th></th>
<th>initial</th>
<th>final (DNS/LES)</th>
<th>ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re$</td>
<td>3500</td>
<td>15000/22600</td>
<td>4.3/6.4</td>
</tr>
<tr>
<td>$Re_\tau$</td>
<td>210</td>
<td>800/1110</td>
<td>3.8/5.2</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td></td>
<td></td>
<td>14.5/27.4</td>
</tr>
</tbody>
</table>

![Graph showing $U_b$ vs $Re$](image-url)
Low speed streaks

Challenges in grid resolution

- $Re = 3500$ and $15000$
Simulations parameters

<table>
<thead>
<tr>
<th>DNS based on $Re_\tau = 800$</th>
<th>LES based on $Re_\tau = 1110$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_x \times L_y \times L_z$</td>
<td>$L_x^+ \times L_y^+ \times L_z^+$</td>
</tr>
<tr>
<td>$6h \times 2h \times 4h$</td>
<td>$6780 \times 2260 \times 4520$</td>
</tr>
<tr>
<td>$N_x \times N_y \times N_z$</td>
<td>$N_x \times N_y \times N_z$</td>
</tr>
<tr>
<td>$384 \times 384 \times 640$</td>
<td>$256 \times 192 \times 384$</td>
</tr>
<tr>
<td>$\Delta x^+$</td>
<td>$\Delta x^+$</td>
</tr>
<tr>
<td>12.5</td>
<td>26.2</td>
</tr>
<tr>
<td>$\Delta y_{min}$</td>
<td>$\Delta y_{min}$</td>
</tr>
<tr>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td>$\Delta y_{max}$</td>
<td>$\Delta y_{max}$</td>
</tr>
<tr>
<td>9.7</td>
<td>28.0</td>
</tr>
<tr>
<td>$\Delta z^+$</td>
<td>$\Delta z^+$</td>
</tr>
<tr>
<td>5.0</td>
<td>11.6</td>
</tr>
</tbody>
</table>

- Acceleration time: $T = 16.4$
- Acceleration time: $T = 27.3, 16.3$ and $10.9$. 
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3. Conclusions
Temporal acceleration

Mean velocity profile variation

- Three $f$ values ($f = 0.2, 0.35$ and $0.5$) with steady results.
Four distinctive stages

- **IT**: initial transient \((3500 < Re < 4200, \text{ or } 0 < tU_{m0}/h < 1)\)
- **WT**: weak transient \((4200 < Re < 12000, \text{ or } 1 < tU_{m0}/h < 12)\)
- **ST**: strong transient \((12000 < Re < 16000, \text{ or } 12 < tU_{m0}/h < 17)\)
- **PS**: pseudo-steady stage \((Re > 16000, \text{ or } tU_{m0}/h > 17)\)
Wall shear stress variation

- $Re_\tau$ (left) and the rate of change of $Re_\tau$ (right)
- Good agreement with Dean and Bradshaw (1976)
**Mean velocity profile variation with $f = 0.2$ and 0.5**

- **WT:** Uniform increase in velocity in the early stage of transient.
- **PS:** It reaches the pseudo-steady velocity at the end of acceleration.

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$f = 0.2$

$f = 0.5$
Mean velocity profile variation

- A downward shift at the IT stage due to a higher $u_\tau$ value.
- An upward shift at the WT stage due to delay in new turbulence generation.

\[ f = 0.2 \]

\[ f = 0.5 \]
Mean velocity

Velocity change in time

- Near-wall region: smaller acceleration due to the no-slip condition
- Centre region: almost constant acceleration.

\[
f = 0.2
\]

\[
f = 0.5
\]
**Velocity fluctuations**

- Near-wall turbulence ($y < 0.2$) responds first to acceleration.
- Turbulence intensities in the core region are smaller than steady values.

\[
\mathbf{u}_{rms}
\]

For $f = 0.2$ and $f = 0.5$.
$v_{rms}$

- Slower response than $u_{rms}$
- $v_{rms}$ in $y < 0.5$ region responded first to acceleration.

$f = 0.2$

$f = 0.5$
Velocity fluctuations

- A similar trend to the response of $v_{rms}$.
- Slower response than $u_{rms}$.

\[ f = 0.2 \]

\[ f = 0.5 \]
Velocity fluctuations

- Fluctuations are normalised by local $u_\tau$.
- Near-wall turbulence responds to the acceleration first.
- Turbulence is transported to the core region.
$u_{rms}$ velocity fluctuations with three $f$ values ($f = 0.2$, $0.35$ and $0.5$)
Maximum $u_{rms}$ location

- For steady, $y^+ = 15$.
- At $Re = 3500$, $y = 0.07$; and $Re = 22600$, $y = 0.015$. 

$u_{rms}$ velocity fluctuations

$u_{rms}$ velocity fluctuations

Maximum $u_{rms}$ location

- For steady, $y^+ = 15$.
- At $Re = 3500$, $y = 0.07$; and $Re = 22600$, $y = 0.015$. 

$u_{rms}$ velocity fluctuations
Maximum $u_{rms}$ location in wall units

- For steady, $y^+ = 15$. 

![Graph showing $y^+$ vs. $Re$ for different cases](image)
Maximum $u_{rms}$ and $v_{rms}$

- $u_{rms}(t) - u_{rms}(0)$ normalised by the steady values.
- An overshoot for $f = 0.5$ case.
Vorticity fluctuations, $\omega'_x$ with three $f$ values ($f = 0.2, 0.35$ and $0.5$)
Streamwise vorticity fluctuations, $\omega'_x$

**Wall $\omega'_x$ value**
- A delay followed by a sudden increase.
- A longer delay for a large $f$ value.
Streamwise vorticity fluctuations, $\omega'_x$

**Maximum $\omega'_x$ value**

- A similar trend to the wall $\omega'_x$ value.
- Weaker than the steady state value.
Streamwise vorticity fluctuations, $\omega'_x$

**Minimum $\omega'_x$ location**

- Two sudden decreases.
- Located further away from the wall than the steady state case.

---

**Graphs**

![Graph 1](image1)

![Graph 2](image2)
Wall-normal vorticity fluctuations, $\omega'_y$

Maximum $\omega'_y$ value

- A similar trend to the wall $\omega'_x$ value.
- Weaker than the steady state value.
Wall-normal vorticity fluctuations, $\omega'_y$

**Maximum $\omega'_y$ location**

- Similar to minimum $\omega'_y$ location: two sudden decreases.
- Located further away from the wall than the steady state case.

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**Diagram**

- **Re** vs $\omega'_y$ for different cases.
- **Time** vs $\omega'_y$ for different cases.

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Wall-normal vorticity fluctuations, $\omega'_y$

- Maximum $\omega'_x$ and $\omega'_y$ location in wall units
  - A similar trend to the wall $\omega'_x$ value.
  - Weaker than the steady state value.
Low speed streaks

$Re = 3500$

$Re = 8000$

$Re = 12000$

$Re = 15000$
$\lambda_2$ iso-surface contours

- $Re = 3500$
- $Re = 9000$
- $Re = 12000$
- $Re = 15000$
Domain size test

Destruction of turbulence at $Re = 9000$

- $6 \times 2 \times 4$
- $12 \times 2 \times 4$
Domain size test

Turbulence structures at $Re = 11000$

6 × 2 × 4

12 × 2 × 4
Active area

Percentage of active area during temporal acceleration

- Destruction of *old* turbulence.
- Generation of *new* turbulence.

![Graph showing the percentage of active area during temporal acceleration.](image)
Conditional average at $Re = 12000$

- Area with/without active flow structures.
- Location for the maximum $u_{rms}$.
Conditional average at $Re = 12000$

Destruction of turbulence structure

- Large changes.
- New and old turbulence.

$v_{rms}$

$w_{rms}$
Destruction of turbulence structure

- New turbulence is much weaker at $Re = 11000$ than the steady flow.
- Generation of new turbulence is almost completed at $Re = 15000$.

\[ Re = 11000 \]

\[ Re = 15000 \]
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Conclusions

Turbulent channel flow with temporal acceleration

- Turbulence responds rapidly to the temporal acceleration after a certain delay until pseudo-steady equilibrium is achieved.
- $u_{rms}$ velocity component responds to the acceleration first, followed by $v_{rms}$ and $w_{rms}$ velocity components.
- Three delays: production, redistribution, and transport.
- Destruction of the initial turbulence and generation of new turbulence.
- Effect of the acceleration parameter.
- Effect of the initial flow condition.
Thank You!
Wall shear stress

Wall shear stress variation
- $Re_\tau$ (left) and the rate of change of $Re_\tau$ (right)
- Good agreement with Dean and Bradshaw (1976)