

Mathematics Self-Assessment Test

1. Find the magnitude of the vector $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.
2. Find the angle between the vectors $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$.
3. Find a vector which is mutually perpendicular to $\mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{d} = -4\mathbf{i} + 3\mathbf{j} + 1\mathbf{k}$.
4. Find the inverse of $A = \begin{pmatrix} 2 & 3 \\ -1 & -1 \end{pmatrix}$.
5. Find the eigenvectors and eigenvalues of $A = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$.
6. Solve $xy' = 4y$.
7. Find a particular solution of $y'' - 2y' + y = 10e^{-2x} \cos x$.
8. Solve Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, 0 < y < b$$

to find the steady-state temperature $u(x, y)$ of a 2D plate subject to the boundary conditions

$$\left. \frac{\partial u}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial u}{\partial x} \right|_{x=a} = 0, \quad 0 < y < b$$

and

$$u(x, 0) = 0, \quad u(x, b) = f(x), \quad 0 < x < a.$$

Be sure to obtain expressions for any coefficients used in superposition.

9. Show that x^2 and x^3 are orthogonal over $[-1, 1]$ with respect to the inner product

$$\langle f | g \rangle = \int_{-1}^1 f(x)g(x) dx$$

10. What is the probability that a single card chosen from a deck is not an ace?
11. A scientist tracked how many cups of coffee she drank every day at work over the course of a year. She used the data to build a probability distribution where the random variable C represents the number of cups of coffee she drank on a given day. Here is the distribution:

$C = \text{num. cups}$	0	1	2	3
$\mathbb{P}(C)$	0.05	0.10	0.75	0.10

Calculate the mean and variance of C .