CH159 – Part 2 Test
Thursday, February 6, 2014

<u>The time allowed for the test is 40 minutes.</u>

Attempt every question giving your answers clearly in the space provided in **black** or **blue** ink.

Graphical calculators are not permitted.

The formula for solving a quadratic equation is: $\left(x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \right)$

Standard Derivatives

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}\sin(ax) = a\cos(ax)$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}\cos(ax) = -a\sin(ax)$$

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} \qquad \frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$$

$$\frac{d}{dx}\ln x = \frac{1}{x} \qquad \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

Standard Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \quad (n \neq -1) \qquad \int x^{-1} dx = \ln x + c$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c \qquad \int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c \qquad \int \ln x\, dx = x\ln x - x + c$$

Taylor Series

$$f(x) = f(a) + (x-a)f'(a) + \frac{1}{2!}(x-a)^2 f''(a) + \frac{1}{3!}(x-a)^3 f'''(a) + \ldots$$

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1. All parts of this question refer to the complex numbers
$c = 2 + 3i$; $d = 1 - i$; $k = -2i$; $m = 4$

(a) plot c, d, k and m on an Argand diagram [3]

(b) evaluate $c - 3d$ [1]

(c) evaluate $k \times c$ [2]

(d) evaluate $d \times c$ [2]

(e) evaluate $c \times c^*$ [2]

2. Evaluate the following, given the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 0 \\ -3 & 1 & 1 \\ 2 & 4 & -1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -3 & 0 \\ 2 & 1 & -1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 5 & -1 \\ 1 & 0 \\ 0 & 3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 2 & 4 \\ -1 & 1 \end{pmatrix}$$

(a) $\mathbf{A} + \mathbf{B}$ [2]

(b) $\mathbf{AC}$ [3]

(c) $|\mathbf{D}|$ [2]

(d) $\mathbf{B}^{\mathrm{T}}$ [1]

3. All parts of this question relate to the following simultaneous equations: $\begin{matrix} 2x + y = 2 \\ x - 2y = 6 \end{matrix}$.

(a) write these simultaneous equations in matrix form [2]

(b) Calculate the inverse of the matrix of coefficients [3]

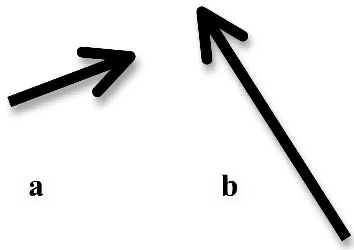
(c) Use the inverse matrix to solve the simultaneous equations [3]

4. Given the matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$:

(a) calculate the eigenvalues of **M** [4]

(b) hence find the two eigenvectors of **M** [4]

5. (a) Given the vectors **a** and **b**, below, draw the vector **a** – **b**



[2]

(b) Given the vectors $\mathbf{v} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{u} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, evaluate the following:

(i) $\mathbf{v} - \mathbf{u}$ [2]

(ii) $2\mathbf{u}$ [2]

(iii) $\mathbf{u} \cdot \mathbf{v}$ [2]

(iv) $\mathbf{u} \times \mathbf{v}$ [3]

6. (a) Write out the Taylor series expansion for $U(x) = \frac{1}{x^{12}} - \frac{2}{x^{6}}$ about the point x=1, giving the first four terms. [5]

(b) Hence evaluate U(1.01) to four decimal places **without using your calculator**. [3]