

Proposal for a Regensburg Warwick Graduate Exchange

Analysis of a Diffuse Interface Model for Two-Phase Flow with Surfactant

This proposal seeks support for initiating a collaboration between Josef Weber, PhD student at the University of Regensburg (supervisors: H. Abels, H. Garcke) and Andrew Kei Fong Lam, PhD student at the University of Warwick (supervisors: B. Stinner, C.M. Elliott) in order to analyse a recently derived phase field model for surfactants in two-phase flow with respect to well-posedness and convergence in the sharp interface limit.

In a collaborative approach between Regensburg (Garcke) and Warwick (Lam, Stinner) novel diffuse interface models for two-phase flow with surfactants have been derived [4]. Essential features are: Allowance for both soluble and insoluble surfactants, thermodynamic consistency, coverage of both cases of instantaneous and dynamic adsorption, and flexibility to realise various isotherms and equations of state. By means of a formal asymptotic analysis, problems with moving boundaries are obtained in the sharp interface limit.

Andrew Lam has been studying a reduced problem since. It accounts for the surfactant densities as unknown fields only while assuming that the interface is stationary and given. Weak well-posedness of quite general problems in both sharp interface and diffuse interface setting is established for one of the models (Model A in [4]). His aim is now to show convergence of the latter one to the former one using ideas from [2] or [3]. For this attempt he will benefit from the expertise on these techniques within the group of H. Abels [1] where he, in particular, will be able to engage in discussions with the PhD student Stefan Schaubek who intensively studied [2]. For other questions, namely, generalisations of the well-posedness results and convergence of the dynamic adsorption model to the instantaneous adsorption model, discussions with H. Garcke are expected to be valuable.

Josef Weber has started with the analysis of one of the fully coupled models (Model C of [4]). Beneficial for him will be discussions on the structure of the governing equations (with Andrew Lam) and suitable analytical methods (with B. Stinner), particularly with respect to nonlinearities related to the surfactant. Moreover, with regards to hypotheses emerging from the sharp interface analysis, e.g., related to the orders of convergence, there is the possibility to perform numerical tests using the software available in the group of B. Stinner.

To get the collaboration going we ask for funding for reciprocal visits of about 6 days each. Andrew Lam will travel to Regensburg beginning of June 2013 where he will be given opportunity to give an Analysis Oberseminar talk. In turn, Josef Weber will come to Warwick beginning of October 2013 around the first week of term when he'll be given the opportunity to give a talk in the Applied PDEs Working Seminar.

We request £350 for travel, £600 for accomodation plus £200 subsidence.

References

- [1] H. Abels, *On a diffuse interface model for two-phase flows of viscous, incompressible fluids with matched densities*, Arch. Ration. Mech. Anal. 194 (2009), 463–506.
- [2] N.D. Alikakos, P.W. Bates, X. Chen, *Convergence of the Cahn-Hilliard Equation to the Hele-Shaw Model*, Arch. Ration. Mech. Anal. 128 (1994), 165–205.
- [3] P. De Mottoni, M. Schatzman, *Geometrical evolution of developed interfaces*, Trans. Am. Math. Soc. 347, no. 5 (1995), 1533–1589.
- [4] H. Gareke, K.F. Lam, B. Stinner, *Diffuse interface modelling of soluble surfactants in two-phase flow*, submitted.