2.10 Turbulent mixing layer / splitter plate.

Consider fast moving fluid above and fluid of zero velocity ~~behind~~ below a splitter. Assume the profile can be approximated by $f(\xi)$ with

$$\frac{df}{d\xi} = \exp\left(-\frac{1}{2}\xi^2\right)$$

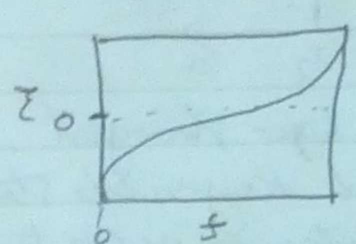


Fig A

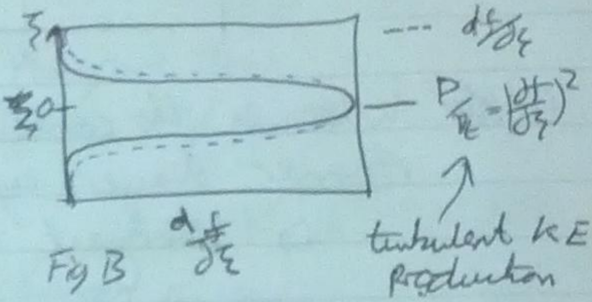


Fig B

(a) Self preservation equation for a mixing layer is

$$\left[\frac{\ell}{U_s}\frac{dU_s}{dx}\right]f^2 - \left[\frac{d\ell}{dx}\right]f'\int_0^{\xi} f\,d\eta - \left[\frac{\ell}{U_s}\frac{dU_s}{dx}\right]f'\int_0^{\xi} f\,d\eta = \frac{1}{R_T}g'$$

Assume $U_s(x) = Ax^m$, $\ell(x) = Bx^n$

(i) How does ℓ change downstream?

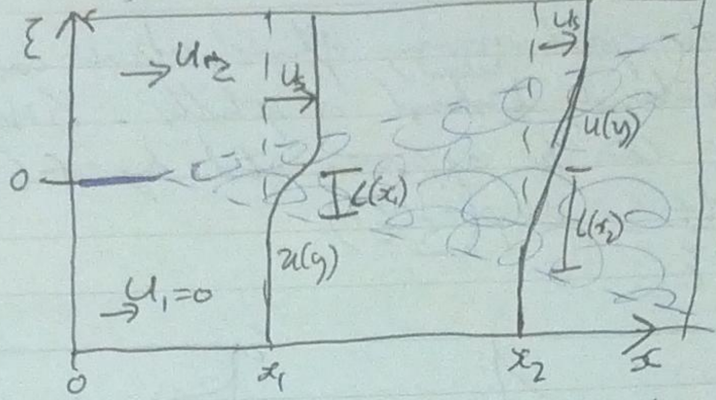
Answer We need $\left[\frac{\ell}{U_s}\frac{dU_s}{dx}\right]$ and $\left[\frac{d\ell}{dx}\right]$ to be independent of x for a self preserving solution, therefore $\ell \sim Bx$,

(ii) How does U_s change with time?

Answer For a mixing layer, the large scale flow is determined by two free-stream velocities U_2 above and U_1 below. There difference $U_s = U_2 - U_1$ doesn't change in time or with x.

Therefore $\frac{dU_s}{dx} = 0 \Rightarrow U_s \sim 1$, U_s constant.

(iii) Sketch how a turbulent mixing layer grows downstream



(iv) Where is the constant stress layer strongest.

Answer Layer of constant Reynolds stress is strongest in the interior of the turbulence.

(v) How does the Reynolds number based on l change with x?

Answer $R_l = \frac{U_s l}{\nu} = (U_2 - U_1)\frac{l}{\nu} = \frac{(U_2 - U_1) Bx}{\nu}$ (const; ν const)

so $R_l \sim x$

(b)

(i) Turbulent KE budget

$$0 = -U_0 \frac{\partial}{\partial x_1} \frac{1}{2}\overline{q^2} - \overline{uv}\frac{dU_1}{\partial x_2} - \frac{d}{\partial x_2}\overline{v\left(\frac{1}{2}q^2 + \frac{P}{\rho}\right)} - \varepsilon$$

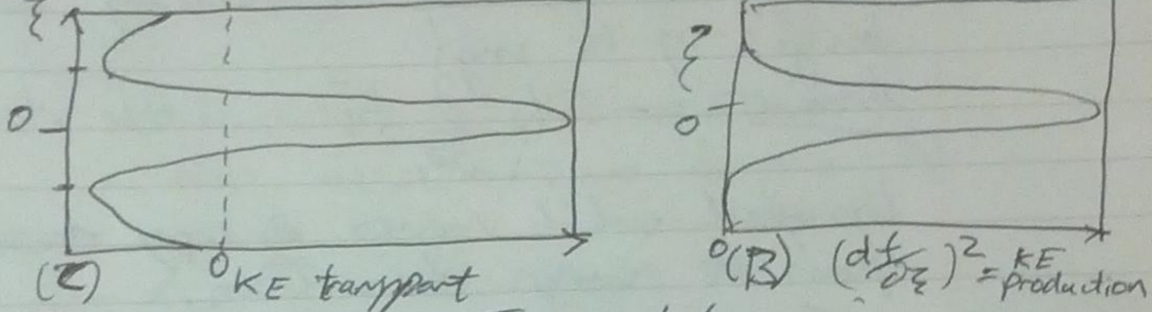
Identify transport terms:

Answer The downstream advective term is

$$U_0 \frac{\partial}{\partial x_1} \frac{1}{2}\overline{q^2}$$

Cross-stream transport term is $\frac{d}{\partial x_2}\overline{v\frac{1}{2}q^2}$ (flux of turbulent KE.)

(ii) Assume cross stream flux of turbulent KE is proportional to $\xi e^{-\frac{1}{4}\xi^2}$. Sketch the transport term.



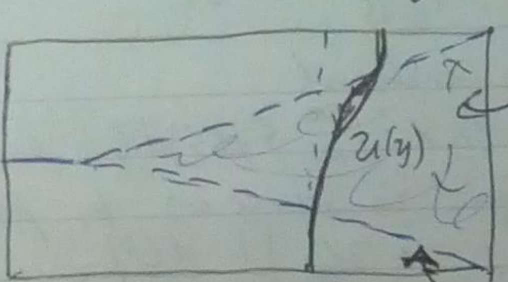
$\frac{1}{2}\overline{vq^2} \sim \xi e^{-\frac{1}{4}\xi^2}$, Transport term is

$$\frac{\partial}{\partial x_2}\frac{1}{2}\overline{vq^2} \approx \frac{\partial}{\partial \xi}\xi e^{-\frac{1}{4}\xi^2}$$

$$= e^{-\frac{1}{4}\xi^2} - \xi^2 e^{-\frac{1}{4}\xi^2}$$

(iii) Explain the role of its changes in sign.

Answer The change in sign is due to the transport of KE across the mixing layer. The sign changes represent where the KE transport switches ~~from~~ between removing energy from the interior where it is being produced in figure B and depositing energy at the outer edges.



(iv) Which term is production of turbulent KE?

Answer The production term is

$$\overline{uv}\frac{dU_1}{dx_2}$$

which exchanges KE between the mean flow and turbulence. There is no contribution ~~from~~ this insignificant to terms in the wings of the profile.

(V) Identify the roles of the remaining terms in the budget of the wings.

<u>Answer</u> $-U_0 \frac{\partial}{\partial x_1} \frac{1}{2}\overline{q^2}$ is the downstream transport which moves energy ~~downstream~~ from the wings downstream.

ε is dissipation, this transfers KE into heat energy.