MA3D1 Fluid Dynamics Support Class 9 - Compressible Flow

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Jorge Lindley email: J.V.M.Lindley@warwick.ac.uk

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1 Compressible Flows

Example 1. ***Energy conservation in compressible flow***

1. *Using the 1D compressible inviscid flow equations*

$$\partial_t u + u\partial_x u = -\frac{1}{\rho}\partial_x p, \tag{1.1}$$

$$\partial_t \rho + \partial_x(\rho u) = 0, \tag{1.2}$$

$$\partial_t S + u\partial_x S = 0 \tag{1.3}$$

prove the energy continuity equation

$$\partial_t\left(\epsilon + \frac{\rho}{2}u^2\right) + \partial_x\left[\rho u\left(\frac{u^2}{2} + h\right)\right] = 0, \tag{1.4}$$

for a 1D polytropic flow, in which the internal energy and the enthalpy are given by expressions

$$\epsilon = \frac{p}{\gamma - 1}, \quad h = \frac{\gamma p}{(\gamma - 1)\rho} = \frac{c_s^2}{\gamma - 1}. \tag{1.5}$$

2. *Without derivation, present a general qualitative argument on the role of viscosity in the energy conservation law.*

Example 2. *Hyperpersonic collision of two gas masses*
Given information: jump conditions across the shocks are (c.f. question 9.3.7):

$$(u_1 - U_1)^2 = \frac{\rho_3}{\rho_1} \frac{(p_3 - p_1)}{(\rho_3 - \rho_1)}, \tag{1.6}$$

$$\rho_3 = \rho_1 \frac{\frac{p_3}{p_1}(\gamma + 1) + (\gamma - 1)}{(\gamma + 1) + \frac{p_3}{p_1}(\gamma - 1)}, \tag{1.7}$$

where u_1, p_1 and ρ_1 are the velocity, the pressure and the density ahead of the shock, U_1 is the shock speed and p_3 and ρ_3 are the pressure and the density behind of the shock. (Here we have chosen the same notations as for the right shock in figure 1)

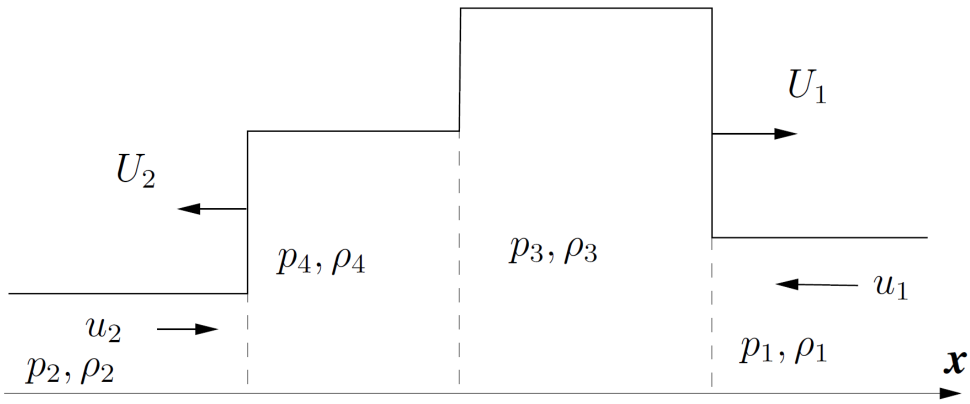

Figure 1: Gas motion after a collision of two clouds. The dashed lines mark positions of the three jumps: two shocks (on the right and on the left) and a contact discontinuity (in the middle). The solid line is the density profile. The pressure profile is similar, except that there is no pressure jump across the contact discontinuity.

Consider two polytropic gas clouds with initial uniform pressures and densities, p_1, ρ_1 and p_2, ρ_2, moving toward each other with velocities $u_1 < 0$ and $u_2 > 0$ respectively (gas 1 is on the right and gas 2 is on the left). At some instant of time the clouds collide, and our goal is to describe their subsequent dynamics assuming that the collision is hypersonic: the relative velocity $u = u_2 - u_1$ is much greater than the initial speed of sound in both clouds.

1. *Explain why, in general, three jumps will form: two shocks running away from the collision point in the opposite directions an a contact discontinuity in between of the shocks, see figure 1.*

2. *The shock waves that form as a result of a hypersonic collision are strong: the pressure behind the shock is much greater than the pressure ahead of the shock. Find the limiting expressions for the ratio of the density behind the shock to the density ahead of the shock when the pressure ratio tends to infinity (c.f. question 9.3.7). Find the densities on the two sides of the contact discontinuity.*

3. *In the strong shock limit, using the mass flow continuity, find the shock speeds U_1 and U_2 in terms of the gas velocities u_1 and u_2 in the reference frame of stationary contact discontinuity.*

4. *Using* (1.6)*, find the pressures behind the shocks in terms of the flow parameters ahead of the shocks and the shock speeds in the strong shock limit. Equating pressures on the two sides of the contact discontinuity, find another condition on the velocities and, using this condition, find U_1 and U_2 in terms of the relative velocity $u = u_2 - u_1$ in the reference frame of stationary contact discontinuity. Find the pressure between the shocks in terms of ρ_1, ρ_2, α and u.*