MA946	Term 1
Introduction to Graduate Probability	15 CATS

Status: Three lectures per week are scheduled for f2f teaching in B3.02: Monday 1–12pm and Wednesday 1–2pm.

Assessment: Oral examination (50%), essay (12 pages) 50%.

Essay: The essay topic can be chosen from a list of basic probability theory ideal for those students who like to learn basic probability theory, or a list of advanced high-profile and hot topics of current research (flexibility for the students to select their papers, reviews, and books according to their needs and interest). Essay topics can be discussed directly with the module leader. It is strongly recommended to select an essay topic within the first two weeks of the term. For questions and discussion about essay topics as well as further questions and guidance email the module leader.

Hand-in time essay: Essay must be submitted as pdf-file via email and a hardcopy version into the pigeon hole of the module leader by Friday 7 January 2022 by 12 pm.
Some suggestions for essay topics (basic and advanced):

1. The Central Limit theorem (basic)
2. The Central Limit Theorem for random dynamical systems (basic)
3. Basic large deviation theory (basic)
4. Brownian motion (scaling, main features and properties) (basic)
5. Laplace transforms and characteristic functions (basic)
6. Large deviations and Gibbs conditioning for finite alphabets (basic)
7. Random walks (basic)
8. Potential theory (basic)
9. Large deviations for interacting Brownian motions (transformed path measures)
10. The random field of gradients (definitions, properties, surface tension)
11. The discrete Gaussian Free Field (random walk representations)
12. The continuum Gaussian Free Field (GFF) (definition, main properties, scaling limit)
13. Large deviation principles in topological vector spaces (general Gärtner-Ellis Theorem)
14. Determinantal point processes (in physics)
15. Determinantal point processes for machine learning
16. Introduction to stochastic Loewner evolution (definition and important properties)
17. An introduction to the renormalisation group theory - an probabilistic approach
18. Gibbs conditioning principle and large deviations
19. The arcsine laws for Brownian motion
20. Markov processes with condensation
21. Loop measures and isomorphism theorems
22. Conformal invariance of planar Brownian motion
23. Jump Markov Processes (martingale property, invariant distributions, recurrence and transience)
24. General Theory of Markov processes (martingales, generator)

25. Girsanov Formula (Brownian motion with drift)

26. Sample path large deviations (random walks, Brownian motion)

Lecture notes will be provided (available online). Below are some suggestions for background and additional material, more will be provided in week 1.

References

[DV-J05] D.J. Daley & D. Vere-Jones: *An introduction to the theory of point processes*, Vol I, Springer (2005).

[DZ97] Amir Dembo and Ofer Zeitouni: *Large Deviations Techniques and Applications*, Springer (1997).

[FK06] Jin Feng and Thomas G. Kurtz, *Large Deviations for Stochastic Processes*, American Mathematical Society (2006).

[Geo12] Hans-Otto Georgii, *Stochastics*, De Gruyter Textbook, 2nd rev. and ext. (2012).

[vdH] Frank den Hollander, *Large Deviations (Fields Institute Monographs)*, (paperback), American Mathematical Society (2008).

[Kal02] Olav Kallenberg: *Foundations of Modern Probability*, 2nd ed. Springer (2002).

[LV00] Gregory Lawler & Vlada Limic: *Random Walk: A Modern Introduction*, Cambridge University Press (2000).

[MP] Peter Moerters and Yuval Peres: *Brownian motion*, Cambridge University Press (2010).

[Str93] Daniel W. Stroock: *Probability - An analytic view*; revised ed. Cambridge University Press1993.

[SV] Daniel W. Stroock & S.R. Srinivasa Varadhan: *Multidimensional Diffusion Processes*, Springer (1979).