

Theme Guide

MA4K8: Maths in Action

2026/2027

General information

This guide contains details of the Maths-in-Action themes offered this year. **Register** your theme choice by

Sunday, 1 November 2026

at the latest on the module **Moodle** page. Each theme will have a restricted number of takers and these will be assigned on a first come first served basis. While references are included in each theme description, Maths-in-Action themes have been chosen to be current and topical. You may combine or adapt the suggested directions within a theme. It is up to you to identify the most appropriate source material. You should discuss with the project organisers if you have questions.

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1 Applied Algebraic Topology

Synopsis

Topological data analysis (TDA) studies the shape of point clouds, networks and images. Its central tool, **persistent homology**, tracks connected components, loops and higher-dimensional holes as a scale changes. Barcodes and persistence diagrams record their appearance and disappearance [1, 2]. Applications include biological and neural data, materials and time series.

Your Brief

Begin with simplicial complexes, boundary maps and homology over a field, then study filtrations and the computation of persistent homology. Possible directions, which may be combined, include:

- **Persistence and stability.** Compute barcodes for sampled circles or other simple spaces using a Vietoris–Rips filtration. Explain a stability theorem and investigate how sampling, noise and outliers affect the results [1, 2].
- **Topology of biological or neural data.** Investigate loops or cavities in a small biological dataset or a synthetic model. For example, explore how persistence can distinguish a circle from a torus, motivated by the topology of grid-cell population activity [4].
- **Multiparameter persistence.** Vary both distance scale and a density threshold. Study a small bifiltration, its rank invariant or one-parameter slices, and explain why a single barcode no longer gives a complete description [5].
- **Topological and spectral information.** As an advanced option, compare persistent homology with a persistent Laplacian. Investigate what nonzero eigenvalues add to the information in its kernel on a small example [6].

You can explain a mathematical construction and supporting theory, and develop reproducible examples. Justify the metric, filtration and coefficient field: long persistence alone does not establish scientific significance. Preparation includes linear algebra and basic topology, with further algebra or spectral theory for the final two routes.

References

Start with [1] or [2]. Oudot’s book [3] develops persistence theory and its connection to quiver representations. [5, 6] offer further extensions.

- [1] Frédéric Chazal and Bertrand Michel. “An Introduction to Topological Data Analysis: Fundamental and Practical Aspects for Data Scientists”. In: *Frontiers in Artificial Intelligence* 4, 667963 (2021). doi: [10.3389/frai.2021.667963](https://doi.org/10.3389/frai.2021.667963). URL: <https://arxiv.org/abs/1710.04019>
- [2] Gunnar Carlsson and Mikael Vejdemo-Johansson. *Topological Data Analysis with Applications*. Cambridge University Press, 2021
- [3] Steve Y. Oudot. *Persistence Theory: From Quiver Representations to Data Analysis*. Vol. 209. Mathematical Surveys and Monographs. American Mathematical Society, 2015. doi: [10.1090/surv/209](https://doi.org/10.1090/surv/209)
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- [6] Zhe Su et al. *Topological Data Analysis and Topological Deep Learning Beyond Persistent Homology - A Review*. Preprint. 2025. arXiv: [2507.19504](https://arxiv.org/abs/2507.19504)

2 Applied Algebraic Geometry

Synopsis

Polynomial maps and constraints describe many statistical and machine-learning models. Their geometry reveals expressivity, identifiability and difficulties in model fitting. This theme follows the **nonlinear algebra** perspective developed in [1, 2]. Its tools include **polynomial ideals, algebraic varieties, elimination, Gröbner bases and tensor decompositions**. Recent work in **neuroalgebraic geometry** applies these ideas to spaces of functions represented by neural networks [3]. This project is suitable for students who enjoy algebra and algebraic geometry.

Your Brief

Learn the basic connection between polynomial equations, ideals and varieties. Suitable directions, which may be combined, include:

- **Algebraic statistics.** Independence of two binary variables gives the equation $p_{00}p_{11} - p_{01}p_{10} = 0$ for their joint probabilities. Use this starting point to study a small conditional independence or latent-variable model. Investigate its defining equations, identifiability or maximum-likelihood estimation [2].
- **Neuroalgebraic geometry.** For a small network with polynomial activations, such as squaring, express its output coefficients as polynomials in the weights. Study the dimension of the represented function family, parameter symmetries and their implications for expressivity or identifiability. Compare different architectures [3, 4].
- **Low-rank models and optimisation.** Study determinantal varieties of rank-constrained matrices and their nearest-point problems. Derive critical-point equations and investigate singularities. Tensor decomposition is an advanced extension, distinguishing tensor rank from border rank [1, 5].

Combine a worked mathematical example with symbolic or numerical investigation. Distinguish the complex variety from the real feasible model, including statistical positivity and normalisation constraints. For networks, distinguish the parameter space, its image and the image's Zariski closure. Linear algebra, polynomial rings and multivariable calculus are useful preparation. The statistics route also needs probability.

References

Start with [1]. For neuroalgebraic geometry, begin with Kathlén Kohn and coauthors' invitation [3].

- [1] Mateusz Michałek and Bernd Sturmfels. *Invitation to Nonlinear Algebra*. Vol. 211. Graduate Studies in Mathematics. American Mathematical Society, 2021. URL: <https://math.berkeley.edu/~bernd/gsm211.pdf>
- [2] Mathias Drton, Bernd Sturmfels, and Seth Sullivant. *Lectures on Algebraic Statistics*. Vol. 39. Oberwolfach Seminars. Birkhäuser, 2009. DOI: [10.1007/978-3-7643-8905-5](https://doi.org/10.1007/978-3-7643-8905-5)
- [3] Giovanni Luca Marchetti et al. "Position: Algebra Unveils Deep Learning - An Invitation to Neuroalgebraic Geometry". In: *Proceedings of the 42nd International Conference on Machine Learning*. Vol. 267. PMLR, 2025, pp. 81759–81773. URL: <https://arxiv.org/abs/2501.18915>
- [4] Kaie Kubjas, Jiayi Li, and Maximilian Wiesmann. "Geometry of polynomial neural networks". In: *Algebraic Statistics 15* (2024), pp. 295–328. DOI: [10.2140/astat.2024.15.295](https://doi.org/10.2140/astat.2024.15.295). URL: <https://arxiv.org/abs/2402.00949>
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3 Climate

Synopsis

Life as it evolved on Earth can exist in a fairly narrow range of temperature and pressure roughly corresponding to the liquid state of water. So, it would not be an exaggeration to say that Earth's climate is a singularly important factor for every living organism on our planet. Climate is a state of the climate system, the ultimate complex system consisting in case of Earth of atmosphere, hydrosphere, cryosphere, geosphere and biosphere. It is a non-equilibrium system evolving via complex interactions between its components in response to external sources of energy from the Sun and the Earth core, and energy loss to outer space. Weather is an instantaneous state of the climate system, which is essentially random due to complexity and open nature of the system. Climate is the ergodic average of weather states over the time window of 30 years, making the old expression "Climate is what you expect, weather is what you get" surprisingly precise mathematically. The statistically averaged observables of the climate system such as air and water temperature and the CO₂ concentration in the atmosphere are expected to evolve deterministically according to laws following from basic physics.

Due to the incredible complexity of the climate systems, the derivation of basic equations governing the evolution of its observables from the first principles is still lacking. Currently scientists resort to building increasingly complicated models capturing various aspects of the climate system and studying them either theoretically or numerically. They start with zero dimensional models of homogeneous, homogeneously irradiated Earth to capture the global energy balance and move up the complexity scale by accounting for the obvious inhomogeneities of the climate system, effects of convection and other energy fluxes within and between different components, etc.

Your Brief

Your task is to study the climate system using any model which captures your fancy. Would you prefer to study the evolution of climate using **low dimensional dynamical systems (O.D.E.s)** whose behaviour (fixed points, limit cycles, stability, etc) can be studied theoretically? Or would you rather try to understand the most realistic climate models (**e.g. P.D.E.s**) and learn about the results of their numerical studies? Perhaps you are interested in paleoclimatology and would like to compare the predictions of climate models with indirect observations of Earth's climate over the scales of millions of years? You might even decide that Earth's climate is too complicated and study the climate of Jupiter dominated by strong zonal jets and awesome persistent storms such as the Great Red Spot instead? Whatever you do, you will be studying one of the most complex and relevant physical systems known to mankind. Good luck!

References

- [1] H. G. Kaper and Hans Engler. *Mathematics and climate*. Philadelphia: Society for Industrial and Applied Mathematics, 2013. ISBN: 978-1-61197-260-3
- [2] F. W. Taylor. *Elementary climate physics*. New York: Oxford University Press, 2005. ISBN: 978-0-19-856733-2 978-0-19-856734-9
- [3] Valentin P Dymnikov and Aleksander N Filatov. *Mathematics of Climate Modeling*. English. OCLC: 879624832. Boston, MA: Birkhäuser Boston, 1997. ISBN: 978-1-4612-4148-5

4 Renewable Energy and Power Systems

Synopsis

Electricity systems must balance supply and demand while remaining stable after disturbances. Wind and solar generation introduce variable supply, while storage, transmission and control offer ways to manage it. Replacing rotating generators with power-electronic interfaces also changes grid dynamics. These questions lead to mathematics involving **differential equations, graph theory, spectral analysis, optimisation and probability**. Small network models can reveal useful principles behind the energy transition.

Your Brief

Begin with power and energy balances, then develop mathematical models of generation, storage or a small network. Possible directions include:

- **Synchronisation and stability.** Use swing equations or coupled-oscillator models to describe generator phases [1]. Find synchronous operating states and analyse their linear stability using a weighted graph Laplacian. Investigate how network structure, damping or a line outage changes the response to a disturbance.
- **Renewables, inertia and control.** Explore a low-dimensional frequency model with changing inertia or feedback representing inverter control [3]. Derive stability conditions and compare constant with time-varying parameters. Distinguish local stability from resilience to large disturbances.
- **Storage and network optimisation.** Formulate a linear programme to meet time-varying demand using renewable generation and a battery, with efficiency and capacity constraints [2]. Extend it to a small transmission network using linearised power-flow equations. Explore trade-offs between storage and transmission, or uncertainty using several weather scenarios.

These directions can be combined, for example by asking whether an economically attractive dispatch is dynamically stable. Derive the model and investigate a theorem, bound or optimisation result alongside small simulations. State modelling assumptions, distinguish power from energy, and examine sensitivity to parameters or data. Useful preparation includes linear algebra, ODEs and optimisation; uncertainty modelling also uses probability.

References

- [1] and [2] introduce the dynamical and optimisation viewpoints; [3] offers a recent extension.
- [1] Florian Dörfler, Michael Chertkov, and Francesco Bullo. “Synchronization in complex oscillator networks and smart grids”. In: *Proceedings of the National Academy of Sciences* 110.6 (2013), pp. 2005–2010. doi: [10.1073/pnas.1212134110](https://doi.org/10.1073/pnas.1212134110)
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- [3] Andreas Kasis, Stelios Timotheou, and Marios Polycarpou. *On the stability properties of power networks with time-varying inertia*. Preprint, revised December 2025. 2025. arXiv: [2301.09202](https://arxiv.org/abs/2301.09202)

5 Collective Dynamics

Synopsis

Collective dynamics play an important role in many applications in life and social sciences. Prominent examples include the motion of large animal groups or pedestrian flows. The underlying mechanisms are similar – interaction laws trigger the synchronisation of the individual’s velocity – the outcomes are complex macroscopic patterns such as rotating mills or lane formation in pedestrian dynamics.

Several interesting questions arise in this context: How does global alignment emerge from local interactions? What is the nature of the alignment and under which conditions will we observe it? Does synchronisation depend on the precise details of the alignment interactions or just general properties (such as symmetry and range)?

Your Brief

You could investigate fish schools, bird flocks or pedestrian flows, or compare the mathematical mechanisms across these settings. Study the corresponding microscopic models, such as the Adler’s equation, Cucker-Smale model or Helbin’s social force model, **differential equations**, **discrete systems**. Discuss alignment in terms of the parameters and the initial conditions using analytical and/or numerical tools. You may also consider to investigate the stability of stationary patterns using linear stability analysis.

References

- [1] George F. Young et al. “Starling Flock Networks Manage Uncertainty in Consensus at Low Cost”. In: *PLOS Computational Biology* 9.1 (Jan. 2013), pp. 1–7. DOI: [10.1371/journal.pcbi.1002894](https://doi.org/10.1371/journal.pcbi.1002894)
- [2] Felipe Cucker and Steve Smale. “Emergent behavior in flocks”. In: *IEEE Transactions on automatic control* 52.5 (2007), pp. 852–862
- [3] Seung-Yeal Ha and Jian-Guo Liu. “A simple proof of the Cucker-Smale flocking dynamics and mean-field limit”. In: *Commun. Math. Sci.* 7.2 (June 2009), pp. 297–325
- [4] Craig W Reynolds. *Flocks, herds and schools: A distributed behavioral model*. Vol. 21. 4. ACM, 1987
- [5] Tamás Vicsek et al. “Novel type of phase transition in a system of self-driven particles”. In: *Physical review letters* 75.6 (1995), p. 1226
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- [7] Sylvain Faure and Bertrand Maury. “Crowd motion from the granular standpoint”. In: *Mathematical Models and Methods in Applied Sciences* 25.03 (Sept. 2014), pp. 463–493. ISSN: 0218-2025. DOI: [10.1142/S0218202515400035](https://doi.org/10.1142/S0218202515400035)
- [8] Bertrand Maury and Sylvain Faure. *Crowds in Equations*. World Scientific (Europe), 2018. DOI: [10.1142/q0163](https://doi.org/10.1142/q0163)
- [9] Emiliano Cristiani, Benedetto Piccoli, and Andrea Tosin. *Multiscale Modeling of Pedestrian Dynamics*. Vol. 12. MS&A. Cham: Springer International Publishing, 2014. ISBN: 978-3-319-06619-6 978-3-319-06620-2. DOI: [10.1007/978-3-319-06620-2](https://doi.org/10.1007/978-3-319-06620-2)
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There are good articles on Wikipedia, and you might want to check out the webpage of the Couzin Lab <https://www.ab.mpg.de/379909/research> and the webpage of the Forschungszentrum in Jülich <https://ped.fz-juelich.de/da/doku.php>.

6 Cryptocurrencies

Synopsis

Cryptocurrencies combine cryptographic protocols, consensus and incentives to record payments without a trusted bookkeeper. Nakamoto's Bitcoin paper [2] proposes a blockchain secured by proof of work to prevent double spending. The mathematics includes **algebra, probability, algorithms and game theory**.

Your Brief

Begin with hash functions, public keys, digital signatures and Merkle trees [1], then read Nakamoto's paper. Possible directions for mathematical investigation include:

- **Cryptographic foundations.** Explain a signature scheme using finite groups or elliptic curves, or analyse Merkle membership proofs and birthday bounds for hash collisions.
- **The Byzantine generals problem.** Study agreement despite dishonest participants [3]. Analyse the three-general impossibility example and a four-general protocol tolerating one traitor in the oral-message model. Explain how signatures change the problem.
- **Proof of work.** Model block discovery using Poisson processes and an attacker's progress using a random walk. Revisit Nakamoto's double-spend calculation: compare analytical and simulated success probabilities for different hash-power shares and confirmation depths.
- **Proof of stake.** Use Ouroboros [4] as a case study. Analyse stake-weighted leader selection and concentration bounds in a simplified model. Compare with proof of work in terms of resource requirements, adversarial assumptions and confirmation guarantees.
- **Economics and incentives.** Derive an equilibrium linking rewards, participation costs and attack incentives [5]. Investigate how fees or the cost of committed stake affect the security budget.

Develop a proof, model or simulation, stating assumptions about communication, authentication and adversarial resources. You could even develop your own cryptocurrency! Preparation includes probability and discrete mathematics, plus algebra or game theory for specific routes.

References

Start with Chapters 1–2 of [1] and [2]; use the remaining papers to explore the directions that interest you.

- [1] Arvind Narayanan et al. *Bitcoin and cryptocurrency technologies: a comprehensive introduction*. Princeton: Princeton University Press, 2016. ISBN: 978-0-691-17169-2. URL: <https://bitcoinbook.cs.princeton.edu/>
- [2] Satoshi Nakamoto. *Bitcoin: A Peer-to-Peer Electronic Cash System*. bitcoin.org, Oct. 2008. URL: <https://bitcoin.org/bitcoin.pdf>
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7 Data and Information

Synopsis

Modern science and technology depends increasingly on the efficient acquisition, storage and processing of vast amounts of data. Quoting [7], by 2025, it is estimated that globally 463 exabytes (10^{18} bytes) of data will be created each day! Examples range from high-precision medical imaging to genomic data analysis and radio telescopes in outer space. While the seemingly endless availability of data is a blessing from a statistical point of view, it poses enormous challenges from a computational perspective. In many cases of interest, however, the data possesses a simple underlying structure; the actual information content is orders of magnitude smaller than the data dimension. An exciting recent development has been the emergence of methods for information acquisition and processing, with a complexity proportional to the information content rather than the larger data dimension. One typical application is a reduction in the amount of radiation a patient is exposed during medical scans.

Your Brief

In this theme you will focus on efficient ways in which data is acquired, processed and stored by taking into account special structure hidden within the data. You will work with sample datasets and use computational tools from **topology, optimization and numerical analysis**. At first, you should study methods of data representation (Fourier analysis, Wavelets, curvelets and shearlets, sparsity). Based on an understanding of the structure of data, you would then look in more detail into one or more of the following aspects:

- Total Variation image processing
- Compressed Sensing: efficient recovery of information from partial observations
- Image Inpainting: accurate prediction of missing data from incomplete measurements (for example, in modeling consumer preferences)
- Deep Learning in medical imaging

References

- [1] Daniel Leycam and Dimitris G. Angelakis. "Topological data analysis and machine learning". In: *Advances in Physics* 8.1 (2023), p. 2202331
- [2] Simon Foucart and Holger Rauhut. "An Invitation to Compressive Sensing". In: *A mathematical introduction to compressive sensing*. Springer, 2013, pp. 1–39
- [3] Antonin Chambolle et al. "An introduction to total variation for image analysis". In: *Theoretical foundations and numerical methods for sparse recovery* 9.263-340 (2010), p. 227
- [4] Stéphane Mallat. *A wavelet tour of signal processing*. Elsevier, 1999
- [5] Gitta Kutyniok and Demetrio Labate. *Shearlets: Multiscale analysis for multivariate data*. Springer Science & Business Media, 2012
- [6] A. Iserles and C. B. Schoenlieb. "Mathematics of Information - The Second Industrial Revolution," in: *Mathematics Today, IMA@50* 50.1 (2013), pp. 53–58
- [7] C. F. Higham and D. J. Higham. "Deep learning: An introduction for applied mathematicians". In: *SIAM Review* arXiv preprint arXiv:1801.05894, 2018 (2019)
- [8] URL: <https://www.visualcapitalist.com/how-much-data-is-generated-each-day/>

8 Epidemiology

Synopsis

The recent pandemic has highlighted the potential of infectious diseases to disrupt daily life, having disastrous consequences for communities. However, even before COVID-19, understanding the transmission and control of infectious diseases has been critical to minimising other outbreaks and preventing morbidity and mortality. Since the early 20th century, mathematical modelling of infections was being developed **O.D.E.s and discrete dynamical systems**, and since then it has been ever increasing in sophistication with new scientific developments and vast improvements in processing power of computers.

Epidemiological modelling can be used to guide strategic decisions to curb infection and optimise how we use the tools at our disposal (vaccines, curative drugs, chemoprophylaxis, control of animals, etc.). An important factor in how impactful an intervention strategy can be is its timing: How much difference might one week's delay make in controlling a new outbreak? When is the best time of year to deploy insect control against a vector-borne disease? How do time-varying social contact patterns (e.g. school term times) influence control of disease?

Your Brief

In this project you could study an infectious disease (in humans or animals), or compare related diseases, investigating how the timing of interventions affects their effectiveness. You should formulate a basic dynamic model of the disease in question and, using your favourite programming language, perform numerical simulations. Use your model to illustrate infection patterns of the disease in an uncontrolled situation and under at least three plausible intervention scenarios with different timings of intervention. How could these types of results be used to influence policy decisions? Are there other factors or considerations that might impact decision making?

Possible directions include:

1. *Infections with seasonal forcing*: e.g. in tropical infectious diseases such as dengue or malaria, or transmission of seasonal influenza in the UK
2. *Outbreak dynamics*: e.g. pandemic influenza or Ebola
3. *How to time interventions to minimise resources*: e.g. what is the best timing between mass treatment rounds against intestinal worms (mass drug administration for soil transmitted helminths)

References

- [1] N.C Grassly and C Fraser. "Seasonal infectious disease epidemiology". In: *Proceedings of the Royal Society B: Biological Sciences* 273.1600 (2006), pp. 2541–2550. doi: [10.1098/rspb.2006.3604](https://doi.org/10.1098/rspb.2006.3604)
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- [3] S.D Mylius et al. "Optimal allocation of pandemic influenza vaccine depends on age, risk and timing". In: *Vaccine* 26.29 (2008), pp. 3742–3749. ISSN: 0264-410X. doi: <https://doi.org/10.1016/j.vaccine.2008.04.043>
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- [6] M.J Tildesley et al. "Optimal reactive vaccination strategies for a foot-and-mouth outbreak in the UK". in: *Nature* 440.7080 (2006), pp. 83–86. URL: <https://www.nature.com/articles/nature04324>

9 Geometry in nature

Synopsis

The **geometry of curves** provides an early introduction to ordinary differential equations. Geometric **partial differential equations** arise in understanding problems in geometry involving hypersurfaces and manifolds, **differential geometry**. They arise naturally in diverse applications including optimal shapes, soap films, biological cells, image processing and many others. The design of objects leads to natural problems in the calculus of variations involving shapes. Blood cells have shapes associated with minimisers of geometric functionals. The function of biological cells depend on processes which can be modelled by partial differential equations on hypersurfaces. The shape of ice crystals may be understood using complex anisotropic surface functionals.

Your Brief

In this project you should investigate any mathematical aspect you fancy including numerical computation or analysis or application involving geometric partial differential equation, for example

- Level set equations/The role of distance function in applications
- Curve shortening/Motion by curvature
- Active contours in imaging
- Biological cell motion/Biomembranes
- Crystal growth and Wulff shapes

or something else!

References

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10 Financial Machine Learning

Synopsis

How can learning algorithms model financial markets and support decisions under uncertainty? Modern financial machine learning connects **stochastic processes, statistical learning, stochastic control, optimisation and graph theory**. Applications include hedging, market microstructure, fixed income and portfolio construction [1, 2].

Your Brief

Possible directions, open to combination and extension, include:

- **Deep hedging.** Learn an adapted trading strategy to minimise a convex risk measure of hedging losses under transaction costs [3]. Study how costs, constraints or model misspecification change the strategy in a small stochastic-volatility or jump model.
- **Limit order books.** Compare a point-process model with a learned predictor or generative model of order-book dynamics [4]. Investigate conditional distributions, queue imbalance, spreads or market impact using synthetic or accessible data.
- **High-frequency trading and execution.** Formulate execution or market making as a stochastic control problem, with inventory risk and trading costs [2]. Derive a Bellman equation and compare reinforcement learning with dynamic programming in a small model.
- **Fixed income and yield curves.** Compare linear factors with autoencoders or neural state-space models for yield curves [5]. Investigate forecasting, filtering and no-arbitrage constraints, distinguishing real-world prediction from risk-neutral pricing dynamics.
- **Networks and portfolios.** Explore directed lead-lag networks, path signatures or clustering for asset relationships [6]. Analyse a simple estimator and test recovery of known dependence in simulated returns before considering portfolio applications.

These are starting points; related applications such as risk forecasting or generative market simulation are welcome. Develop a mathematical argument alongside experiments. Use chronological validation, address leakage and repeated strategy selection, and include costs when evaluating trading performance [1]. Preparation includes probability, linear algebra and statistics, with optimisation, stochastic calculus or control for particular directions.

References

The books provide background; the papers develop specific applications.

- [1] Marcos López de Prado. *Advances in Financial Machine Learning*. Wiley, 2018. ISBN: 978-1-119-48208-6
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11 Optimal Transport

Synopsis

Optimal transport asks how to move one distribution of mass into another at minimum cost. It provides a geometry for comparing probability distributions, with applications to images, machine learning and cell biology. Its tools connect **linear programming, convex duality, probability, partial differential equations and geometry**. For discrete distributions with masses a_i and b_j , the basic problem minimises $\sum_{i,j} c_{ij} \pi_{ij}$ over nonnegative transport plans whose row and column sums are a and b [1, 2].

Your Brief

Begin with finite transport problems and the distinction between a transport map and a transport plan. Possible directions include:

- **Transport geometry and duality.** Derive the dual linear programme and interpret its potentials. Study Wasserstein distances for discrete or one-dimensional distributions; compare displacement interpolation with simply averaging densities [2].
- **Entropic regularisation.** Derive Sinkhorn's matrix-scaling iterations for a transport problem with an entropy penalty. Investigate the trade-off between smoothing, approximation error and numerical stability on small point clouds or images [1].
- **Dynamics and gradient flows.** Explore the continuity-equation formulation of transport. As an advanced extension, study how diffusion can arise as a gradient flow of entropy in Wasserstein geometry, using a simple Gaussian or one-dimensional example [2].
- **Cell populations and trajectories.** Use transport to couple synthetic cell populations and explore proposed developmental paths. Investigate sensitivity to the cost, regularisation and sampling; recent multistage methods motivate extensions [3]. Explain which biological assumptions are needed: an optimal coupling alone does not identify the true ancestry of cells.

Combine or adapt these directions around a mathematical question. Explain a theorem or derive an algorithm, then investigate examples. Linear algebra, probability and optimisation suffice for discrete problems; continuous transport and gradient flows require further analysis.

References

Start with the discrete theory in [1]; [2] develops the analytical foundations.

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12 Scientific Machine Learning

Synopsis

Scientific machine learning combines data with models of physical processes. Can a neural network approximate a PDE solution, learn a solution operator, or reproduce dynamics while respecting conservation laws? These questions connect **differential equations, approximation theory, functional analysis, optimisation and numerical analysis**. Study accuracy, stability and generalisation alongside quality of fit [1].

Your Brief

Begin with a familiar ODE or PDE for which an analytical solution or a reliable numerical reference is available. Possible directions include:

- **Physics-informed neural networks.** Approximate a solution by a network and train using a loss built from the differential-equation residual and boundary or initial conditions [2]. For a one-dimensional Poisson or heat equation, investigate when a small residual controls solution error. Vary sampling and loss weights, and compare accuracy and computational cost with a finite-element method [5].
- **Neural operators.** Study maps from initial data, forcing or coefficients to PDE solutions [3]. Explain a Fourier neural operator or a simpler basis expansion. Test its behaviour on unseen inputs and different discretisations, distinguishing an architecture defined on functions from its finite-grid implementation.
- **Learning dynamics with physical structure.** Learn a Hamiltonian for an oscillator or pendulum and obtain dynamics from its derivatives [4]. Compare with an unconstrained learned vector field. Analyse conservation in continuous time and energy drift after numerical time stepping; preserving a learned Hamiltonian need not preserve the true physical energy.

Directions may be combined, for example to study how physical constraints affect error. Include a derivation or error analysis and a modest computational comparison. Separate training and prediction costs, and test beyond training samples. Useful preparation includes linear algebra, multivariable calculus, differential equations and numerical methods.

References

For context, see [1]; [5] compares PINNs and FEM in solution time, evaluation time and accuracy.

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13 Mathematics of Large Language Models

Synopsis

Large language models predict the next token (a word or part of a word) from its context. Many use the transformer architecture [1], whose **self-attention** mechanism mixes information between token vectors. Viewing these vectors as an **interacting particle system**, with network depth playing the role of time, connects attention to **dynamical systems, differential equations, probability and optimal transport** [2]. When do token representations cluster, how stable are these patterns, and how do architectural choices affect them?

Your Brief

Begin with softmax attention, query/key/value matrices, residual connections and normalisation. Possible mathematical directions include:

- **Particle dynamics and clustering.** For a simplified model with fixed parameters and two or three tokens on the circle, determine equilibria and their stability. With more tokens, investigate how attention temperature affects clustering. An advanced extension is *metastability*: clusters persist for a long time before merging [3].
- **Differential equations and mean-field limits.** Interpret a suitably scaled residual layer as an Euler step for an ODE. Separately, describe many interacting tokens by a probability measure and derive a nonlocal transport equation. Explore a simple stationary state or compare discrete layers with the continuous-depth model [2, 4].
- **Optimal transport and gradient flows.** Compare ordinary softmax attention with Sinkhorn normalisation, which makes the attention matrix doubly stochastic. Investigate the assumptions under which an energy or gradient-flow interpretation is possible, and its consequences for the dynamics [5, 4].

Explain a mathematical result and develop your own analysis or numerical investigation using small synthetic examples. State your assumptions, including whether causal masking (preventing attention to later tokens) is omitted. Distinguish layer evolution from parameter training, and discuss the relevance to trained language models. Linear algebra, probability and ODEs provide a starting point; the mean-field and optimal-transport routes require further analysis.

References

Start with [1] and the modelling sections of [2]; the remaining papers offer more advanced directions.

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14 Diffusion Models and Flow Matching

Synopsis

Diffusion models are at the core of generative AI, and provide the technology that is behind the generation of AI images and videos. How can a model generate structured data from random noise? Diffusion models gradually corrupt data and learn how to reverse this process [1, 2]. Their mathematical foundations include **Markov chains, stochastic differential equations, Fokker–Planck equations and numerical analysis**. A central object is the *score*, $\nabla \log p_t(x)$, of a time-dependent density. Related flow-matching methods learn a vector field transporting a simple distribution into a target distribution [3].

Your Brief

Begin with Gaussian distributions and a simple forward noising process. Possible directions, which may be combined, include:

- **Denoising and learning the score.** Derive Gaussian forward transitions and reverse conditionals given the original sample. Explain the connection between predicting added noise and estimating the score. Test a denoising objective on a small synthetic dataset [1].
- **Time reversal and stochastic sampling.** Study an Ornstein–Uhlenbeck process and its evolving mean and covariance. Derive its reverse-time dynamics under appropriate regularity assumptions. Compare numerical samples with an exact Gaussian solution, then investigate a Gaussian mixture, time-step error or an approximate score [2].
- **Deterministic transport and flow matching.** Derive the continuity equation for a distribution transported by an ODE. Construct a flow between simple distributions and investigate how its choice affects numerical sampling. Compare a diffusion’s probability-flow ODE with its SDE: matching marginal distributions does not mean matching individual trajectories [2, 3].

Combine mathematical analysis with reproducible examples in one or two dimensions. Separate approximation of the score or vector field from numerical integration error. Analytically known scores can support a substantial project without training a large model. Useful preparation is probability, multivariable calculus and ODEs; stochastic routes introduce Itô calculus and stochastic integration.

References

Start with [6] for an accessible mathematical introduction with code and PDE connections; [4] surveys the wider field. For SDE/ODE details and proofs see [5], and for flow matching see [3].

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15 Mathematics of AI Safety and Alignment

Synopsis

Alignment is a key concern in the development of powerful AI systems, in particular with a view towards emerging recursive self-improving systems. How can AI pursue intended objectives, remain reliable and accept intervention? These questions connect **probability, optimisation, decision theory, geometry and control**, with explicit assumptions needed to assess safety claims.

Your Brief

Explore and combine these directions through mathematical analysis and examples:

- **Reward hacking.** Construct a finite Markov decision process where optimising a proxy reward worsens the intended reward. Investigate consistent rankings of policies [1].
- **Certified robustness.** Bound perturbations preserving a classifier’s prediction using Lipschitz estimates or Gaussian randomised smoothing; compare certificates on a small network [2].
- **Uncertainty and risk.** Derive conformal prediction coverage under exchangeability, or study conformal risk control. Investigate informativeness and sensitivity to distribution shift [3].
- **Interpretability.** Analyse how small networks represent more sparse features than their hidden dimension, using the geometry of superposition and interference [4].
- **White-box estimation.** Explore the [ARC White-Box Estimation Challenge 2026](#): estimate mean activations of random ReLU networks using their weights. Start with Gaussian integrals, then compare moment approximations with Monte Carlo for accuracy and cost [7].
- **Safe control.** Use a control barrier function and quadratic programming to keep ODE trajectories inside a safe set. Investigate sensitivity to modelling error [5].
- **Human oversight.** In an off-switch game, how do objective uncertainty and assumptions about human decisions affect incentives to accept intervention [6]?

Background includes linear algebra and probability, with optimisation, game theory or ODEs as needed. In addition to the references below, look up organisations such as ARC (Alignment Research Center) or METR (Model Evaluation & Threat Research).

References

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16 Opinion Dynamics

Synopsis

Opinions play a crucial role in many social phenomena and global challenges, with local interactions among individuals creating interesting dynamics for the system as a whole. Spread of fake news, propaganda and election interferences have made it clear there is an essential need to know more about these dynamics. Added to that, the digital age has instigated a growth in social media, enhancing the need to better understand how influences or media coverage affect the formation of opinions and how information is spread. Aided by advancements in computer processing power, recent years have seen substantial refinement and realism in the modelling of opinion dynamics.

Opinion dynamic models have been developed to capture the interplay of different elements, including (but not limited to) individual predisposition, the influence of peer interactions and information exposure, **discrete and continuous dynamical systems, iteration and O.D.E.s**. Pertinent questions include: How does the presence or not of external information (a mechanism mimicking a sort of mass media broadcast,) impact the long-term behaviour of the system? What is the effect of adding or removing contacts on the dynamics? Is it possible to devise interventions that can promote the spread of truthful information, whilst countering the spread of misinformation and disinformation?

Your Brief

This topic has a broad range of application areas, including politics (e.g. voting behaviour), finance and business, and spread of behaviours driven by online sharing. After reading broadly about differing opinion dynamic model frameworks, explore applications that interest you, or compare their mathematical mechanisms. You may also consider investigating extensions to the foundational models that introduce more complex processes, such as stubbornness, bias, repulsive behaviour, power evolution, and the impact of memory.

References

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17 Quantum Computing

Synopsis

In December 2001 physicists at IBM's Almaden Research Centre in California made a vial of designer molecules act like a 7 quantum-bit (qbit) computer implementing a program to carry out on Shor's factorisation algorithm. The program's output, that $15 = 3 \times 5$, was hardly earth-shaking, but its implications for the future of computing were enormous. It proved that quantum phenomena, usually confined to the world of atoms and photons, have the potential to transform our lives through the processing of information.

Whereas **factorizing large numbers** on classical computers takes exponential time (every extra digit roughly doubles the run-time) for the fastest known algorithms, a quantum computer can theoretically do it in polynomial time and thereby render useless the most encryption currently used for Internet security. It is unlikely that the approach used in the IBM experiment can be scaled up, but many other ideas for creating larger quantum computers are being explored and may one day transform computing beyond all imagination.

Your Brief

The main focus of your brief could be any mathematical aspect of quantum computing, such as

- the mathematics that describes the particular quantum phenomena and building blocks that are exploited by quantum computers,
- the mathematics of quantum information theory,
- the **mathematics of quantum error correcting**. Noise is a huge problem with quantum computing, but because of the no clone theorem, classical error correcting methods can't be used.
- exploring how ZX-calculus can be used in quantum circuit optimisation,
- the mathematics informing algorithms that take advantage of the special features of quantum computers to make them more efficient than classical computers,
- computational complexity of quantum algorithms and classical algorithms and the theoretical bounds of computability.

References

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18 Traffic and Autonomous Driving

Synopsis

It is widely believed that in the near future, vehicle transportation systems will be very different from what we have today. The emergence of new technologies and systems such as vehicle connectivity, automation, shared mobility, and advanced sensing are rapidly changing mobility and accessibility. There is realistic expectation that it will soon be possible to build safer, more efficient, reliable, accessible, and sustainable transportation systems. While the demand on the nation's motorways will surely continue, mathematics has the potential to play a significant role in reducing accidents, journey times and emissions.

Your Brief

There are two main, interconnected, threads in this theme: Traffic flow and autonomous vehicles. Both of these, particularly the second, involve a vast array of distinct mathematical challenges featuring **differential equations**, **machine learning**, **geometry**, **data**, **optimisation**. After reading broadly about the topic, develop mathematical questions around the areas that interest you and their connections. Possibilities include:

- Mathematics of traffic jams, including so-called phantom jams. Here you can examine continuous PDE models or discrete models, or both. Nonlinearity will be important. You could explore the connection to fluid flow and to statistical phase transitions. You can also consider real-time control strategies.
- A topic which bridges the two main area is understanding the impacts of connected and automated vehicles on the traffic flow.
- Autonomous vehicle design incorporates hardware, software as well as mathematical analysis. 3D sensing and modelling, and deep learning are potential areas with good mathematical content.

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