Recommended Syllabus

This is the recommended syllabus for the module detailed below. The module should contain all the topics listed below in some form, but be aware that there may be additional material covered that can also be examined.

MA132 Foundations

1. Number Systems

   Number systems: Natural numbers, integers. Rationals and real numbers. Existence of irrational numbers. Complex Numbers.

   Polar and exponential form of complex numbers. De Moivre's Theorem, n'th roots and roots of unity.

   Euclidean algorithm; greatest common divisor and least common multiple.

   Prime numbers, existence and uniqueness of prime factorisation. Infiniteness of the set of primes.

   Modular arithmetic. Congruence, addition and multiplication modulo n.

2. Language and Proof

   Proof by induction. Well-ordering Principle.

   Proof by contradiction.

   Basic set theory: $\cap, \cup$, Venn diagrams and de Morgan's Laws. Cartesian product of sets, power set.

   Logical connectives $\wedge$, $\vee$, $\Rightarrow$ and their relation with $\cap$, $\cup$ and $\subseteq$. Quantifiers $\forall$ and $\exists$.

3. Sets, functions and relations

   Injective, surjective and bijective functions. Inverse functions.

   Relations: equivalence relations, order relations.

4. Polynomials

   Multiplication and long division of polynomials.

   Euclidean algorithm for polynomials.

   Remainder theorem; a degree n polynomial has at most n roots.

   Algebraic and transcendental numbers. Fundamental theorem of Algebra (statement only).

5. Counting

   Cardinalities, including infinite cardinalities.

   Cardinality of the power set of X is greater than cardinality of X. Russell's paradox.

   Countability of the rational numbers, uncountability of the reals.

   Transcendental numbers exist!

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