Recommended Syllabus

This is the recommended syllabus for the module detailed below. The module should contain all the topics listed below in some form, but be aware that there may be additional material covered that can also be examined.

MA106 Linear Algebra

Compulsory part of syllabus

1. The vector space $\mathbb{R}^n$, including a geometric description of vector addition in $\mathbb{R}^2$.

2. Fields. Definition of a vector space $\mathbf{V}$ over a field. The space spanned by a subset of $\mathbf{V}$. Linear dependence and independence. Bases. Dimension. Subspaces. Dual spaces and dual bases.

3. Linear maps $f : \mathbf{V} \to \mathbf{W}$. Isomorphism of vector spaces. Any n-dimensional vector space over $\mathbb{F}$ is isomorphic to $\mathbb{R}^n$. Examples of linear maps, including differentiation and integration as linear maps on spaces of functions or polynomials.

4. Matrices. Algebraic operations on matrices. Reduction of a matrix using row and column operations. Application to the solution of linear equations. Rank. Row rank = Column rank.

5. The relation between linear maps and matrices. the matrix of a linear map with respect to a given basis. Change of basis changes A to PAQ^{-1}. The kernal and image of $f : \mathbf{V} \to \mathbf{W}$. The rank and nullity of f.

6. Determinants, defined by $\sum_{\sigma \in S_n} \operatorname{sign} \sigma \left(\prod a_{i,\sigma(i)} \right)$. Det(AB) = Det(A)Det(B) (proof either in general or in the cases $n = 1, 2, 3$). Submatrices, minors, cofactors, the adjoint matrix. Rules for calculating determinants. The inverse of a matrix. Ax=0 has non-zero solution if and only if det(A)=0. Determinantal rank.

7. Eigenvalues and eigenvectors. Definition and examples. Their geometric significance. Diagonalisation of matrices with distinct eigenvalues.

8. Inner product spaces and isometries. Euclidean spaces. Orthogonal transformations and matrices.

Last updated 5th January 2018