

# Recommended Syllabus

This is the recommended syllabus for the module detailed below. The module should contain all the topics listed below in some form, but be aware that there may be additional material covered that can also be examined.

## MA131 Analysis I

1. Inequalities: rules for manipulating inequalities; inequalities and powers; inequalities and absolute values; Bernoulli's inequality; triangle inequality.
2. Sequences: monotonic sequences; bounded sequences; subsequences; tending to infinity; null sequences (and algebra of); convergent sequences (and algebra of, boundedness, uniqueness of limit); sandwich theorems; shift rules; standard limits; ratio test; limits and inequalities; recursively-defined sequences.
3. The real numbers: infinitely many rationals/irrationals in any open interval; rationals/irrationals and terminating/recurring/non-recurring decimals; numbers with more than one decimal representation; sets and upper/lower bounds; supremum and infimum; completeness axiom (in the form "every non-empty set bounded above has a supremum"); consequences of completeness; existence of  $k$ th roots; Bolzano-Weierstrass theorem; Cauchy sequences (contracting sequences as example).
4. Series: partial sums; convergence and divergence; sum rule; shift rule; null sequence test, epsilon as a series and the limit of a sequence.
  - (a) Series with positive terms: boundedness condition, comparison tests, ratio test, integral test.
  - (b) Alternating series: alternating series test, absolute convergence, absolute value form of ratio test, conditional convergence, rearrangements.

*Last updated 12th November 2008*

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## MA131 Analysis II

### 1. Continuity

Definition of sequential continuity A function sequentially discontinuous everywhere A function sequentially continuous at just one point Definition of continuity Equivalence of continuity and sequential continuity Sums, products and quotients of continuous functions Continuity of composition Continuity of standard functions

### 2. Properties of continuous functions

The Intermediate Value Theorem Solving equations Fixed point theorem The Inverse Function Theorem The Max-Min Existence Theorem

### 3. Continuous Limits

Definition of continuous limit Uniqueness of the value of a limit Sequential definition of a continuous limit Squeeze rule for limits of functions Definition of one-sided continuous limit Infinite limits and limits at infinity

### 4. Differentiability

Definitions of differentiability Differentiability implies continuity Continuity does not imply differentiability A function that is only differentiable and continuous at one point Sums, products and quotients The chain rule The inverse rule

### 5. Properties of differentiable functions

Derivative is zero at max and min Rolle's Theorem The Mean Value Theorem Non-zero derivative implies strictly monotonic Inverse function theorem for differentiable functions

### 6. Higher order derivatives

Definition of higher order derivatives Definition of  $C^k$  space Definition of derivatives on closed intervals

### 7. Power Series

Definition of radius of convergence Existence of radius of convergence Formula for the radius of convergence Ratio test and root test for power series Differentiation of power series

### 8. Taylor's Theorem

Taylor's Theorem Cauchy's version of Taylor's Theorem Estimating errors L'Hopital's Rule The Binomial series

9. The Classical Functions of Analysis

The exponential function The logarithm Sine and Cosine

10. Upper and Lower Limits

Limsup and liminf Cauchy's Test

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