

ALGEBRA IN CONTEXT

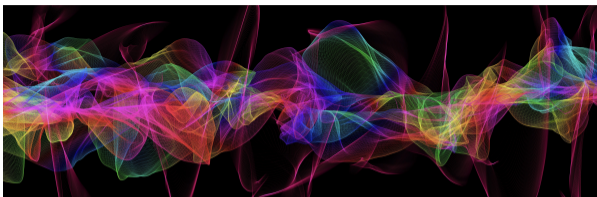
OR WHAT I DID ON MY HOLIDAYS

Nicholas Jackson

8 May 2026



It is September 1997/2002/2014/2025 and I am writing a postgraduate dissertation...



- A 30-CATS Postgraduate Award run by Jo Kukuczka from the Academic Development Centre, open to teaching staff at the University of Warwick.
- Taught component: Four seminars (February to July):
 - 1 Curriculum Design as a Social Practice
 - 2 Re-Imaging Curriculum
 - 3 Crafting Curriculum
 - 4 Critiquing Curriculum
- Project: Critically evaluate these techniques and use them to design a two-module mini-programme, explaining your design choices.

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- Viewpoints: “product” vs “process” vs “praxis”
- **Product**: Curriculum is a product to be delivered to students, with focus on content and meeting pre-set objectives
- **Process**: Curriculum as the interaction of teachers, students and knowledge, with focus on enabling learning and experimentation
- **Praxis**: Curriculum as a vehicle for human emancipation and egalitarian social change, with focus on raising students’ awareness of inequality through critical pedagogy

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- Question: How do we do this in maths?

CONNECTED CURRICULUM FRAMEWORK

- Dilly Fung, *A Connected Curriculum for Higher Education* (2017)
- Six 'dimensions':
 - ① Students connect with researchers and with the institution's research.
 - ② A through-line of research activity is built into each programme.
 - ③ Students make connections across subjects and out to the world.
 - ④ Students connect academic learning with skills for the workplaces.
 - ⑤ Students learn to produce outputs – assessments directed at an audience.
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UDL (UNIVERSAL DESIGN FOR LEARNING)

- Student-centred, build in support for those at the margins
- What is necessary for some usually benefits everyone
- Reframe barriers from student problems to environmental limitations

MY THEORY THAT I HAVE, WHICH IS MINE

My project:

- *Algebra in Context*
- Based on *MA267 Groups and Rings*, a 15 CATS, self-contained abstract algebra module for second year joint degree maths students (term 1). This evolved from *MA249 Algebra II*, sister module to *MA268 Algebra 3*
- Two 15-CATS modules:
 - 1 *MA2xx Algebra in Context 1: Groups* (term 1)
 - 2 *MA2yy Algebra in Context 2: Rings and Algebras* (term 2)
- Standard treatment of second year algebra topics (groups, rings and fields) interlaced with seminar-style discussions of interdisciplinary applications and the historical context and development of the subject.
- Assessments: traditional problem sets, mini-essays, STACK quizzes, timed final exam

BLOOMING TAXONOMY

Standard framework for classifying learning objectives and activities:

BLOOM'S TAXONOMY

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2 Comprehension

3 Application

4 Analysis

5 Synthesis

6 Evaluation

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Maths-specific variant framework due to Smith et al (1996)

MATHEMATICAL ASSESSMENT TASK HIERARCHY (MATH)

- GROUP A Factual knowledge and routine procedures
- GROUP B Using existing mathematical knowledge in new ways
- GROUP C Evaluation, justifying and interpreting

LEARNING OBJECTIVES

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- 3 Use existing concepts and theorems to construct illustrative examples and counterexamples, and rigorously prove further results about algebraic structures such as groups, rings and algebras. (Bloom 5; MATH C)
- 4 Critically discuss the historical and wider scientific and social context of algebraic structures such as groups, rings and algebras in mathematics and other disciplines. (Bloom 6; MATH C)

ASSESSMENT

- STACK quizzes ($5 \times 2\% = 10\%$): Randomly-generated, automatically marked, retakeable online quizzes with an 80% pass mark. Full (2%) credit for a pass, zero otherwise. (mostly formative)

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| ILO | Quizzes | Essays | Problems | Exam |
|-----|---------|--------|----------|------|
| 1 | Y | N | Y | Y |
| 2 | N | Y | Y | Y |
| 3 | N | N | Y | Y |
| 4 | N | Y | N | Y |

- Contact hours:
 - Two 1-hour lectures, covering mathematical content
 - One 1-hour seminar-style discussion, covering history and interdisciplinary applications
 - Optional support classes/office hours
- Semi-flipped model with supplementary reading on both mathematical content and class discussion topics

SYLLABUS: ALGEBRA IN CONTEXT 1: GROUPS

| Week | Topic | Theory | Context | Assessment |
|------|--------------|---|--|--------------------|
| 1 | Groups | Definitions, basic properties | Early/mediæval history | |
| 2 | Symmetry | Isomorphism, cyclic, dihedral and matrix groups | Symmetry in chemistry | Quiz 1 |
| 3 | Permutations | Permutations, subgroups, Cayley's Theorem | Abstract group concept, the 19th century, bell-ringing | Problems 1 |
| 4 | Subgroups | Cosets, Lagrange's Theorem Cauchy's Theorem | Number Theory | Essay 1, Quiz 2 |
| 5 | Quotients | Normal subgroups, quotients, direct products | Conjugation in linear algebra, Rubik's Cube | Problems 2 |

SYLLABUS: ALGEBRA IN CONTEXT 1: GROUPS

| Week | Topic | Theory | Context | Assessment |
|------|-----------------|--|--|-----------------|
| 6 | Homomorphisms | Kernels and images, Noether's Isomorphism Theorems | Women in maths, Germany, Noether's Theorem in physics | Quiz 3 |
| 7 | Classification | Presentations, small finite groups, abelian groups | Classification of Finite Simple Groups | Problems 3 |
| 8 | Actions | Groups acting on sets, orbits and stabilisers, conjugacy | Symmetry in music and art | Essay 3, Quiz 4 |
| 9 | Finite groups | Conjugacy in S_n and A_n , Sylow's Theorems, classification up to order 31 | Combinatorics and counting (Not) Burnside's Lemma, Polya's Enumeration Theorem | Problems 4 |
| 10 | Representations | Linear representations, characters | Particle physics | Essay 4, Quiz 5 |

SYLLABUS: ALGEBRA IN CONTEXT 2: RINGS AND ALGEBRAS

| Week | Topic | Theory | Context | Assessment |
|------|------------------|---|---|--------------------|
| 1 | Rings | Definitions, examples, Sun Tzu's Remainder Theorem | History: China and India | |
| 2 | Integral Domains | Zero divisors, units, division rings, fields | Cryptography | Quiz 1 |
| 3 | Quotients | Homomorphisms, ideals, quo- tients, Isomorphism Theorems | Error-correcting codes, information theory | Problems 1 |
| 4 | Divisibility | Prime and irreducible elements | CRC checksums | Essay 1, Quiz 2 |
| 5 | Domains | Euclidean domains, PIDs and UFDs | Algebraic Number Theory, quaternions and octonions | Problems 2 |

SYLLABUS: ALGEBRA IN CONTEXT 2: RINGS AND ALGEBRAS

| Week | Topic | Theory | Context | Assessment |
|------|------------------|--|---|-----------------|
| 6 | Polynomials | Irreducible polynomials, roots of unity | History of solving equations, Sumeria to the Renaissance | Quiz 3 |
| 7 | Field Extensions | Tower Law, splitting fields, the Galois group | Finite fields | Problems 3 |
| 8 | Galois Theory | The Galois Correspondence, solving equations by radicals | Geometric constructions: circle-squaring, angle trisection, cube doubling | Essay 3, Quiz 4 |
| 9 | Algebras | Vector spaces, algebras, modules | Lie algebras in particle physics | Problems 4 |
| 10 | Categories | Categories, objects, morphisms and functors | Algebraic topology, computer science | Essay 4, Quiz 5 |

Some historical topics in algebra:

- Sumeria (cuniefom tablets c.1500–2000 BCE, eg Plimpton 322)
- Egypt (Rhind papyrus c.1500 BCE)
- Greece (c.600–200 BCE)
- China (c.300 BCE onwards, eg *Nine Chapters on the Mathematical Art*, Sun Zi's Remainder Theorem)
- India (c.400 CE onwards, eg Aryabhata, Brahmagupta, Bhaskara II)
- Arabia (c.800–1200 CE, eg Al-Khwarizmi, Omar Khayyam)
- Women in mathematics (eg Noether's Isomorphism Theorems)

DESIGN MANIFESTO

- Nothing is apolitical: the status quo is a choice, and in some cases this status quo dates back centuries
- Adopt a “praxis” style approach to curriculum: “[to help] these young fellas be the best versions of themselves”
- Discuss the social and historical context, and decolonise the subject by considering non-western contributions (eg China, India, Egypt, Sumeria, ...)
- Use of examples (qv LCT concept of a “semantic wave”) to help embed theoretical concepts – anti-Bourbakist approach
- Traditionally pure maths has avoided discussion of applications (qv Hardy and Littlewood) but we actively want to examine interdisciplinary connections
- Avoid the “great man” model of history: “we are all standing on the shoulders of giants”.

I LIKE YOUR MANIFESTO, PUT IT TO THE TEST-OH!

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