The Rhind Papyrus

APPROXIMATE DATE

1550 BCE

ORIGINAL AUTHOR(S)

Unknown

PLACE OF ORIGIN

Egypt (Thebes)

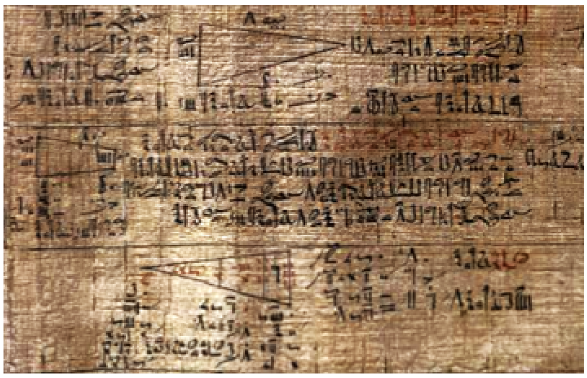
Historical Context

The Rhind Papyrus is a practical compendium on how to solve common mathematical problems in Egyptian society. It would have been used as a sort of textbook to train scribes, who were expected to be mathematically literate. This is reflected in it's original title - 'The correct method of reckoning for grasping the meaning of things and knowing everything, obscurities and all secrets' [1]. Nowadays, we refer to it as the Rhind Papyrus, named after the Scottish lawyer who purchased it in 1858 in Thebes, where it was discovered [2].

A scribe named Ahmes produced a copy of the papyrus, dated 1650 BCE, so the text is sometimes also referred to as the Ahmes Papyrus [3]. Although it is from ancient Egypt, it is not written in heiroglyphs, rather in a form of shorthand. It's place of discovery is widely believed to have been a tomb, possibly so the owner could carry their credentials with them to the afterlife. Papyrus is a material that rots and burns easily, so there are few documents remaining from that period [1], making the Rhind papyrus hugely important!

Content Overview

The enitre paprus is in 3 pieces, 2 of which are stored in the British Museum. Orignally the entire document would have been written on a scroll roughly 17ft (5.18m) long, so it won't be surprising that it is the largest know mathematical text anywhere in the ancient world [1]. Similar to the Nine Chapters on the Mathematical Art, it revolves around a set of 84 problems, which are each solved in a manner that provides a general solution for such problems. The actual mathematics used would equate to roughly GCSE level in today's terms. There is a focus on arithmetic, algebra and geometry, as these areas were the most useful for daily calculations such as taxes, exchanging goods, flood levels of the Nile and managing building works.



The papyrus is compactly organised and was used as a 'cheat sheet' for 550 BCE administrative exams. The text written in red ink are section titles [1]. Image source: The British Museum

Highlight topics

- Egytpian fractions
- Areas of shapes
- Approximation of pi

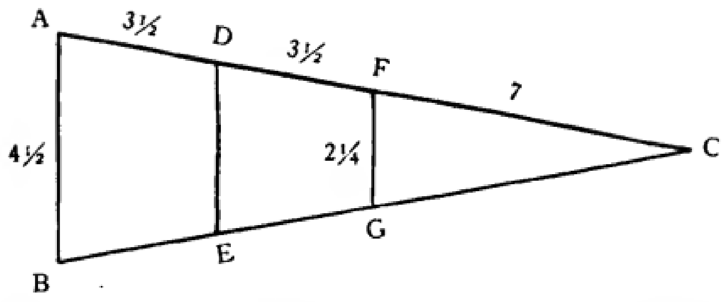
Egyptian fractions

To fully understand how the problems in the Rhind Papyrus are phrased, it is important to note how the Ancient Egyptians dealt with fractional quantities. Rather than having the numerator of the fraction be any number, they exclusively used ‘unit fractions’ (i.e fractions with a numerator of 1). Any fraction is then written as a sum of these unit fractions. This can be done in multiple different ways however, any one unit fraction may only be used once within such a sum. [4]

The Rhind Papyrus contains a table of unit fraction decompositions for fractions of the form 2/n, for odd n up to 101 [4]. This is called a 2/n table.

Areas of shapes

The Papyrus’ second chapter is on geomtry. It covers basic volumes and areas such as the volume of cylindrical and rectangular granaries, comparing the area of a circle with its circumscribing square and the area of ‘truncated triangles’ [5].



Here, a ‘truncated triangle’ really just refers to an isosceles triangle that has been sectioned in the above manner [5]. Image source: Mathematical Association of America

Pictured to the left is the triangle from problem 53 of the Rhind Papyrus, in which the objective is to calculate the area of each section of the triangle. The solution given in the Papyrus actually contains some numerical errors, however nowadays this can easily be solved by identifying that ABC, DEC and FGC are similar triangles.

Problems 45 and 46 bear similarity to Chapter 4 of the Jiuzhang Suanshu - Shao Guang - where the reader is instructed to calculate a missing dimension of a rectaungular granary, given the volume of grain it contains. [5]

An Egyptian Approximation of Pi

By examining the steps given in the Rhind Papyrus on how to calculate the area of a circle, we can determine what approximation of pi would have been used by the Ancient Egyptians. Below is problem 50 from the Papyrus; to make sense of the units, 1 khet is a length of 100 cubits (52.5m) and 1 setat is equal to 1 square khet [6].

“Example of a roundfield of diameter 9 khet. What is its area? Take away 1/9 of the diameter, namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. Therefore it contains 64 setat of land.” [5]

More generally, let us take the diameter as $2r$, with r being the radius of the circle. Following this method we get the expression for the area of a circle radius r as $\frac{256}{81}r^2$. So we can see that the Ancient Egyptians used pi to be $\frac{256}{81}$, roughly 3.16.

References

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九章算术

The Nine Chapters on the Mathematical Art (Jiuzhang Suanshu)

APPROXIMATE DATE

200 BCE

ORIGINAL AUTHOR(S)

Unknown

PLACE OF ORIGIN

China

★★★★★

Historical Context

A Unlike Euclid's Elements, to which it is often compared, the Nine Chapters is less concerned with proof and more focused on how to solve practical problems. This is due to the fact that while slaves were used in ancient Greece, in China the Zhou Dynasty, which ended in 256 BCE, introduced a system of governance similar to feudalism [1]. Farming was a large part of this system, hence many mathematical problems to do with area, proportion and solving linear equations became important to understand for the majority of the population. However in Greece mathematics was only accessible to the elite, meaning there was less necessity to solve real-life problems and more room for a philosophical approach to mathematics. 'Suanshu', which appears in the title, literally means 'the art of calculation'. Alongside being a mathematical aid in agriculture, it was also used to train imperial servants. Current verisons of the book are based on a 263 CE edition and commentary by mathematician Liu Hui [2].

Content Overview

The book is structured around a set of 246 practical problems, which are solved in turn and then the methods of solution are explored and given in general terms [2]. Each of the nine chapters focusses on a different area of problem solving that would be useful in day-to-day life as a farmer, citizen or member of the civil service.

1. Fang Tian - calculating areas of shapes commonly used for farming.
2. Su Mi (Millet and Rice) - ratio, proportion and exchanging goods
3. Cui Fen - distribution by proportion; useful for tax paying, and arithmetic and geometric progressions
4. Shao Guang- calculating unknown dimensions from a known volume or area
5. Shang Gong - area and volume of shapes used in architecture
6. Jun Shu (Fair levies) - wages and tax (a continuation of chapter 3)
7. Ying Bu Zu - solving linear equations algorithmically, not with algebra
8. Fang Cheng (rectangular arrays - using matrices to solve linear equations
9. Gou Gu (Right Triangles) - solving geometric problems using right triangles

[3]

We now have a copy of the Nine Chapters in the university library!

Highlight topics

Ying Bu Zu

Ying Bu Zu, which translates literally to 'too much and not enough' is an algorithmic method of solving linear equations. Nowadays in modern mathematics, one may immediately jump to using algebra, however algebra as we know it originated in Baghdad in c.820 CE, with Al-Khwarizmi's book 'Kitab al-Jabr wa-l-Muqabala' [4]. The algorithm is based on the manipulation and treatment of fractions, namely the processes of creating a common denominator (tong) and cross multiplying (qi) [5].

Let's examine this problem, from chapter 7 :
“An item is purchased jointly; everyone contributes 8 [coins], the excess is 3; everyone contributes 7, the deficit is 4. Tell: The number of people, the item price, what is each?” [5]

So we have that 8 coins per person gives us 1 item and 3 coins excess, and that 7 coins per person plus an additional 4 coins gives us 1 item exactly. We begin by using qi to cross-multiply the excess with the defecit; multiply the quantities in the first statement by 4 and the second by 3. So, 32 coins per person gives us 4 items and an excess of 12 coins, and 21 coins per person plus an additional 12 coins gives us 3 items. Adding these new equations, we can see that our excess and defecit cancel out:

$$32 + 21\ coins = 7\ items \qquad \longrightarrow \qquad 1\ item = \frac{53}{7}\ coins$$

Now, we can clearly see that to buy 1 item requires 7 people to contribute 53 coins each.

Fang Cheng

Fang cheng in modern day China is taken to mean equations or functions, however it directly translates to 'rectangular arrays', which is how it is used within the Nine Chapters. In other words - matrices! In chapter 8 we are introduced to a method of solving a 3x3 system of linear equations, which you will recognise as being incredibly similar to Gaussian elimination, despite predating it by over 1500 years! [6]

Example from chapter 8:
“Now there are 3 bundles of top grade cereal, 2 bundles of medium grade cereal and 1 bundle of low grade cereal, [which yield] 39 dou [of grains] as shi; 2 bundles of top grade cereal, 3 bundles of medium grade cereal and 1 bundle of low grade cereal [yield] 34 dou as shi; 1 bundle of top grade cereal, 2 bundles of medium grade cereal and 3 bundles of low grade cereal [yield] 26 dou as shi. Find the measure [of grains] in each bundle of the top, medium and low grade cereals.” [3]

The solution provides instructs the reader to create a 4x3 rectangular array and use the operations of scalar multiplication and zhi chu, or 'direct subtraction', on the columns until the array is in triangular form [6]. After this is achieved the desired quantities are easily found by substitution.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \\ 26 & 34 & 39 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 6 & 3 \\ 2 & 9 & 2 \\ 3 & 3 & 1 \\ 26 & 102 & 39 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 5 & 2 \\ 3 & 1 & 1 \\ 26 & 24 & 39 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 0 & 3 \\ 6 & 5 & 2 \\ 9 & 1 & 1 \\ 78 & 24 & 39 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 3 \\ 4 & 5 & 2 \\ 8 & 1 & 1 \\ 39 & 24 & 39 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 3 \\ 20 & 5 & 2 \\ 40 & 1 & 1 \\ 195 & 24 & 39 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 3 \\ 0 & 5 & 2 \\ 36 & 1 & 1 \\ 99 & 24 & 39 \end{bmatrix}$$

So, 1 bundle of low grade is $2\frac{3}{4}$ dou.

References

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[4] Garge, S. (2022), 'The Compendious Book on Calculation by Completion and Balancing', Available at: https://alchetron.com/The-Compendious-Book-on-Calculation-by-Completion-and-Balancing

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The edition of the Nine Chapters we have in the library is:
Shen, K. (1999) *The Nine Chapters on the Mathematical Art : Companion and Commentary*. Translated by Crossley, J. and Lun, A. Oxford: Oxford University Press

Make sure to go and have a look!

كتاب الجبر و المقابلة

The Kitab al-Jabr wa-l-Muqabala

APPROXIMATE DATE
820CE

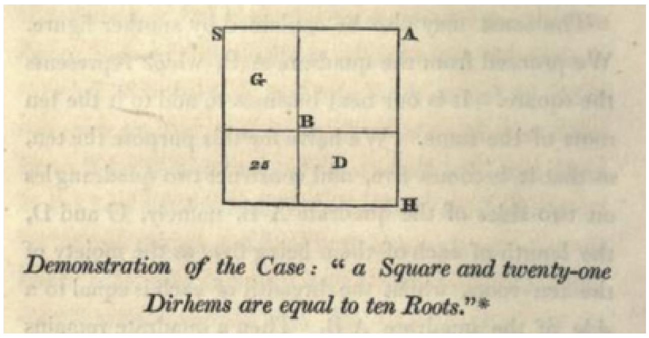
ORIGINAL AUTHOR(S)
Muhammad Al-Khwarizmi

PLACE OF ORIGIN
Baghdad

★★★★★

Historical Context

Al-Khwarizmi is widely regarded as the father of algebra. In fact, the Latin translation of his name is where we get the word 'algorithm' from. He worked in Baghdad as a scholar for "House of Wisdom" (Dār al-Ḥikma) [1]. This was a vast library and place of intelligence, supported by the Islamic government. The Kitab al-Jabr is probably his most famous work, as it is the origin of the algebra that we now learn at school. Specifically, 'algebra' is derived from 'al-Jabr' (الجبر), which translates literally as forcing or restoring [2]. Although it is known as a guide to solve equations, the original Arabic script has no equations, instead it is written entirely in prose [3].



Geometry was a large part of Islamic culture, not only in their art, but as a useful tool to understand the world. Al-Kwarizmi uses meany geometric proofs in the Kitab al-Jabr [4]. Image source: University of California (J. L. Cox)

Content Overview

The Kitab al-Jabr is most well known for introducing algebraic methods of solving equations. Like the Rhind Papyrus and the Nine Chapters, the Kitab al-Jabr explores problems that would be useful for everyday citizens, not just academic scholars. The equations explored in the book often relate to practical scenarios, for example distributing inheritance [4].

Al-Khwarizmi also dealt with quadratic equations, which he classified into 6 categories, with the aim of avoiding negative numbers [5]. Since the book is entirely in prose, each element of a quadratic equation is referred to by a name:

- regular numbers, represented in the left as c, are called 'dirhems'
- x is a 'root'
- x^2 is a 'square' [4]

The 6 categories of quadratics

$$bx = c \qquad ax^2 + c = bx$$

$$ax^2 = c \qquad ax^2 + bx = c$$

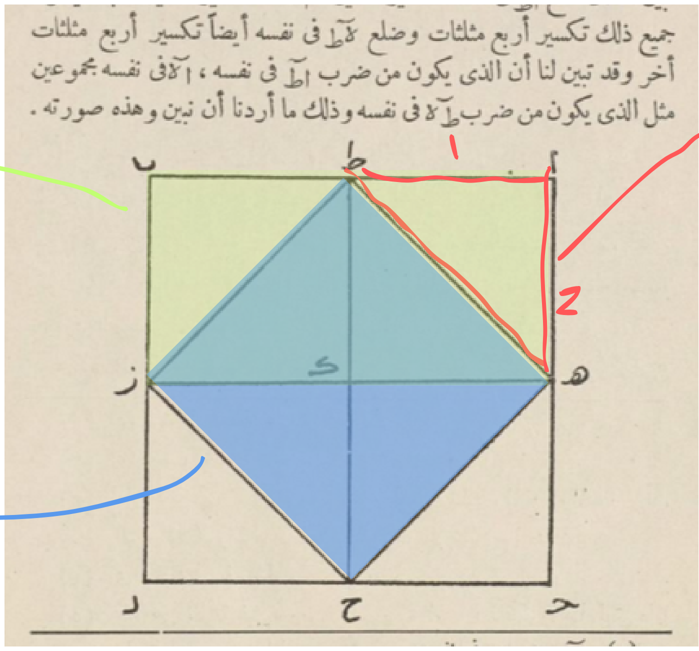
$$ax^2 = bx \qquad bx + c = ax^2$$

Highlight topics

A geometic proof

The green area represents the sum of the squares of side 1 and side 2.

The Blue area represents the square of the hypotenuse. By dividing each smaller square diagonally into 2, we can clearly see that both areas are equal in size.



Al-Khwarizmi's proof that "...in every rectangular triangle the two short sides, each multiplied by itself and the products added together, equal the product of the long side multiplied by itself" [4]. Image source: Columbia University Library

The rectangular (right-angled) triangle. In this case, the geometric proof only works if it is an isosceles right-angled triangle.

We know from Pythagoras' theorem that this fact is true for all right-angled triangles, however a geometric proof for a non-isosceles right triangle would be a lot more complicated!

Solving an $'ax^2 + bx = c'$ quadratic

Consider this problem from the Kitab al-Jabr:
"I have multiplied one- third of thing and one dirhem by one-fourth of thing and one dirhem, and the product was twenty." [4]

The reader is first instructed on how to multiply out the brackets: *"You multiply one- third of thing by one- fourth of thing; it is one-half of a sixth of a square. Further, you multiply one dirhem by one-third of thing, it is one- third of thing ; and one dirhem by one-fourth of thing, it is one-fourth of thing ; and one dirhem by one dirhem, it is one dirhem. "* [4]

We then use the action of Al-Muqabala (المقابلة), which translates as balancing or corresponding [2]. This is the action of subtracting the same positive number from both sides of the equation.

Finally, the book describes an algorithm to find the roots of the equation. Once all the coefficents are positive whole numbers, we "*Halve the number of the roots, and multiply it by itself ... Add this to the numbers, that is, to two hundred and twenty-eight ... Extract the root of this ... Subtract from this the moiety of the roots" [4]*. The word 'moeity' means half and to 'extract the root' means to take a square root. This is essentially a more specialised version of the quadratic formula that doesn't deal with negatives.

$$(\frac{1}{3}x+1)(\frac{1}{4}x+1) = 20$$

$$\frac{1}{12}x^2 + \frac{1}{3}x + \frac{1}{4}x + 1 = 20$$

-1 -1

$$\frac{1}{12}x^2 + \frac{7}{12}x = 19$$

$$x^2 + 7x = 228$$

$$x = \sqrt{\frac{49}{4} + 228} - \frac{7}{2} = 12$$

References

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解見題之法

The Kai kendai no ho

APPROXIMATE DATE

1683 CE

PLACE OF ORIGIN

Japan

ORIGINAL AUTHOR(S)

Seki Takakazu

★★★★★

An ink portrait of Seki Takakazu. Image source: Japan Academy Archives in Tokyo

Historical Context

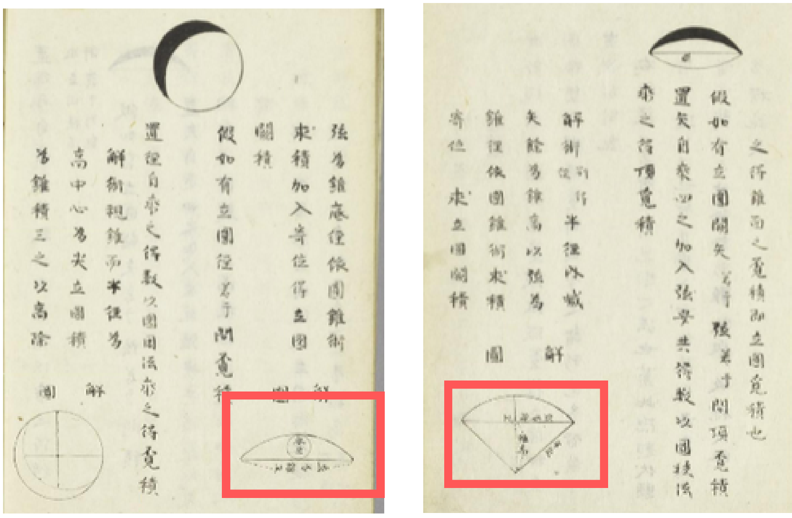
Chinese mathemtics had been used in Japan from the 7th century CE, including the Nine Chapters [1]. During the Tokugawa period (1603–1867) in Japan, citizens had restricted contact with foreigners, limited to interactions with Chinese and Dutch trade ships in Nagasaki. This meant books containing Chinese, and possibly some Western, mathematical ideas had the opportunity to secretly be passed into Japan [2].

Seki Takakazu (1642-1708) was a mathematical prodigy from a young age, and is considered the founder of the Japanese tradition of mathematics (Wasan) [2]. He created a more efficient tabular system of notation, which replaced the traditional method of using Chinese coutning rods, such as was used in the Nine Chapters [1].

Seki's reknown can be summed up by the title engraved on his tombstone - 'The Arithmetical Sage'. He is also known to have pre-discovered many ideas from Western mathematics: he was the first person to study determinants and discovered Bernoulli numbers before Bernoulli [3].

The Kai kendai no ho is far more advanced than the other 3 books explored within this project, requiring an understanding of differential geometry. It is part of a trio of books referred to as the Sanbu Sho (三部書), which is considered one of Seki's major works [4]. Kai kendai no ho translates as 'Methods of solving explicit problems'. The Kai indai no ho - 'Methods of solving implicit problems' - is also part of the Sanbu Sho, along with the Kai fukudai no ho, in which Seki demonstrates methods of calculating determinants [5].

Content overview



These pages of the Kaiken dai no ho, show Seki's calculations for the volume of a spherical cap. Image source: National Diet Library

Whilst the other books in this project have a heavy focus on the use of maths for solving practical, everyday problems, the Kai kendai no ho is solely a theoretical work. This book specifically explores geometrical problems, including a particularly difficult one surrounding a spiral-like curved line called Wanse, or an Archimedian spiral [4]. Seki also demonstrates how to calculate the surface area of a sphere and the perimeter of an ellipse.

References

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Thank you to Dr Weiyi Zhang for his help unravelling the calculations from the Kai kendai no ho.