On The Bounce

Mathematical Modelling of a Bouncing Fluid Droplet

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Introduction

The problem of droplet impacts has been studied for over 100 years, with Lord Rayleigh (1876) [1] being one of the first, examining capillary jet phenomena. As technology developed, better experiments could be performed, such as those by Jayaratne and Mason (1964) [2], which was one of the first papers to show the phenomenon of droplets bouncing on baths of the same fluid. Their experiments involved firing a stream of near uniform droplets at a still bath with varying incident angles, but never normal as otherwise incoming drops would collide with outgoing drops. They found that for different drop sizes, charges, and impact velocities and angles, coalescence could be observed instead of bouncing.

Since then, the field has continued to grow, partly due to further technological advances, such as those allowing for single droplets to be produced. An important discovery by Couder et al. (2005) [3] showed that if the bath was oscillated at certain frequencies and amplitudes, then bouncing of the droplet could be sustained for longer periods of time, on the order of multiple days. This discovery gave way to models for solid spheres as well as fluid droplets impacting fluid baths, such as Blanchette (2016) [4], being produced.

Here, we examine the case of a fluid droplet impacting a solid surface. Prior works, like Okumura et al. (2003) [5], give simple models for the droplet motion and deformation as well as scaling laws for the contact time.

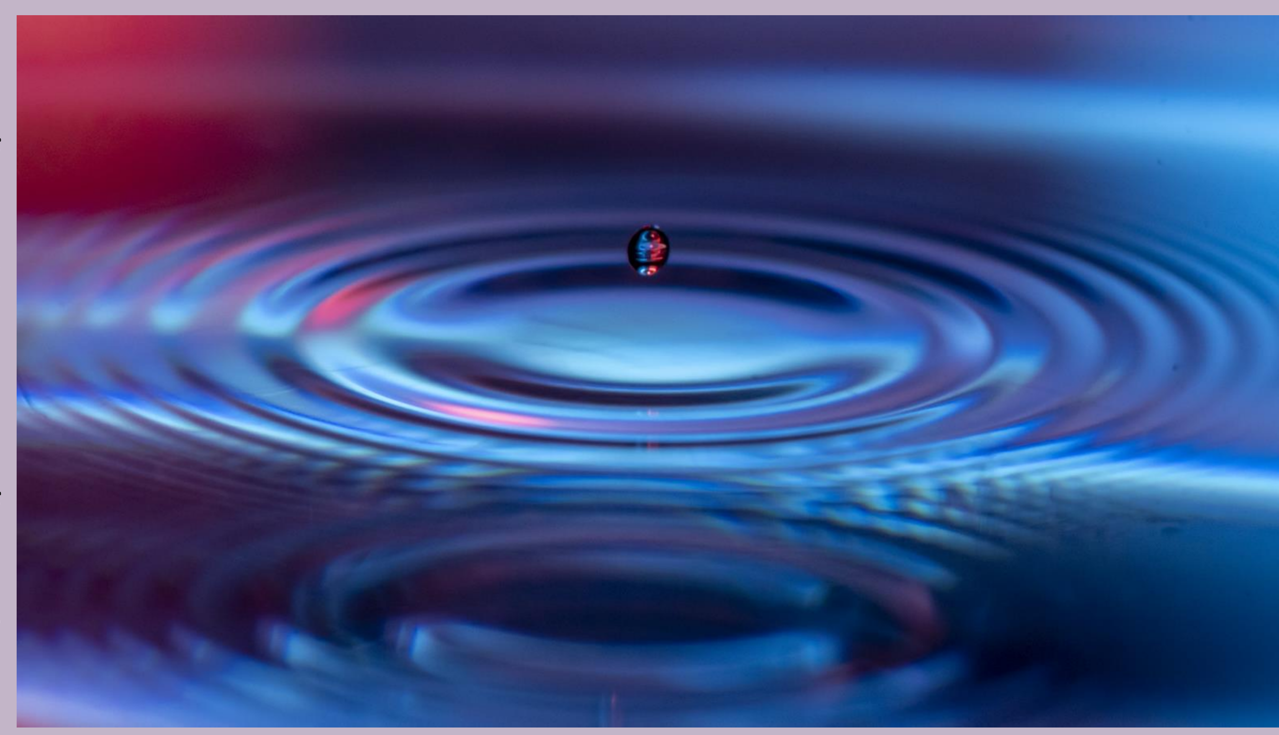


Figure 1 of Alventosa, L., Cimpeanu, R., & Harris, D. 2023. Inertio-capillary rebound of a droplet impacting a fluid bath. Journal of Fluid Mechanics, 958, A24.

Model

To model this problem, we consider two cases, the first where the **droplet falls under gravity** and experiences drag, and the second where the **droplet is in contact with the surface** and experiences an upward contact force. We base the model on that of Blanchette's paper [4], modifying this to account for a solid surface instead of a pool. To simplify, we **nondimensionalise** with length scale, R, being the initial droplet radius and time scale being $\mathrm{T} = \sqrt{\rho_d R^3/\sigma}$, where p_d is the droplet density and σ is the surface tension between the droplet and air. The nondimensional equations for the centre of mass position, z, and vertical radius, r_v, in time, t, are

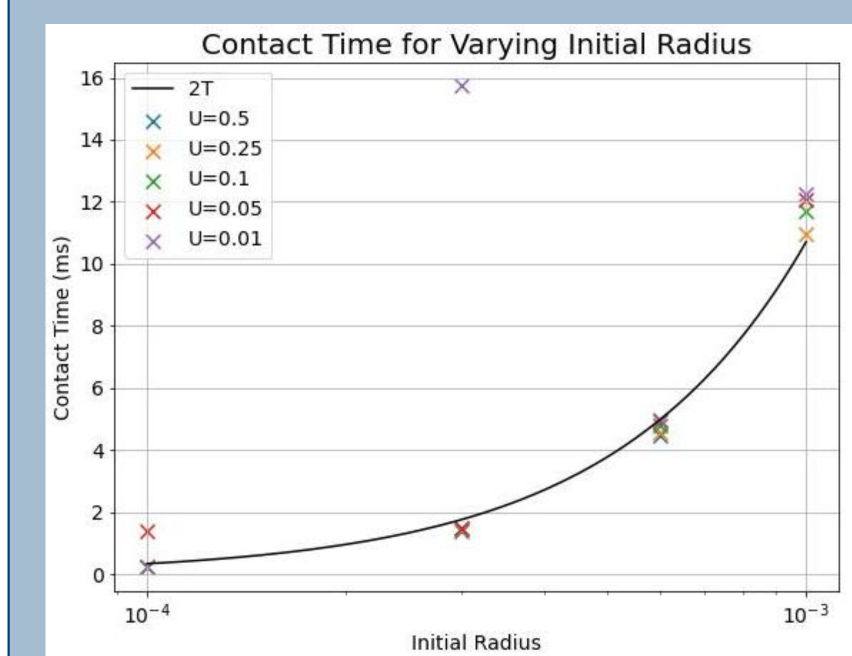
$$\frac{d^2 z}{dt^2} + \frac{9\mu_a T}{2R^2\rho_d}\left(1 + \frac{Re_a}{6}\right)\frac{dz}{dt} = -Bo_d + f,$$

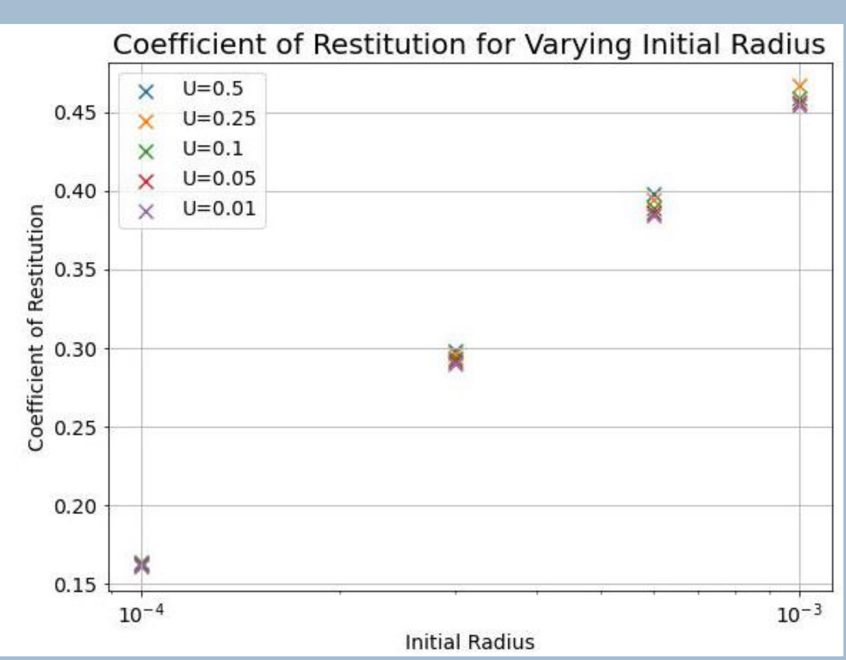
$$\frac{d^2 r_v}{dt^2} + 3.8Oh_d\frac{dr_v}{dt} - 5.84(r_v - 1) = -f,$$

where μ_a is the air viscosity, Re_a is the Reynolds number for air, Bo_d and Oh_d are the Bond and Ohnesorge numbers of the drop, and f is the contact force. We impose the condition $z - r_v = 0$ during contact to determine the contact force, as in [4]. Since this is a two-dimensional system of ODEs, **we can use standard ODE solvers in Python to solve these numerically** and plot results.

Results

Using this model, and solving in Python, we can determine certain quantities from the data. We look at the **contact time** for the bounce (determined by the time for which $z - r_v \leq 0$) and the **coefficient of restitution** (calculated using the local maximum and minimum velocity near the bounce time). We can then **vary the droplet radius and velocity** and see how it affects these quantities. This has been plotted below.



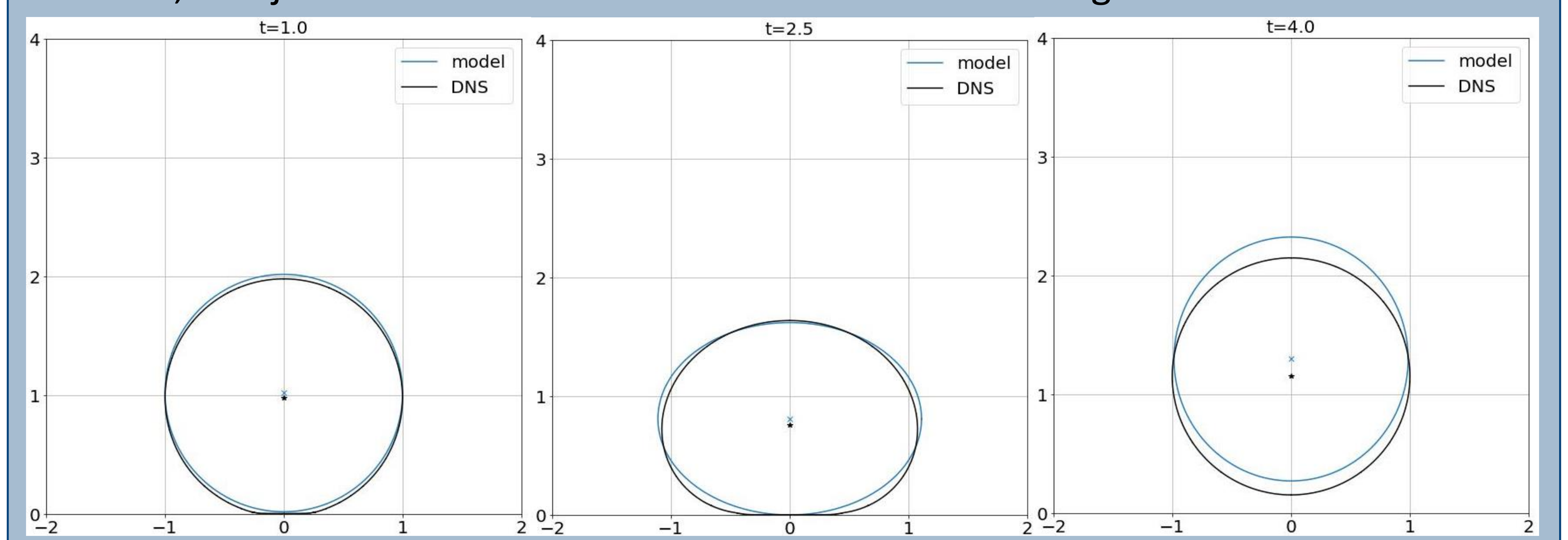


In the **first** plot, we see how the initial radius, in metres, and velocity, in metres per second, affect the **contact time**. Since the time scale depends on the initial radius, we plot dimensional time on the vertical axis. We see that the **initial velocity has very little effect on the contact time**, which is expected as the contact time is expected to scale as the time scale, $T = \sqrt{\rho_d R^3/\sigma}$, as seen in [4] and [5]. We also **plot $2T$** as this varies for the radius. We have **good agreement** with this. We also observe that the **contact time increases with the initial radius**, as expected from the scaling and it captures the $\frac{3}{2}$ power. Interestingly, we see that for $R = 0.3mm$ and $U = 0.01m/s$, we get a much larger contact time, which suggests the model had a problem for this case either due to this being a boundary case or that a numerical error occurred due to the ODE solver used.

In the **second** plot, we observe how the initial radius and velocity affect the **coefficient of restitution** (ratio of outgoing and incoming velocities). We observe that **larger drops conserve more energy** in the bounce and that the droplet **initial velocity has only a small impact** compared to the radius. This is not unexpected as larger drops would deform more so we would have a larger restoring force, resulting in larger outgoing velocities as the energy is transferred from spring to kinetic energy.

Interface Plots

Using the **model as well as direct numerical simulations** (DNS, which solves the full Navier-Stokes equations), with data provided by Dr. Cimpeanu, we can plot the interface shape of the droplet for **10 units of time**. Below, we have **plotted the interface for each model at three different times** (1.0, 2.5, 4.0), **using air as the surrounding fluid and an oil droplet** having density $917.2\,kg/m^3$, viscosity $0.084\,Pa\,s$, initial radius $0.5\,mm$ and initial velocity $0.25\,m/s$, and its centre starting $1mm$ above the surface. The three plots show initial contact, the middle of contact, and just after contact with nondimensionalised lengths.



In the **first** plot, we see **good agreement** between the two models for both the interface and the centre of mass, with the simulation being slightly lower. This is most likely due to slight **discrepancies in the drag terms** of the two models.

In the **second** plot, we see that they again take similar shapes, with the **centre of mass again being slightly lower for the simulation**. This is in part due to the **assumption of spheroidal (a spheroid is a volume of revolution of an ellipse) deformation of the droplet in the model**, which is not the case for the DNS. Thus, the droplet becomes bottom heavy for the simulation, and in turn has a lower centre of mass. We see **both drops extend horizontally to similar widths** as well.

In the **third** plot, we see that the model is again above the simulation, which suggests this model conserves more energy, as the drag term is very small and in fact negligible in size at most times. However, the shapes of the two **interfaces are very similar**, which shows the model behaves correctly. In fact, during the rebound, **we observe the drop extending vertically in both**, which shows that the model behaves as we would hope.

Conclusions

Overall, we see that **the model captures the drop motion and deformation well**, whilst being **relatively simple**, where differences to the DNS can be attributed to the approximations made. This is beneficial as the entire Python code used runs in less than a minute, whereas the simulations can take days, or even weeks for more complex problems. This allows for **insights to be made quickly and then they can be used to inform parameter choices for the simulations**, which would save valuable computational time and resources.

References

[1] Rayleigh, Lord 1879. On the capillary phenomena of jets. Proc. R. Soc. Lond. 29 (196–199), 71–97.
[2] Jayaratne, O.W. & Mason, B.J. 1964. The coalescence and bouncing of water drops at an air/water interface. Proc. R. Soc. Lond. A 280 (1383), 545–565.
[3] Couder, Y., Fort, E., Gautier, C.-H. & Boudaoud, A. 2005a. From bouncing to floating: noncoalescence of drops on a fluid bath. Phys. Rev. Lett. 94 (17), 177801.
[4] Blanchette, F. 2016. Modeling the vertical motion of drops bouncing on a bounded fluid reservoir. Phys. Fluids 28 (3), 032104.
[5] Okumura, K., Chevy, F., Richard, D., Quéré, D. & Clanet, C. 2003. Water spring: a model for bouncing drops. Europhys. Lett. 62 (2), 237.

Code and Video

https://github.com/WilliamGillow/oil_droplet_bounce