Unravelling wrinkles in ice shelves

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Introduction

- Arctic shelves can display unusual wave like surface rolls, first observed in 1876 (see Fig 1)
- Origins remain uncertain but one potential mechanism treats the ice shelf like a thin beam of viscous fluid which buckles under sea ice pressure.
- Here we investigate this with different internal ice structures extending the work done in [N.B22]

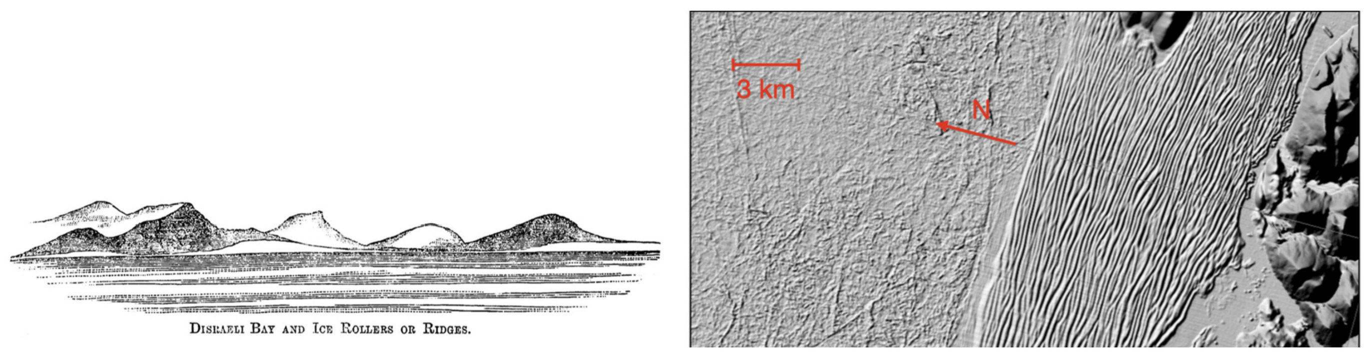


Fig. 1: Sketch of Ice Rolls [GS77] and Ellesmere Ice Shelf Remnant [Co18]

Governing Equations

For our model we consider a 2-dimensional cross section of the ice shelf and assume that all properties parallel to the direction of the coast remain uniform. We also assume the pressure is applied to both sides of the sheet for simplicity.

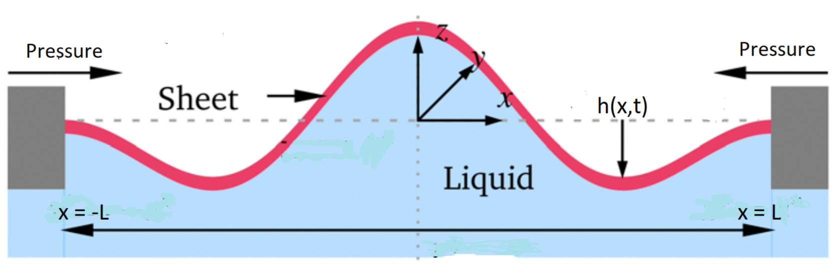


Fig. 2: Ice shelf compressed under sea ice pressure, modified from [M23]

- Start from Navier-Stokes equations who describe the conservation of mass and momentum
- We then discard terms who account for inertia and rescale under the assumption that the shelf is much longer than it is thick
- The resulting equations are considerably simplified but still complex enough to require the help of a computer to solve and we must still decide what rheology (way in which deformation occurs) our ice should have.

Viscous Case

To simplify the proceedings we could assume that ice has a Newtonian rheology, that the stress and strain are linearly related.

$$\tau = \mu\dot{\gamma} \tag{1}$$

With τ: shear stress $\dot{\gamma}$: strain rate μ: viscosity constant

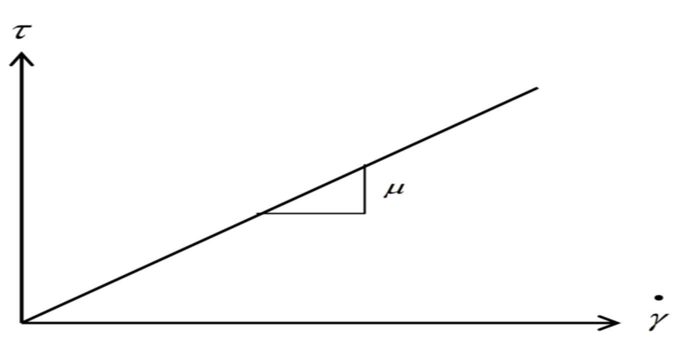


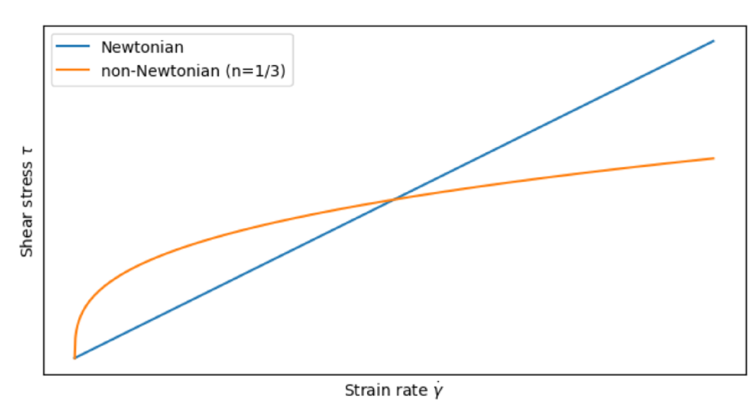
Fig. 3: Graph of Newtonian Viscosity ([Has17]

This has the advantage of making our equations far easier to solve but we may lose accuracy as this simplification might not account for behaviour we see in ice deformation in practice.

Power Law

Research suggests that ice may indeed have a non-Newtonian rheology, that its stress and strain are related by a power law (typically $n \approx 1/3$) [KW10]

$$\tau = \mu(\dot{\gamma})\dot{\gamma}^n \tag{2}$$



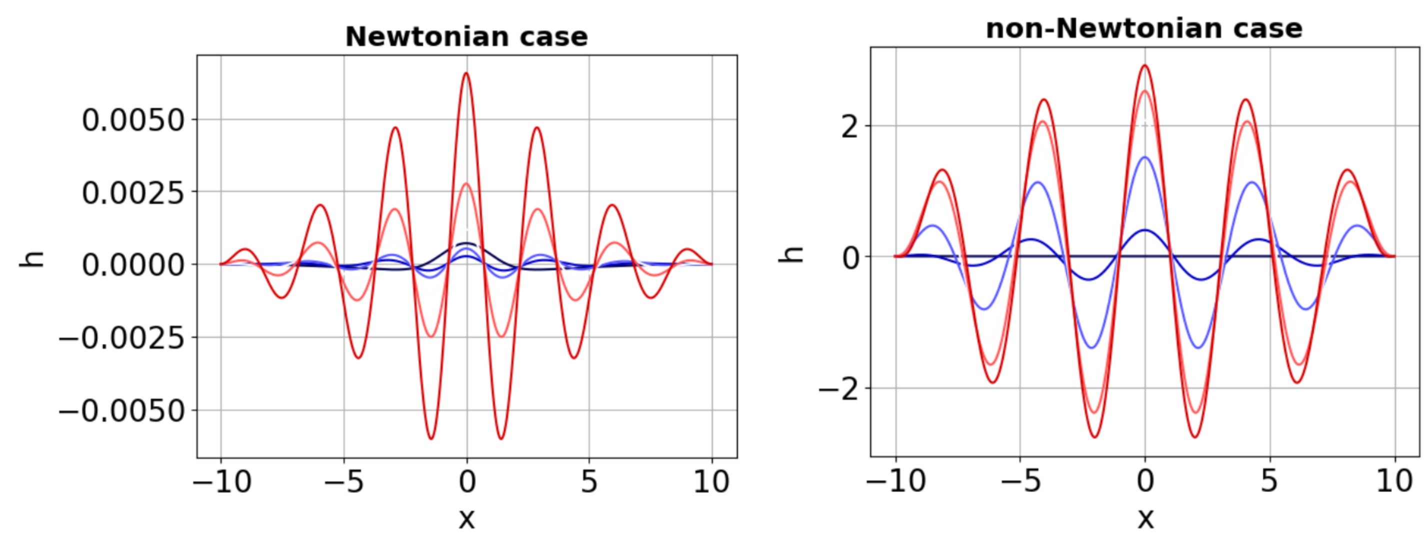
This graph demonstrates the relationship between shear stress and strain rate, fluids that exhibit this behaviour are also known as pseudoplastic. (Here K is a constant value)

Asymptotic Expressions

- For non-Newtonian case the equation is still hard to solve computationally
- We use asymptotic expressions which are like approximations when some variables are very large or small
- We take two cases: when sea ice pressure is very high and very low (near zero)

With Sea Ice Pressure

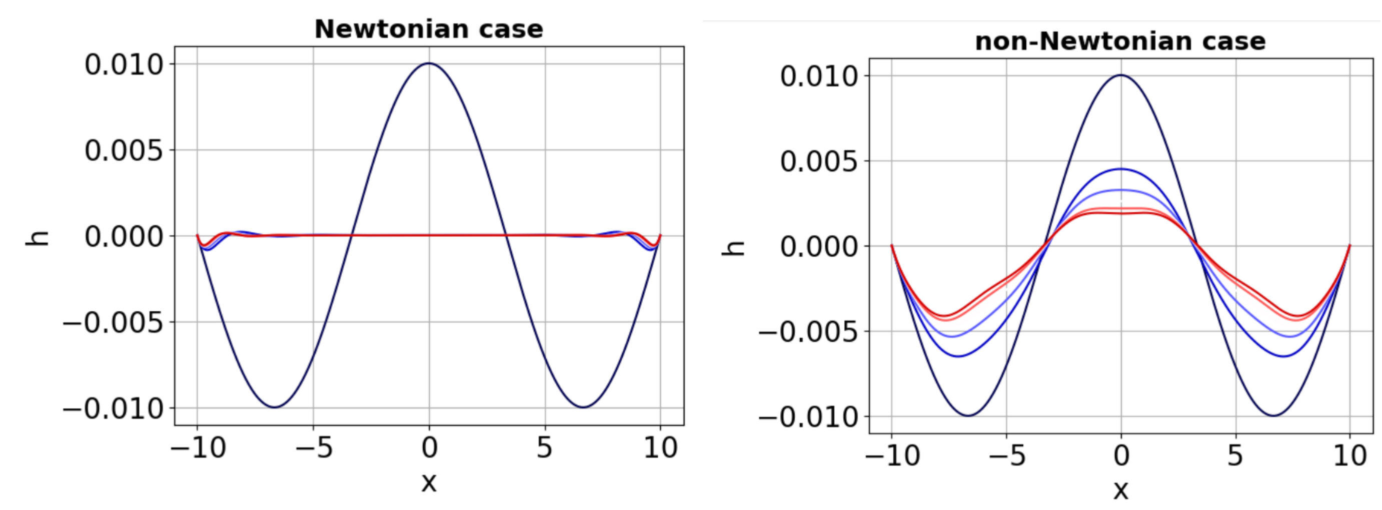
When sea ice pressure is present, corresponding temporally to the initial phase of the buckle formation when sea ice presses against the thicker ice shelf, we solve for the Newtonian case and for the non-Newtonian case using our first asymptotic expression.
The simulation is ran for 40 time units, time going from blue to red.



Here we see the non-Newtonian case causes the buckle to grow much faster than the Newtonian case. This is because the relevant asymptotic expression is similar to the Newtonian case but with a reduced viscosity

Without Sea Ice Pressure

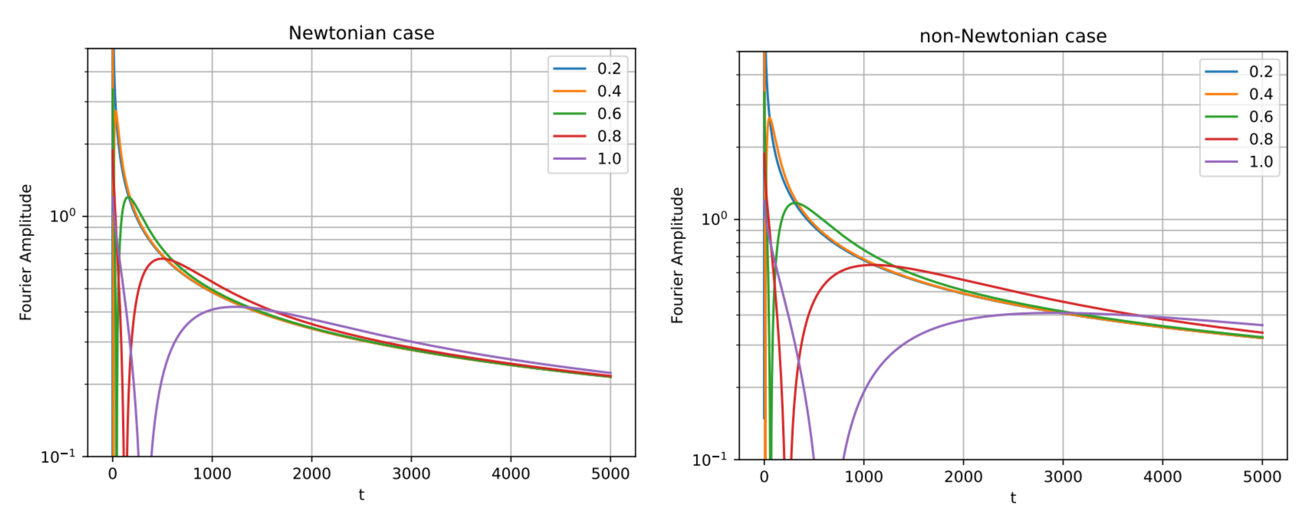
When sea ice pressure is removed, corresponding to later temporally after buckles have already formed, we solve for the Newtonian case with pressure terms removed and use our second asymptotic expression for the non-Newtonian case.
The simulation is ran for 40 time units, time going from blue to red.



We can see that for the Newtonian case the buckles smooth quickly but the Power Law case sees far slower relaxation in shape and size.

Decay Analysis

These graphs plot a variety of different wave numbers (something similar to frequency but spatial instead of temporal, inversely proportional to wavelength) and show how their amplitudes decay for 5000 time units, the graph also has a logarithmic y-axis (n=0.8).



We see that as wave number increases the amplitude decays slower for early time, theory suggests this evolves like $3k^{-4}$. For the non-Newtonian case the decay occurs slower overall, aligning with the buckle decay plot.

Conclusions

- Viscous buckling is a plausible mechanism for the formation of surface rolls.
- A non-Newtonian rheology causes roll decay to occur slower potentially explaining their persistence after potentially several extended periods without sea ice pushing against the ice shelves.
- Higher wavelength waves decay earlier under both rheologies

Acknowledgements

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