STATISTICS OF GALAXY CLUSTERS IN LARGE N-BODY SIMULATIONS

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Introduction

Have you ever wondered what our home really looks like? The **structure of our Universe** can be studied using N-body simulation data generated by huge supercomputers. We studied how well this simulation data matches with theoretical models. **Observational data** is very limited because of the time it takes for light from distant objects to reach us. The scientific community consider N-body **simulation data** to be accurate to test models [1].

Theoretical predictions of cluster abdunances

The **Press-Schechter formalism** is a mathematical model proposed in 1974 that can predict the number of galaxy clusters of a certain mass within a given volume of the Universe [3]. The **Sheth-Tormen approximation** builds on the Press-Schehchter model. The **differential number density** is given by

$$\frac{dn}{dM} = F(v)\frac{\rho_m}{M}\frac{d\ln(\sigma^{-1})}{dM}$$

where σ^2 is the **variance of linear density fluctuations**, $\nu = 1.686/\sigma$, M is the mass of the cluster, and the rest are relevant constants. $F(\nu)$ is then one of the following mass functions:

Sheth-Tormen $$F_{ST} = 0.322\sqrt{\frac{2a}{\pi}}\nu\exp(-\frac{a\nu^2}{2})[1+(a\nu^2)^{-0.3}]$$

Press-Schechter $$F_{PS} = \sqrt{\frac{2}{\pi}}\nu e^{\frac{-\nu^2}{2}}$$

N-body simulations

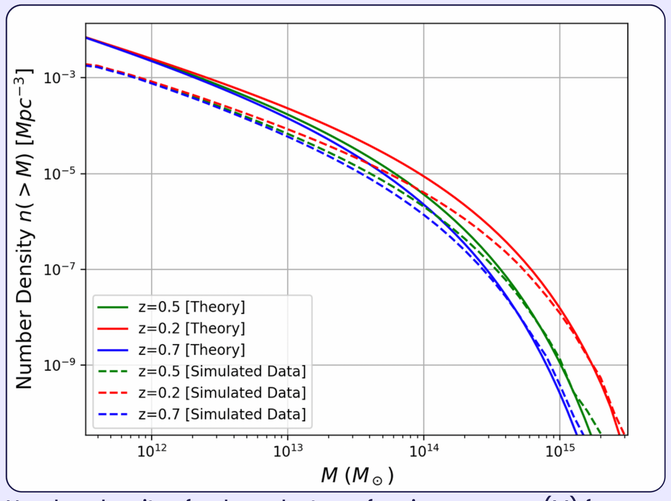
We analysed data from the **MultiDark-Planck (MDPL) simulation** project [2]. SQL was used to query relevant data from the relational database. The **parameters** for the theoretical model were set to match with what the supercomputer used to generate the data.

Number of galaxy clusters of each mass query

```sql
SELECT
    CONCAT(CAST(FLOOR(LOG(halomass)*100)/100
AS DECIMAL(10,2)))
    AS mass_category,
    COUNT(*) AS galaxy_count
FROM
    mdpl2.sag
WHERE
    snapnum = 52
GROUP BY
    mass_category;
```

An SQL query counting galaxy clusters of different masses

Theory v. Simulation

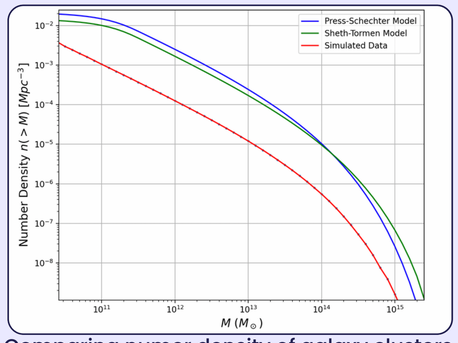


Number density of galaxy clusters of various masses (M) for different redshift values (z) predicted by Press-Schechter

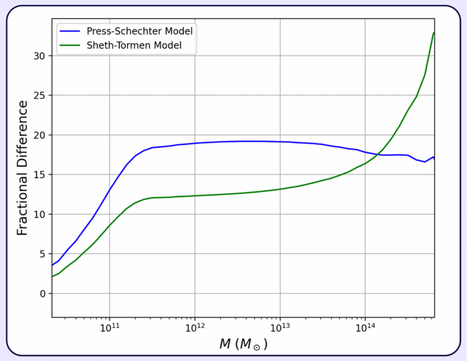
The graph above demonstrates the **convergence** of the simulated data and theoretical model for larger masses.

However we notice that when comparing mass functions at **redshift z=0.1**, the prediction is off by a factor of 10.

This plot illustrates that whilst the Press-Schechter model has a **systematic error** for number desnity, the Sheth-Tormen's error increases.



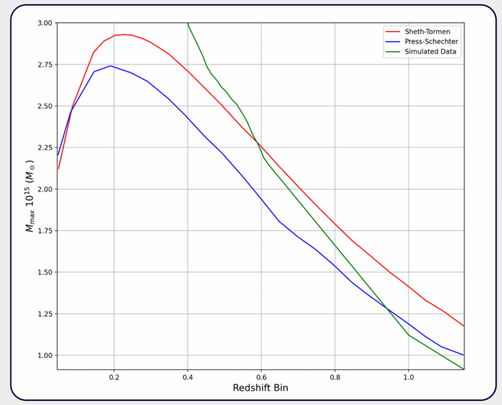
Comparing numer density of galaxy clusters of various masses for different models (z=0.1)



The fractional error between the theoretical models and data for various masses (M)

The LARGEST galaxy cluster

Where in the cosmic timeline do you expect the **largest galaxy cluster** to be? It may be surprising that theory actually predicts it at a **redshift between 0.18 and 2.** This is also backed up by data from the **Hubble telescope.**

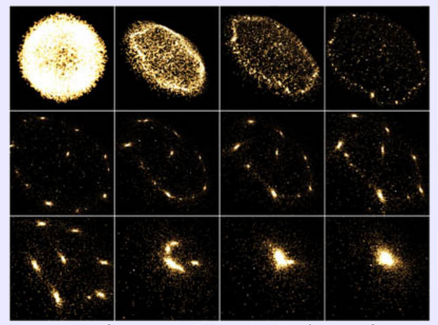


The heaviest galaxy clusters at each redshift

Conclusion

The simulation data matches up strongly with theoretical models with relatively low error.

However there are certain instances where the two don't match up. Particularly for **low redshift regions** which corresponds to the latter parts of the simulation.



Frames from a 3D rendering of 16,384 body simulation (NVIDIA)

This opens up an area for further investigation in fine tuning the **cosmic expansion** factor of N-body simulations .

References & Acknowledgements

[1] Boekholt, T., Portegies Zwart, S. On the reliability of N-body simulations. Comput. Astrophys. 2, 2 (2015).
[2] MultiDark simulations: the story of dark matter halo concentrations and density profiles; Klypin, Yepes, Gottlöber, Prada, Heß, (2016)
[3] Primordial Non-Gaussianity and Extreme-Value Statistics of Galaxy Clusters; Sirichai Chongchitnan, Joseph Silk (2011)

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