Mathematics of Crowd Management

A URSS project by Hector Palmer, supervised by Dr Susana Gomes

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Introduction

Collective dynamics is a vast and varied area of mathematics with a rife range of applications to real-world problems, from bird flocks to traffic flow models and crowd management. My project, to begin with, focused on creating a model of crowd dynamics, based on the Helbing Social Force theory, using the specialised agent-based modelling software Netlogo. With a working model in place, I was able to explore a variety of different scenarios, such as movement in corridors, evacuation scenarios, and, ultimately, how group behaviour within crowds affects evacuation speeds.

The Model

For the model, I followed the Helbing Social Force theory, which posits that people, like particles, are subject to external "social" forces, which govern their motion in crowd scenarios. These forces consist of attractive forces, such as attraction to the desired destination or to other members of the group, and repulsive forces, like repulsion from walls, obstacles and strangers, provided these objects are in the person's field of vision.

$$\vec{F}_\alpha(t) := \vec{F}^0_\alpha(\vec{v}_\alpha, v^0_\alpha \vec{e}_\alpha) + \sum_\beta \vec{F}_{\alpha\beta}(\vec{e}_\alpha, \vec{r}_\alpha - \vec{r}_\beta) + \sum_B \vec{F}_{\alpha B}(\vec{e}_\alpha, \vec{r}_\alpha - \vec{r}^\alpha_B) + \sum_i \vec{F}_{\alpha i}(\vec{e}_\alpha, \vec{r}_\alpha - \vec{r}_i, t)$$

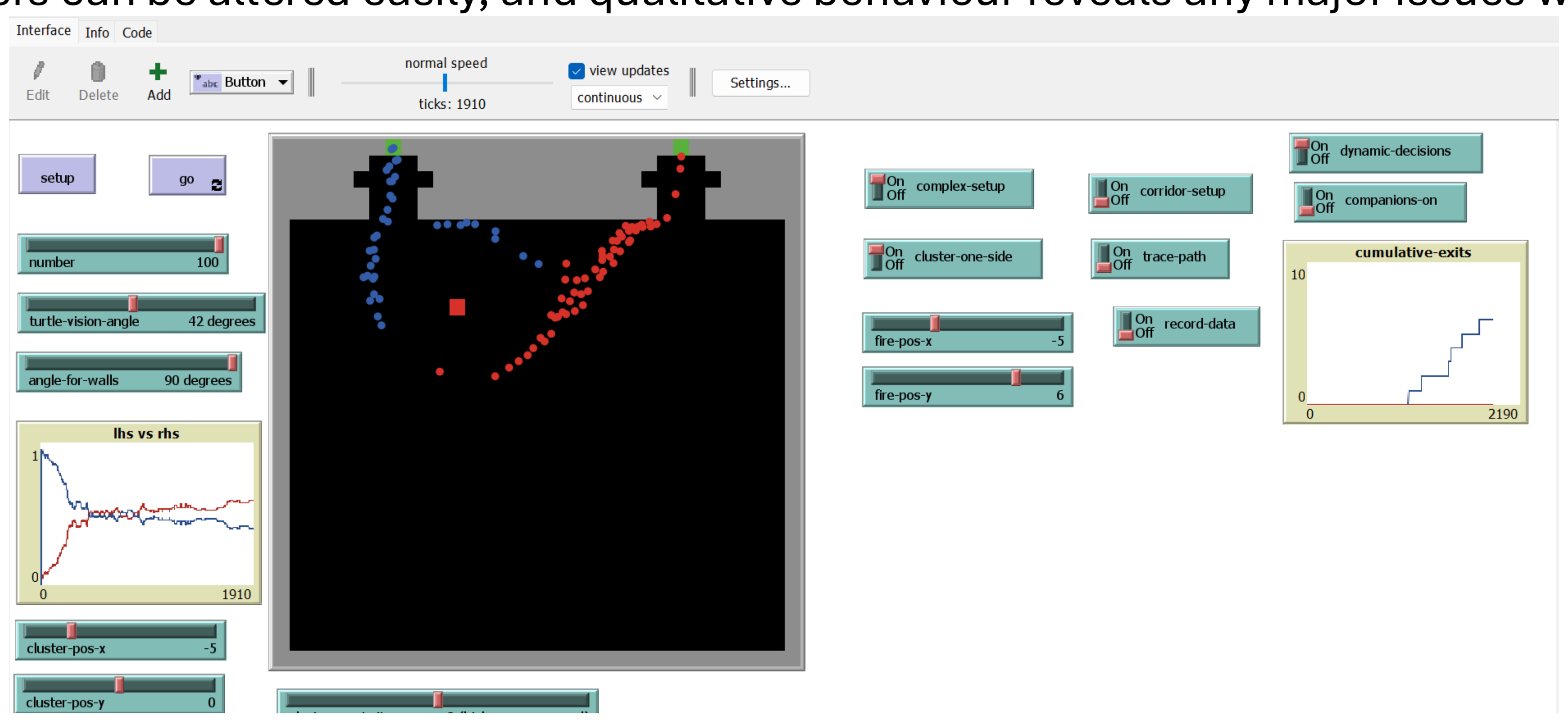
$$\frac{d\vec{w}_\alpha}{dt} := \vec{F}_\alpha(t) + \eta(t)$$

Left: equation for resultant force for agent α at time t, from Helbing, D. and Molnar, P., 1995. Social force model for pedestrian dynamics. *Physical review E*, *51*(5), p.4282; **right:** acceleration of agent α at time t, with normally distributed random fluctuations $\eta(t)$

In my interpretation of the model, repulsive forces were modelled as exponential functions of the distance between two people or a person and a wall, which decrease with increasing distance. Attractive forces were simply modelled as linear functions of distance; when group behaviour was incorporated into the model, attractive forces towards other members of the group only came into play beyond a certain threshold distance. To implement the model in code, these equations were discretised in time; as such, at each time step, the attractive and repulsive forces were calculated, then used to update each person's acceleration, which was in turn used to update velocity and position accordingly.

The Software

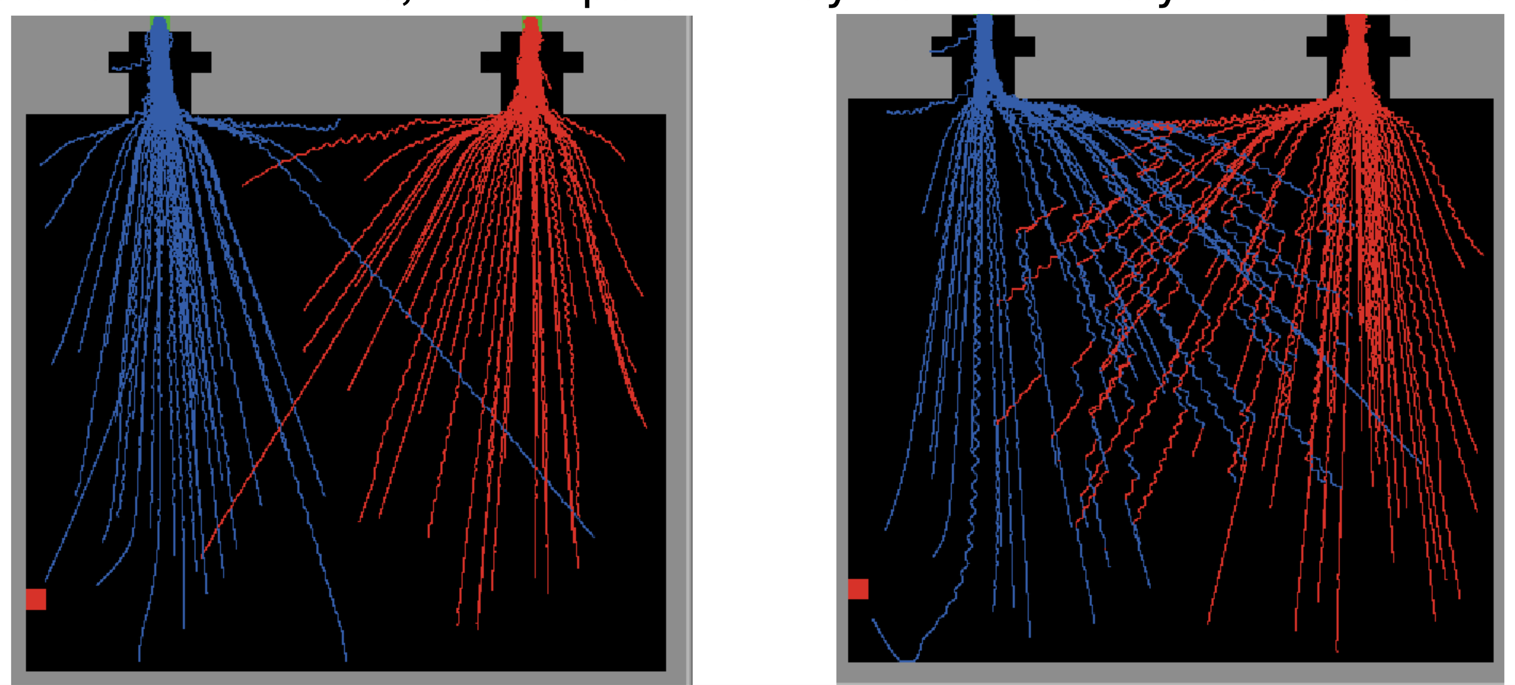
Throughout the project, I used the specialised agent-based modelling software Netlogo to run simulations of the model. Employed widely across biology, the social sciences and environmental studies, Netlogo allows users to watch simulations and observe how the system evolves over time. With this interactive interface, model parameters can be altered easily, and qualitative behaviour reveals any major issues with the model.



Above: screenshot of the user interface mid-simulation, including buttons, switches, sliders for parameters, plots and a visualisation of the model.

Decision-Making in Evacuations

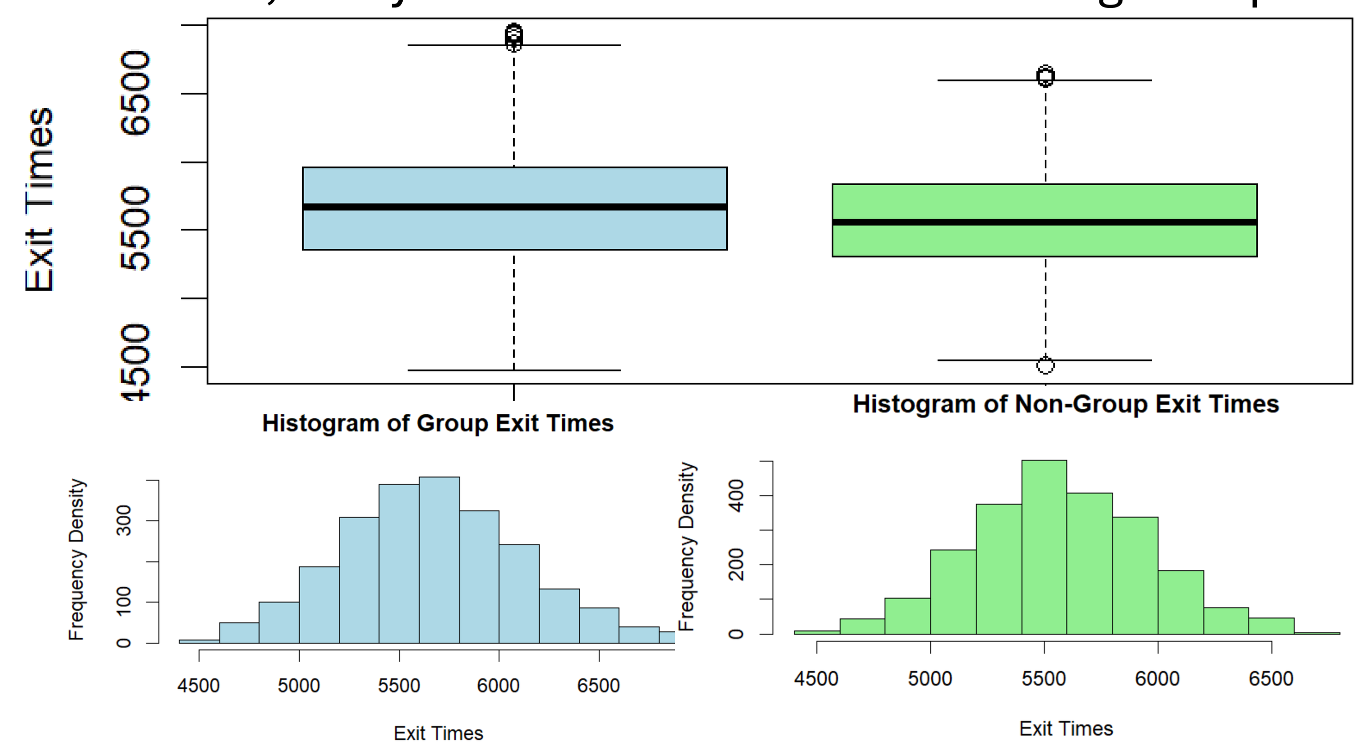
With my simulations displaying reasonable qualitative behaviour, I looked to apply my model to evacuation scenarios, particularly in the case of fire, which is shown as a single red patch in the screenshots below. To explore these scenarios, I had to consider the factors that people might take into account when making split-second decisions in emergencies, and used these to attribute a "score" to each exit; these factors included proximity to each exit, crowd density near each exit, and proximity of each exit to the fire. People were able to change their minds during the evacuation based on the changing situation; for instance, if one exit appeared less crowded at a certain time, then a person may be more likely to choose that exit.



Left: screenshot showing the path taken by each person when they stick with their initial decision; **Right:** screen showing the path taken by each person when they can change their mind at each time step

Experiment: Do Groups Slow Down Evacuations?

Having established a decision-making algorithm, I chose to investigate the effect of group behaviour on evacuation times. To do this, I ran 2500 simulations where all people behaved as individuals, and 2500 simulations where some people were part of small groups who aimed to stick together throughout the evacuation; every 100 simulations I would change the position of the fire. In terms of the number of people opting for each exit, the results were very similar in the group vs non-group experiments, however, when comparing the exit times of the group vs non-group experiments, the group evacuations were on average 2.6% slower than the non-group. This result reflects the general consensus that groups do indeed slow down evacuations, however my model did not incorporate other benefits of group behaviours, for instance more efficient sharing of information or a reduced sense of panic due to "safety in numbers".



Extensions to the Model and Reflections

Owing to the highly complex and varied crowd scenarios which arise on a daily basis, there are many ways in which the model could be extended, from considering a wider range of factors in the decision-making process to exploring more realistic settings in which evacuations can occur. My evacuation simulations took place in a large, square room with only two exits; with Netlogo, a system of grey walls could be added to the environment to replicate the ground floor of a shopping centre, for example. Furthermore, my model assumed that to every person could see both exits from the start. In many scenarios, this isn't the case; take an underground station, for example, where people are guided to the exit by signs before they reach the exit itself, which would of course change the decision-making process. These are just two ways in which the model could be extended, but a working social force model has laid a solid foundation for further exploration. In conclusion, I have thoroughly enjoyed working on this project; while at times tweaking the model and parameters has been somewhat frustrating, I am very satisfied to have created a model that "works", and to have gained insight into a field of study which actively keeps us safe in our day-to-day lives. I would like to extend my thanks to Dr Susana Gomes, who has provided constant and invaluable support throughout the project.