

# 1 Navier Stokes

$$\rho_t \mathbf{u} + \nabla \cdot (\rho(\mathbf{u} \otimes \mathbf{v}) - \mathbb{T}(\mathbf{u}, \nabla \mathbf{u})) = \mathbf{s}(\mathbf{u}) \quad \text{in } \Omega = [0, L_x] \times [0, L_y] \quad (1)$$

where  $\Omega \in \mathbb{R}^2$ ,  $\mathbf{u} = (u, v)$ ,  $\rho(\mathbf{u}, \mathbf{v})$  is the unknown vector,  $\rho$  is density,  $\rho \mathbf{v}$  momentum, and  $\rho e$  total energy. The velocity is  $\mathbf{v} = (v_x, v_y)$ ,  $p$  pressure, and  $T$  is temperature. The inviscid and diffusive fluxes are:

$$\mathbf{F}(\mathbf{u}) = \begin{bmatrix} \rho \mathbf{u} \\ \rho \mathbf{u} \otimes \mathbf{v} \\ \rho(e + \frac{1}{2} \mathbf{v} \cdot \mathbf{v}) \end{bmatrix}, \quad \mathbb{T}(\mathbf{u}, \nabla \mathbf{u}) = \begin{bmatrix} 0 \\ \tau_{11} & \tau_{12} \\ \tau_{21} & \tau_{22} \\ \tau_{13} - \lambda \nabla \cdot \mathbf{u}, \tau_{23} + \lambda \nabla \cdot \mathbf{u} \end{bmatrix}$$

with  $\tau_{11} = \mu(v_x + v_x)$  and  $\tau_{22} = \mu(v_y + v_y)$ . The discontinuity tensor  $\tau$  is:

$$\tau = \begin{bmatrix} (2\mu + \lambda)\nabla \cdot \mathbf{u} - \lambda \nabla_x u & \mu \partial_x v - \lambda \nabla_x v \\ \mu \partial_y u - \lambda \nabla_y u & (2\mu + \lambda)\nabla \cdot \mathbf{u} \end{bmatrix} \quad (2)$$

$\lambda$  is thermal conductivity coefficient, and the term  $\lambda \nabla \cdot \mathbf{u}$  represents the heat flux.



<http://dune-project.org/>

Model

Method

## 1 Spatial Discretization

$$\mathcal{R}(\mathbf{u}_K, \varphi) = \int_K (f(\mathbf{u}_K) \cdot \nabla \varphi + \mathbf{s}_K(\mathbf{u}_K) \varphi) + \int_{\partial K} \tilde{\mathbf{F}}(\mathbf{u}) \cdot \mathbf{n}_K \varphi + \int_K (A(\mathbf{u}_K) \cdot \nabla \mathbf{u}_K \cdot \nabla \varphi - \mathbf{s}_K(\mathbf{u}_K) \varphi) - \int_{\partial K} \tilde{A}(\mathbf{u}_K) \cdot \mathbf{n}_K \varphi + \int_{\partial K \cap \Gamma} \left( \frac{1}{\eta} + \beta_1 \beta_2 \cdot \mathbf{n}_K \right) A(\mathbf{u}_K) \cdot \mathbf{u}_K \cdot \nabla \varphi + \int_{\partial K \cap \Gamma} (\mathbf{u}_K \cdot \tilde{\mathbf{v}}) A(\mathbf{u}_K) \cdot \nabla \varphi + \int_{\partial K \cap \Gamma} (\mathbf{u}_K \cdot \tilde{\mathbf{v}}) A(\mathbf{u}_K) \varphi + \theta_1(\mathbf{u}_K, \varphi) + \theta_2(\mathbf{u}_K, \varphi) \quad (1)$$

$$\tilde{A}(\mathbf{u}) = \begin{cases} \frac{1}{\eta} \mathbf{v} \cdot \mathbf{u} + \lambda_1 (A(\mathbf{u}) \cdot \mathbf{v}) \cdot \mathbf{v} & \text{HR2} \\ (A(\mathbf{u}) \cdot \nabla \mathbf{u} - \mathbf{s}_K(A(\mathbf{u})) \cdot \mathbf{u}) \cdot \mathbf{v} & \text{CDG1} \\ \frac{1}{\eta} A(\mathbf{u}) \cdot \nabla \mathbf{u} + 2\lambda_1 (A(\mathbf{u}) \cdot \mathbf{v}) \cdot \mathbf{v} & \text{CDG2} \end{cases} \quad (2)$$

## 2 Temporal Discretization

# Warwick/NAIS Dune School

20-24 June 2011

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:30	registration	Introduction to Dune	Introduction to Dune-Fem	Finite Element Methods	Discontinuous Galerkin Methods
10:30-11:00		coffee	coffee	coffee	coffee
11:00-12:30	Getting started	Introduction to Dune	Introduction to Dune-Fem	Finite Element Methods	Discontinuous Galerkin Methods
12:30-14:00	lunch in street	lunch in street	lunch in street	lunch break	lunch in street
14:00-15:30	Generic programming in C++	Parallelization in Dune	Adaptive finite volume methods	Finite Elements on Surfaces	Further aspects of Dune
15:30-16:00	tea	tea	tea	tea	
16:00-18:00	Generic programming in C++	Parallelization in Dune	Adaptive finite volume methods	Finite Elements on Surfaces	
evening	drinks		dinner		

Registration: Mathematics Research Centre  
 Courses: **A0.02** Zeeman Building

